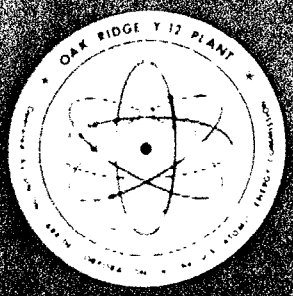
Y-CDC-13

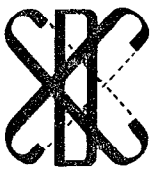


UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT
operated for the
U.S. ATOMIC ENERGY COMMISSION

THE CRITICALITY OF CUBIC ARRAYS OF FISSILE MATERIAL

J. T. Thomas
Oak Ridge Y-12 Plant

CRITICALITY DATA CENTER



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UC-46
Criticality Studies

Y-CDC-10

THE CRITICALITY OF CUBIC ARRAYS OF FISSILE MATERIAL

J. T. Thomas
Union Carbide Corporation
Nuclear Division
Oak Ridge Y-12 Plant

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operated for the U.S. ATOMIC ENERGY COMMISSION
by UNION CARBIDE CORPORATION-NUCLEAR DIVISION
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FOREWORD

The Criticality Data Center was established under the auspices of the U.S. Atomic Energy Commission for the development of methods allowing extension and application of data derived from experiments and from analyses to problems in nuclear criticality safety, as well as for the review and evaluation of the data themselves. A necessary part of this program is a medium whereby information germane to the intent of the Center is made available. This report series has been inaugurated for that purpose.

The first five reports were published by and identified with the Oak Ridge National Laboratory. Subsequent reports, however, issued from the Y-12 Plant, are identified by a number sequence including the prefix Y-CDC.

Inquiries should be directed to E. B. Johnson, P.O. Box Y, Oak Ridge, Tennessee 37830.

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THE CRITICALITY OF CUBIC ARRAYS OF FISSILE MATERIAL

J. T. Thomas

ABSTRACT

Calculated criticality data of water-reflected arrays containing subcritical components of ^{235}U, ^{239}Pu, and ^{233}U are presented in tabular form. The calculations were performed with the KENO Monte Carlo code and used the Hansen-Roach 16-group neutron cross-section sets. The response of the array neutron multiplication factor to changes in the mass of the units provides a systematic measure of subcriticality for nuclear criticality safety specifications. The effect of concrete replacing water as a reflector was investigated as well as the neutron interaction between arrays when separated and reflected by concrete. A number of practical applications to criticality safety problems is suggested and guidance is provided for the storage of fissile materials.

I. INTRODUCTION

The problem of the characterization of the criticality of subcritical components of a particular fissile material arranged in arrays is basic to nuclear criticality safety practices. Once the criticality properties of the systems are known, reliable guidance can be had for common activities encountered in process cycles such as handling, storage and transport. In practice, the rule and not the exception appears to be satisfaction with the assurance of arbitrary subcriticality for specific problems as they arise. Usually, the effort and expense to complete the description of criticality for a material is avoided for economic reasons. While this is expedient, it is shortsighted and delays the development of credible, uniform nuclear criticality safety practices.

In an effort to begin the formulation of a uniform basis for criticality safety evaluations, Monte Carlo calculations of water-reflected arrays were performed for a variety, but limited number, of fissile materials. The fissile material is assumed to have spherical geometry and to be assembled into cubic arrays. It has been shown[1] that for sufficiently large arrays (64 units or more) changing the spherical mass into any other shape results in an array reactivity loss. Further, rearrangement of the cubic cells into other than a cubic array does not lead to an increase in array reactivity. The methods and calculational techniques utilized in this work, which are applicable to arrays of 64 or more units, have minimized the number of calculations necessary to characterize the criticality of fissile materials. Other fissile materials than those considered may be examined in a similar manner. The response of the neutron multiplication factor of arrays to changes in the mass of the unit is explored and furnishes a consistent method for evaluating and compensating for conditions that may be expected to augment the array reactivity. For example, the substitution of concrete in place of water as a reflector may increase the reactivity of an

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1. J. T. Thomas, "Uranium Metal Criticality, Monte Carlo Calculations and Nuclear Criticality Safety," Y-CDC-7, UCC, Oak Ridge Y-12 Plant (1970).

array by as much as 13%. This may be compensated by a uniform mass reduction of about 40% and the system will be returned to its initial multiplication factor. Consideration is also given to neutron coupling that takes place between arrays separated and reflected by concrete. The influence of concrete thickness in such assemblies is investigated. Under conditions providing maximum-reactivity coupling between two arrays, the effect on the system reactivity is explored as a function of the separation of the arrays.

The KENO Monte Carlo code[2] and the Hansen-Roach 16-group neutron cross-section sets[3] are used to define criticality. Validation of the code and cross-section sets, as well as some of the techniques developed and used, are presented in the Appendixes and by cited references. There are many materials considered in physical forms for which no clean, critical experiments have been performed. This is a deficiency not likely to be rectified in the foreseeable future, but should not be a deterrent to examining their criticality by calculation. Where the code and cross sections reproduce experimental results, greater reliance may be placed on their application to other calculated configurations of the same materials. The necessity to validate the code and cross sections for each material cannot be overemphasized, especially where nuclear criticality specifications are the end result. In the absence of such validating calculations, a suitable margin of subcriticality should be employed to preclude criticality in the application of the results.

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2. G. E. Whitesides and N. F. Cross, "KENO-A Multigroup Monte Carlo Criticality Program," CTC-5, Oak Ridge Computing Technology Center (1969).
3. Gordon E. Hansen and William H. Roach, "Six and Sixteen Group Cross Sections for Fast and Intermediate Critical Assemblies," LAMS-2543, Los Alamos Scientific Laboratory (1961).

II. CALCULATED CRITICALITY DATA

The usual approach to exploration of criticality for uniform arrays of fissile material when a given mass of fissile material as a unit in each cell is considered is to determine the number of units and their necessary spacing. Unfortunately, this procedure calls for a great number of calculations, even for a single physical form of fissile material, in order to characterize the critical parameters. One can achieve the same results, at least in a manner suitable for nuclear safety specifications, by considering the number of units and their spacing as fixed and finding the mass of the unit required to produce criticality. There is no loss in generality nor in the applicability of the results if the analysis is confined to examining cubic arrays of spherical units centered in cubic cells. The simplicity of cubic arrays permits the geometry and dimensions of each array to be completely described by specifying a single dimension and attaching a subscript to denote the number of cells along an edge of the array. The most convenient dimension is that of the half edge of the cubic cell, a_n. Thus, the center spacing of the spherical units is $2a_n$, the edge dimensions of an array are $n \cdot 2a_n$, and the total number of units in the array, N, is n^3.

It has been demonstrated elsewhere[4,5] that the NB_N^2 method provides suitable guidance to the calculation of cubic arrays having N greater than 27. Given two critical arrays of identical units of radius r_c which have been determined by validated calculations, say a_n and $a_{n'}$, the requirement of the method, namely that NB_N^2 be equal to $N'B_{N'}^2$, can be interpreted as requiring the same fraction of neutrons leaking from the two arrays. From the dimensions of an array one can express the geometric buckling as

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4. J. T. Thomas, "A Method for Estimating Critical Conditions of Large Arrays of Uranium," Proceedings of Nuclear Criticality Safety, December 13-15, 1966, SC-DC-67-1305, p. 189 (1967).
5. J. T. Thomas, "Remarks on Array Criticality Techniques," Proc. of the Livermore Array Symposium, Sept. 23-25, 1968, CONF-680909, p. 67 (1968).

$$B_N^2 = \frac{3\pi^2}{[2na_n + 2\lambda]^2} \quad , \tag{1}$$

where the terms are as defined above and λ is an extrapolation distance. By simple manipulation, this may be converted to

$$NB_N^2 = \frac{3\pi^2 n}{4 a_n^2} \left\{ 1 - \sqrt{\frac{4 \lambda^2 NB_N^2}{3 \pi^2 N}} \right\}^2 \quad . \tag{2}$$

Now the quantity $\lambda^2 NB_N^2$ is a constant, independent of N, and is evaluated in Appendix B. The above expression can be written, therefore, as a relationship between the spacings, a_n, required for criticality of different arrays ($N \geq 64$) for spherical units of radius r_c,

$$a_n = a_{n'} \frac{n'}{n} \left(\frac{\sqrt{N} - C}{\sqrt{N'} - C} \right) \quad . \tag{3}$$

The result of these relations is apparently that calculations need be done only for arrays having a fixed number of units, N, but for different cell dimensions, a_n, to determine the required radius of fissile unit, r_c, establishing criticality. An explicit example is the critical radii, r_c, of U(93.2) metal spheres in 64-unit, water-reflected arrays shown in Fig. 1 as a function of the half-cell dimension. The radii, r_c, have been normalized to r_o, the critical radius for unreflected U(93.2) metal at a density of 18.76 g U/cm^3. It is to be noted that the range of a_4 extends beyond that of practical interest but is included for completeness. Each of the points shown is the result of Monte Carlo calculation.

The data appearing in Fig. 1 may be substituted directly in Eq. (2) to obtain NB_N^2 as a function of a_n, a particularly useful procedure when it is desired to determine r_c for a given set of cell dimensions as a function of N. The data are repeated in Fig. 2 where the lines for various n are prescribed by Eq. (2). The radii of spheres from which the criticality of water-reflected arrays are readily specified

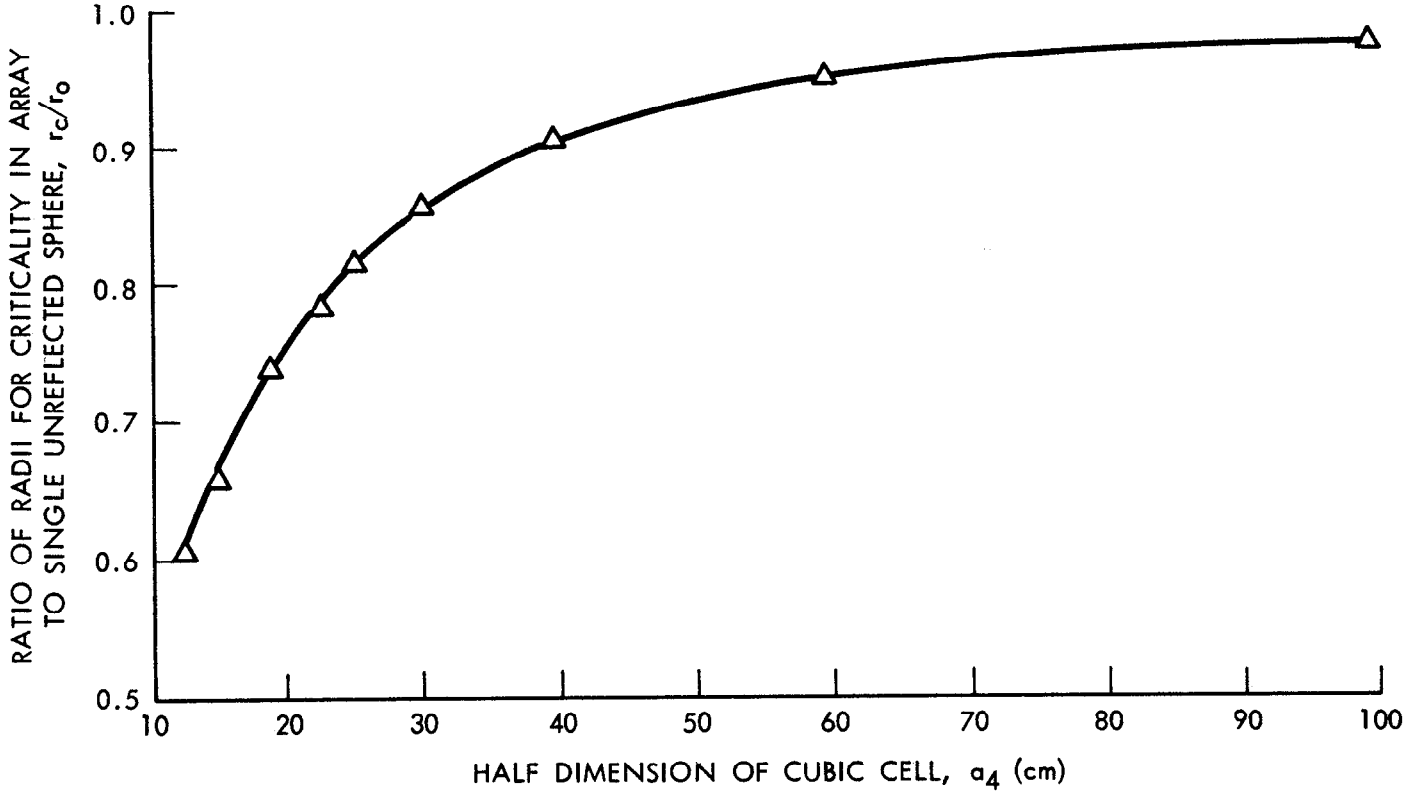

Fig. 1. Ratio of the Radius Required to Produce Criticality in a Reflected Cubic Array of 64 Units to That of an Unreflected U(93.2) Metal Sphere as a Function of the Half Cell Dimension in the Array.

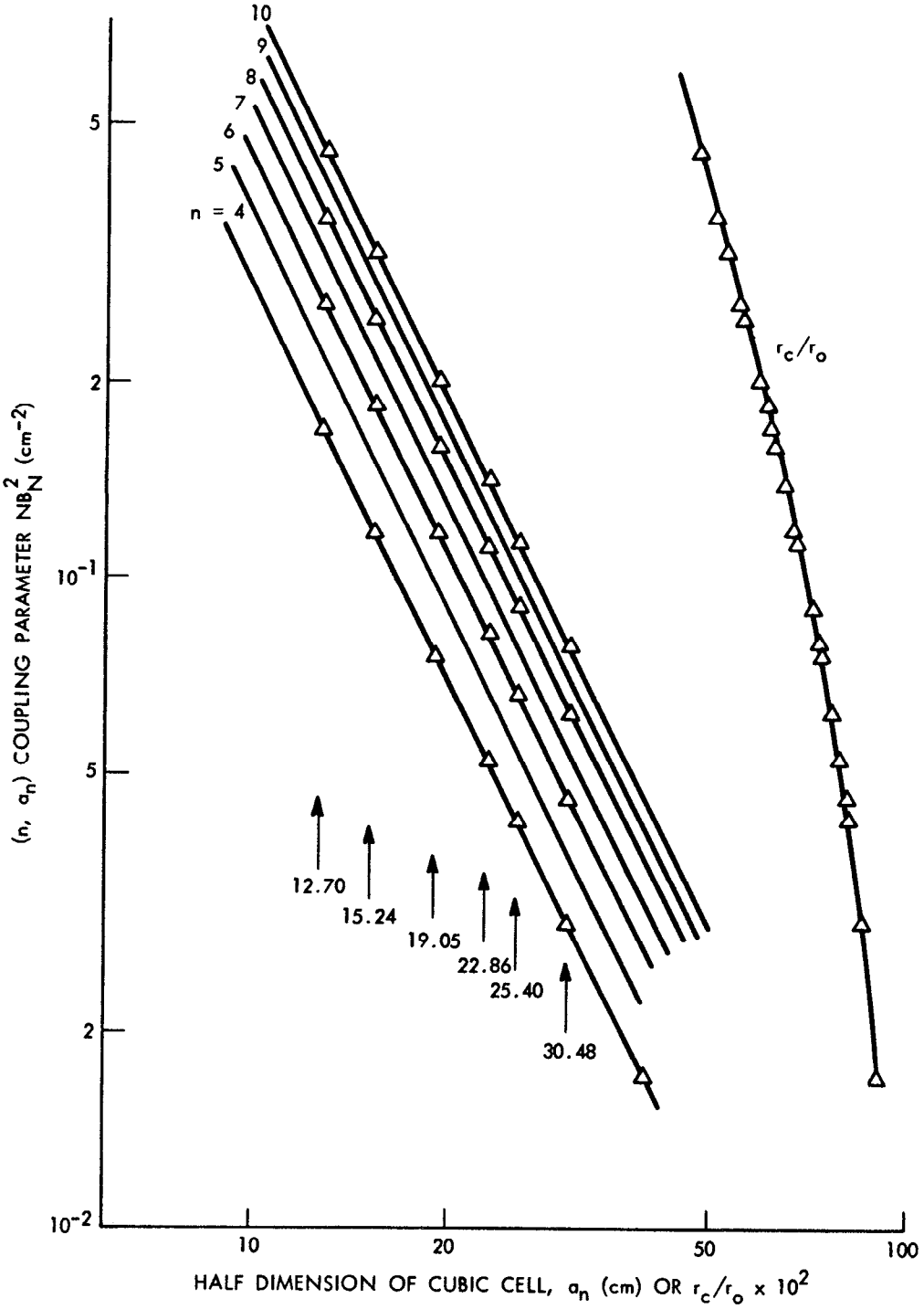

Fig. 2. The Number of Units Along the Edge of Water-Reflected Cubic Arrays and the Parameter NB_N^2 are Shown as a Function of the Ratio of the Radius Producing Criticality in the Array to the Radius of an Unreflected U(93.2) Metal Sphere and of the Half Cell Dimension.

are represented to the right of the figure. Noted on Fig. 2 are six cell sizes selected to display the criticality of U(93.2) metal in tabular form. Arrays ranging from 64 to 1000 units in integral steps of n are presented in Table 1 for each cell dimension. The radii shown were taken from Fig. 2 and the neutron multiplication factors given were determined by Monte Carlo calculations. The calculations evidence the adequacy of the procedure to characterize the criticality of fissile materials over a range of practical interest.

The criticality of various fissile materials was explored by the above technique. The materials selected were oxides, mixed with varying amounts of water, and metals. The results of the study of water-reflected arrays, presented in Table 2, summarize the critical radii in a format similar to Table 1.

A few remarks regarding the material composition chosen for this study should be made to assist in their interpretation. It will be observed that the fissile materials are described by their principal constituents. Other isotopes present in small quantities were considered to be one of the main constituents depending upon their neutron production or absorption properties. The hydrogen-to-fissile material ratio in the oxides was determined by assuming the volumes of oxide and water in the mixture were preserved. Although the calculations were performed for the dioxides, they may be used to define mass limits for other oxides. It is believed that the stated fissile material concentration and atomic ratios facilitate this broader application. The fissile material concentration and hydrogen content should not be exceeded in such applications.

In addition to those Monte Carlo calculations characterizing the criticality of the fissile materials, the criticality of each array in Table 2 containing an even-number of cells was also calculated by the KENO Monte Carlo code and the NB_N^2 results were verified to within one standard deviation except those entries of uranium oxide having less than 93.2 wt % ^{235}U. Each KENO calculation tracked 30×10^3 neutrons and resulted in a standard deviation of $\pm$ 0.005 at a multiplication factor of unity.

Table 1. Monte Carlo Calculations of Water-Reflected Arrays of U(93.2) Metal Spheres Defined from Fig. 2.

Number of Units in Cubic Array, N	Radius of Spherical Unit in the Given Cubic Cell Dimension and the Calculated Array Neutron Multiplication Factor					
	25.4 cm	k_{eff}	σ ($\times 10^3$)	30.48 cm	k_{eff}	σ ($\times 10^3$)
64	5.320	1.007	6	5.780	0.991	5
125	5.025	--		5.485	--	
216	4.760	0.995	5	5.265	1.009	5
343	4.536	--		4.984	--	
512	4.385	1.004	5	4.816	0.991	5
729	4.230	--		4.671	--	
1000	4.100	1.009	5	4.503	0.993	6
	38.1 cm	k_{eff}	σ ($\times 10^3$)	45.72 cm	k_{eff}	σ ($\times 10^3$)
64	6.480	1.006	5	6.880	1.006	5
125	6.100	--		6.590	--	
216	5.811	0.994	5	6.330	1.004	6
343	5.579	--		6.090	--	
512	5.435	0.991	6	5.883	0.993	5
729	5.229	--		5.778	--	
1000	5.096	0.993	5	5.637	1.009	5
	50.80 cm	k_{eff}	σ ($\times 10^3$)	60.96 cm	k_{eff}	σ ($\times 10^3$)
64	7.160	1.007	5	7.520	1.001	6
125	6.840	--		7.225	--	
216	6.620	1.004	5	7.058	1.004	4
343	6.384	--		6.804	--	
512	6.250	1.001	5	6.698	1.007	6
729	6.066	--		6.525	--	
1000	5.875	0.998	5	6.416	1.006	5

Table 2. Spherical Radius in Centimeters of Fissile Material Required for Criticality in Water-Reflected Cubic Arrays.

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
Uranium Metal; H/U = 0; 18.76 g U/cm^3; 100 wt % ^{235}U						
64	5.187	5.537	6.286	6.591	6.863	7.237
125	4.886	5.173	5.930	6.275	6.548	7.118
216	4.691	5.044	5.667	6.064	6.401	6.761
343	4.433	4.768	5.439	5.828	6.170	6.639
512	4.284	4.614	5.298	5.636	6.067	6.501
729	4.070	4.475	5.130	5.509	5.906	6.337
1000	4.014	4.388	5.007	5.400	5.745	6.198
Uranium Metal; H/U = 0; 18.76 g U/cm^3; 93.2 wt % ^{235}U, 6.8 wt % ^{238}U						
64	5.320	5.780	6.480	6.880	7.160	7.520
125	5.025	5.485	6.100	6.590	6.840	7.225
216	4.760	5.265	5.811	6.330	6.620	7.058
343	4.536	4.984	5.579	6.090	6.384	6.804
512	4.383	4.816	5.435	5.883	6.250	6.698
729	4.230	4.671	5.229	5.778	6.066	6.525
1000	4.100	4.503	5.096	5.637	5.875	6.416
Uranium Dioxide; H/U = 0.4; 8.370 g U/cm^3; 93.2 wt % ^{235}U, 6.8 wt % ^{238}U						
64	7.147	7.805	8.642	9.365	10.043	10.654
125	6.598	7.257	8.176	8.834	9.465	10.200
216	6.172	6.910	7.780	8.502	8.876	9.886
343	5.912	6.557	7.668	8.121	8.642	9.520
512	5.690	6.298	7.271	7.815	8.354	9.107
729	5.446	6.050	7.051	7.641	8.080	8.802
1000	5.266	5.868	6.850	7.440	7.888	8.574
Uranium Dioxide; H/U = 3; 4.566 g U/cm^3; 93.2 wt % ^{235}U, 6.8 wt % ^{238}U						
64	7.309	8.067	9.195	10.177	10.507	11.414
125	6.880	7.550	8.600	9.450	9.900	10.750
216	6.467	7.149	8.200	9.000	9.611	10.397
343	6.160	6.846	7.840	8.624	9.065	9.940
512	5.883	6.549	7.445	8.295	8.734	9.666
729	5.688	6.336	7.335	8.073	8.478	9.315
1000	5.504	6.111	7.097	7.833	8.218	9.162

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 10; 2.054 g U/cm^3; 93.2 wt % ^{235}U, 6.8 wt % ^{238}U					
64	7.548	8.416	9.537	10.360	10.947	11.660
125	7.010	7.850	8.900	9.750	10.300	11.050
216	6.712	7.426	8.409	9.312	9.873	10.745
343	6.335	7.070	8.043	8.925	9.443	10.290
512	6.082	6.797	7.708	8.642	9.125	10.013
729	5.868	6.561	7.425	8.316	8.820	9.720
1000	5.677	6.369	7.288	8.064	8.533	9.452
	Uranium Dioxide; H/U = 20; 1.150 g U/cm^3; 93.2 wt % ^{235}U, 6.8 wt % ^{238}U					
64	7.700	8.465	9.679	10.544	11.082	11.950
125	7.175	7.910	9.000	9.800	10.500	11.275
216	6.784	7.494	8.487	9.479	10.018	10.885
343	6.475	7.175	8.120	8.967	9.618	10.451
512	6.184	6.910	7.809	8.677	9.245	10.128
729	6.021	6.669	7.542	8.370	8.982	9.828
1000	5.805	6.541	7.435	8.093	8.641	9.576
	Uranium Dioxide; H/U = 0.4; 8.369 g U/cm^3; 80 wt % ^{235}U, 20 wt % ^{238}U					
64	7.300	8.079	9.202	9.914	10.404	11.260
125	6.800	7.545	8.590	9.290	9.800	10.670
216	6.453	7.174	8.100	8.952	9.408	10.290
343	6.146	6.825	7.756	8.540	9.016	9.807
512	5.909	6.556	7.468	8.247	8.732	9.555
729	5.688	6.354	7.200	7.992	8.460	9.270
1000	5.496	6.145	6.983	7.747	8.203	9.026
	Uranium Dioxide; H/U = 3; 4.570 g U/cm^3 80 wt % ^{235}U, 20 wt % ^{238}U					
64	7.650	8.600	9.541	10.445	10.962	11.845
125	7.125	7.980	8.910	9.800	10.310	11.235
216	6.735	7.520	8.494	9.420	9.907	10.817
343	6.405	7.175	8.092	8.946	9.450	10.325
512	6.139	6.877	7.850	8.682	9.185	10.048
729	5.886	6.624	7.560	8.415	8.874	9.720
1000	5.675	6.390	7.316	8.148	8.619	9.499

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 10; 2.057 g U/cm^3; 80 wt % ^{235}U, 20 wt % ^{238}U					
64	7.748	8.726	9.772	10.699	11.227	12.030
125	7.215	8.115	9.140	10.100	10.590	11.480
216	6.865	7.672	8.722	9.610	10.095	11.016
343	6.510	7.287	8.309	9.114	9.709	10.598
512	6.267	6.994	7.995	8.884	9.368	10.289
729	6.030	6.759	7.722	8.605	9.081	9.972
1000	5.823	6.525	7.462	8.302	8.803	9.707
	Uranium Dioxide; H/U = 20; 1.152 g U/cm^3; 80 wt % ^{235}U, 20 wt % ^{238}U					
64	7.810	8.640	9.774	10.675	11.194	12.163
125	7.255	8.045	9.125	10.025	10.500	11.500
216	6.860	7.596	8.635	9.531	10.042	10.970
343	6.545	7.266	8.225	9.107	9.604	10.500
512	6.252	6.956	7.916	8.795	9.275	10.202
729	6.066	6.741	7.632	8.496	9.000	9.873
1000	5.874	6.508	7.404	8.219	8.715	9.611
	Uranium Dioxide; H/U = 0.4; 8.372 g U/cm^3; 70 wt % ^{235}U, 30 wt % ^{238}U					
64	7.590	8.373	9.482	10.386	10.900	11.679
125	7.025	7.810	8.875	9.750	10.270	11.050
216	6.670	7.418	8.415	9.287	9.833	10.721
343	6.335	7.049	8.036	8.855	9.366	10.206
512	6.077	6.763	7.745	8.555	9.100	9.958
729	5.850	6.534	7.470	8.253	8.766	9.639
1000	5.668	6.327	7.246	8.010	8.539	9.412
	Uranium Dioxide; H/U = 3; 4.574 g U/cm^3; 70 wt % ^{235}U, 30 wt % ^{238}U					
64	7.775	8.707	9.747	10.786	11.320	12.206
125	7.250	8.090	9.100	10.125	10.690	11.540
216	6.917	7.698	8.707	9.651	10.188	11.148
343	6.559	7.294	8.309	9.198	9.774	10.765
512	6.315	7.031	7.991	8.886	9.423	10.351
729	6.084	6.795	7.749	8.622	9.135	10.080
1000	5.862	6.575	7.487	8.300	8.804	9.765

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 10; 2.059 g U/cm^3; 70 wt % ^{235}U, 30 wt % ^{238}U					
64	7.973	8.788	9.886	10.855	11.476	12.454
125	7.375	8.185	9.260	10.235	10.765	11.750
216	6.956	7.737	8.791	9.745	10.277	11.256
343	6.636	7.385	8.365	9.275	9.814	10.773
512	6.342	7.089	8.085	8.948	9.480	10.443
729	6.120	6.858	7.785	8.640	9.198	10.125
1000	5.924	6.608	7.554	8.384	8.882	9.828
	Uranium Dioxide; H/U = 20; 1.153 g U/cm^3; 70 wt % ^{235}U, 30 wt % ^{238}U					
64	7.926	8.772	9.944	10.775	11.484	12.373
125	7.390	8.225	9.275	10.150	10.785	11.740
216	7.003	7.794	8.815	9.771	10.315	11.287
343	6.650	7.406	8.400	9.292	9.814	10.822
512	6.393	7.118	8.090	8.996	9.540	10.479
729	6.138	6.858	7.812	8.649	9.180	10.143
1000	5.924	6.657	7.579	8.403	8.914	9.886
	Uranium Dioxide; H/U = 0.4; 8.377 g U/cm^3; 50 wt % ^{235}U, 50 wt % ^{238}U					
64	8.226	9.189	10.414	11.414	11.978	13.072
125	7.690	8.465	9.745	10.700	11.255	12.920
216	7.232	8.080	9.216	10.190	10.785	11.795
343	6.860	7.644	8.750	9.709	10.220	11.291
512	6.583	7.376	8.404	9.360	9.919	10.929
729	6.318	7.119	8.136	9.018	9.603	10.620
1000	6.135	6.853	7.845	8.747	9.288	10.280
	Uranium Dioxide; H/U = 3; 4.580 g U/cm^3; 50 wt % ^{235}U, 50 wt % ^{238}U					
64	8.292	9.192	10.350	11.452	12.106	13.093
125	7.725	8.535	9.685	10.750	11.400	12.350
216	7.210	8.073	9.206	10.213	10.789	11.831
343	6.881	7.658	8.750	9.723	10.325	11.375
512	6.581	7.354	8.415	9.386	9.943	10.968
729	6.345	7.092	8.136	9.018	9.612	10.746
1000	6.085	6.833	7.858	8.757	9.314	10.321

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 10; 2.063 g U/cm^3; 50 wt % ^{235}U, 50 wt % ^{238}U					
64	8.216	9.135	10.446	11.433	12.000	13.027
125	7.660	8.520	9.750	10.715	11.260	12.270
216	7.234	8.089	9.211	10.174	10.761	11.759
343	6.860	7.700	8.792	9.716	10.290	11.305
512	6.592	7.376	8.427	9.390	9.960	10.940
729	6.345	7.146	8.127	9.018	9.612	10.602
1000	6.124	6.931	7.857	8.766	9.318	10.298
	Uranium Dioxide; H/U = 20; 1.156 g U/cm^3; 50 wt % ^{235}U, 50 wt % ^{238}U					
64	8.152	8.999	10.250	11.190	11.745	12.573
125	7.540	8.400	9.570	10.465	11.060	11.952
216	7.140	7.973	9.027	10.030	10.557	11.543
343	6.797	7.560	8.666	9.534	10.150	11.025
512	6.511	7.293	8.296	9.214	9.775	10.744
729	6.309	7.020	8.010	8.955	9.522	10.413
1000	6.094	6.783	7.769	8.636	9.163	10.166
	Uranium Dioxide; H/U = 0.4; 8.382 g U/cm^3; 30 wt % ^{235}U, 70 wt % ^{238}U					
64	9.536	10.622	12.091	13.431	14.182	15.492
125	8.800	9.995	11.250	12.500	13.260	14.500
216	8.343	9.317	10.655	11.856	12.582	13.897
343	7.903	8.904	10.143	11.256	11.970	13.237
512	7.572	8.501	9.725	10.859	11.539	12.809
729	7.299	8.226	9.423	10.422	11.133	12.375
1000	7.052	7.889	9.068	10.111	10.768	11.992
	Uranium Dioxide; H/U = 3; 4.587 g U/cm^3; 30 wt % ^{235}U, 70 wt % ^{238}U					
64	8.903	9.928	11.289	12.497	13.115	14.239
125	8.275	9.255	10.510	11.700	12.290	13.400
216	7.821	8.762	9.950	11.055	11.731	12.898
343	7.427	8.330	9.464	10.556	11.200	12.250
512	7.145	7.984	9.090	10.154	10.810	11.915
729	6.849	7.722	8.892	9.864	10.440	11.484
1000	6.606	7.432	8.517	9.459	10.093	11.199

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 10; 2.067 g U/cm^3; 30 wt % ^{235}U, 70 wt % ^{238}U					
64	8.624	9.616	10.846	11.981	12.574	13.636
125	8.000	8.950	10.150	11.235	11.785	12.900
216	7.548	8.419	9.648	10.651	11.237	12.334
343	7.175	8.036	9.128	10.143	10.724	11.830
512	6.867	7.681	8.778	9.781	10.386	11.408
729	6.642	7.416	8.460	9.432	10.044	11.070
1000	6.375	7.170	8.191	9.100	9.724	10.764
	Uranium Dioxide; H/U = 20; 1.158 g U/cm^3; 30 wt % ^{235}U, 70 wt % ^{238}U					
64	8.457	9.448	10.638	11.657	12.234	13.200
125	7.910	8.780	9.960	10.910	11.500	12.485
216	7.421	8.285	9.466	10.401	10.965	11.987
343	7.070	7.910	8.967	9.877	10.500	11.494
512	6.787	7.562	8.638	9.572	10.154	11.141
729	6.507	7.290	8.325	9.225	9.885	10.809
1000	6.303	7.051	8.074	8.990	9.537	10.524
	Plutonium Metal; H/Pu = 0; 19.70 g Pu/cm^3; 100 wt % ^{239}Pu					
64	3.670	3.920	4.184	4.414	4.460	4.580
125	3.530	3.790	4.015	4.250	4.350	4.460
216	3.399	3.641	3.915	4.156	4.277	4.375
343	3.280	3.528	3.794	4.046	4.183	4.312
512	3.175	3.432	3.723	3.986	4.120	4.285
729	3.060	3.339	3.654	3.906	4.050	4.230
1000	2.970	3.259	3.576	3.844	3.919	4.183
	Plutonium Dioxide; H/Pu = 0.4; 8.731 g Pu/cm^3; 100 wt % ^{239}Pu					
64	5.408	5.911	6.556	6.950	7.160	7.698
125	5.080	5.545	6.200	6.575	6.875	7.395
216	4.800	5.288	5.950	6.482	6.651	7.152
343	4.592	5.089	5.726	6.146	6.475	6.951
512	4.420	4.890	5.546	5.992	6.375	6.705
729	4.266	4.770	5.373	5.814	6.210	6.660
1000	4.130	4.655	5.208	5.704	5.985	6.512

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array With Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Plutonium Dioxide; H/Pu = 3; 4.707 g Pu/cm^3; 100 wt % ^{239}Pu					
64	6.329	6.863	7.737	8.256	8.576	9.185
125	5.875	6.415	7.250	7.805	8.160	8.750
216	5.504	6.120	6.924	7.536	7.900	8.513
343	5.264	5.866	6.594	7.189	7.560	8.176
512	5.119	5.685	6.368	7.014	7.389	7.965
729	4.860	5.490	6.156	6.777	7.182	7.785
1000	4.710	5.312	5.989	6.580	6.992	7.616
	Plutonium Dioxide; H/Pu = 10; 2.101 g Pu/cm^3; 100 wt % ^{239}Pu					
64	6.966	7.688	8.624	9.342	9.795	10.514
125	6.480	7.200	8.100	8.800	9.250	10.000
216	6.104	6.814	7.719	8.478	8.890	9.662
343	5.859	6.552	7.385	8.120	8.540	9.317
512	5.617	6.296	7.134	7.811	8.265	9.050
729	5.418	6.075	6.867	7.596	8.028	8.811
1000	5.247	5.856	6.617	7.360	7.812	8.584
	Plutonium Dioxide; H/Pu = 20; 1.173 g Pu/cm^3; 100 wt % ^{239}Pu					
64	7.315	8.050	9.032	9.927	10.473	11.186
125	6.790	7.545	8.500	9.375	9.710	10.550
216	6.458	7.200	8.120	9.009	9.270	10.136
343	6.118	6.832	7.728	8.666	8.939	9.730
512	5.912	6.544	7.409	8.334	8.657	9.471
729	5.679	6.372	7.182	8.082	8.424	9.180
1000	5.463	6.156	6.974	7.812	8.151	8.954
	Plutonium Metal; H/Pu = 0; 19.74 g Pu/cm^3; 94.8 wt % ^{239}Pu, 5.2 wt % ^{240}Pu					
64	3.700	4.003	4.251	4.444	4.526	4.707
125	3.598	3.895	4.125	4.325	4.450	4.605
216	3.429	3.731	4.013	4.226	4.306	4.523
343	3.318	3.626	3.878	4.123	4.228	4.424
512	3.191	3.458	3.750	4.013	4.168	4.319
729	3.123	3.411	3.703	3.978	4.104	4.275
1000	3.024	3.318	3.655	3.928	4.041	4.227

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Plutonium Dioxide; H/Pu = 0.4; 8.732 g Pu/cm^3; 94.8 wt % ^{239}Pu, 5.2 wt % ^{240}Pu					
64	5.539	6.025	6.693	7.200	7.478	7.764
125	5.185	5.685	6.325	6.875	7.160	7.500
216	4.900	5.447	6.075	6.619	6.869	7.386
343	4.704	5.187	5.810	6.335	6.636	7.105
512	4.546	5.046	5.622	6.130	6.462	6.920
729	4.392	4.878	5.445	6.012	6.300	6.795
1000	4.262	4.694	5.281	5.866	6.112	6.675
	Plutonium Dioxide; H/Pu = 3; 4.708 g Pu/cm^3; 94.8 wt % ^{239}Pu, 5.2 wt % ^{240}Pu					
64	6.478	7.163	8.036	8.525	8.965	9.697
125	6.050	6.690	7.505	8.150	8.510	9.225
216	5.773	6.350	7.175	7.835	8.223	8.943
343	5.488	6.090	6.846	7.525	7.896	8.575
512	5.278	5.844	6.610	7.293	7.658	8.341
729	5.085	5.670	6.390	7.056	7.452	8.127
1000	4.898	5.493	6.197	6.834	7.234	7.918
	Plutonium Dioxide; H/Pu = 10; 2.101 g Pu/cm^3; 94.8 wt % ^{239}Pu, 5.2 wt % ^{240}Pu					
64	7.536	8.285	9.311	10.160	10.544	11.541
125	7.020	7.775	8.750	9.600	10.000	10.950
216	6.668	7.395	8.361	9.209	9.631	10.484
343	6.307	7.056	7.980	8.806	9.184	10.066
512	6.050	6.800	7.692	8.541	8.920	9.891
729	5.850	6.570	7.443	8.298	8.658	9.486
1000	5.632	6.295	7.202	7.974	8.450	9.252
	Plutonium Metal; H/Pu = 0; 19.74 g Pu/cm^3; 80 wt % ^{239}Pu, 20 wt % ^{240}Pu					
64	3.880	4.142	4.384	4.618	4.684	4.889
125	3.675	3.950	4.210	4.435	4.550	4.750
216	3.524	3.804	4.164	4.351	4.515	4.612
343	3.402	3.654	4.053	4.235	4.347	4.564
512	3.334	3.598	3.921	4.142	4.302	4.511
729	3.222	3.456	3.825	4.086	4.226	4.428
1000	3.139	3.384	3.765	4.019	4.183	4.391

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Plutonium Dioxide; H/Pu = 0.4; 8.733 g Pu/cm^3; 80 wt % ^{239}Pu, 20 wt % ^{240}Pu					
64	5.758	6.310	6.990	7.410	7.666	8.104
125	5.385	5.900	6.615	7.095	7.300	7.825
216	5.153	5.673	6.331	6.865	7.151	7.628
343	4.942	5.439	6.076	6.601	6.853	7.420
512	4.775	5.266	5.892	6.427	6.704	7.250
729	4.617	5.103	5.715	6.273	6.525	7.110
1000	4.386	4.955	5.530	6.050	6.398	6.932
	Plutonium Dioxide; H/Pu = 3; 4.709 g Pu/cm^3; 80 wt % ^{239}Pu, 20 wt % ^{240}Pu					
64	6.939	7.603	8.514	9.245	9.612	10.287
125	6.490	6.900	8.075	8.750	9.210	9.810
216	6.130	6.831	7.722	8.342	8.832	9.556
343	5.852	6.510	7.350	8.043	8.526	9.184
512	5.617	6.276	7.128	7.851	8.300	8.970
729	5.427	6.075	6.876	7.560	8.073	8.775
1000	5.230	5.837	6.676	7.298	7.795	8.513
	Plutonium Dioxide; H/Pu = 10; 2.102 g Pu/cm^3; 80 wt % ^{239}Pu, 20 % ^{240}Pu					
64	8.292	9.283	10.420	11.441	11.935	13.008
125	7.750	8.740	9.785	10.750	11.250	12.300
216	7.256	8.160	9.341	10.269	10.788	11.928
343	7.000	7.875	8.890	9.800	10.325	11.354
512	6.735	7.583	8.503	9.455	10.009	11.020
729	6.498	7.335	8.316	9.099	9.693	10.692
1000	6.278	7.050	8.070	8.849	9.386	10.390
	Uranium Metal; H/U = 0; 18.40 g U/cm^3; 100 wt % ^{233}U					
64	4.034	4.371	4.736	5.013	5.179	5.395
125	3.865	4.162	4.520	4.795	4.975	5.225
216	3.666	3.983	4.393	4.657	4.817	5.115
343	3.577	3.892	4.270	4.550	4.739	5.005
512	3.415	3.749	4.116	4.472	4.688	4.862
729	3.316	3.600	4.005	4.311	4.500	4.770
1000	3.202	3.512	3.926	4.242	4.416	4.733

Table 2 (Cont'd)

Number of Units in Cubic Array	Radius of Spherical Unit in Array with Cubic Cell Dimension of					
	25.40	30.48	38.10	45.72	50.80	60.96
	Uranium Dioxide; H/U = 0.4; 8.214 g U/cm^3; 100 wt % ^{233}U					
64	5.609	6.151	6.899	7.462	7.770	8.247
125	5.250	5.755	6.485	7.050	7.350	7.900
216	4.957	5.513	6.225	6.799	7.074	7.653
343	4.746	5.250	5.908	6.510	6.825	7.420
512	4.575	5.038	5.732	6.316	6.645	7.241
729	4.410	4.914	5.526	6.138	6.462	7.074
1000	4.254	4.777	5.362	5.870	6.276	6.859
	Uranium Dioxide; H/U = 3; 4.501 g U/cm^3; 100 wt % ^{233}U					
64	6.226	6.881	7.651	8.337	8.740	9.400
125	5.785	6.457	7.200	7.875	8.260	8.900
216	5.503	6.132	6.904	7.544	7.935	8.622
343	5.194	5.854	6.573	7.210	7.644	8.260
512	5.038	5.621	6.373	6.987	7.378	8.060
729	4.815	5.409	6.120	6.750	7.200	7.830
1000	4.670	5.200	5.960	6.597	7.013	7.625
	Uranium Dioxide; H/U = 10; 2.030 g U/cm^3; 100 wt % ^{233}U					
64	6.588	7.307	8.194	8.960	9.281	10.175
125	6.150	6.800	7.650	8.400	8.800	9.625
216	5.828	6.497	7.295	8.022	8.489	9.231
343	5.544	6.160	6.930	7.637	8.120	8.834
512	5.304	5.914	6.685	7.380	7.823	8.539
729	5.130	5.715	6.453	7.155	7.650	8.280
1000	4.952	5.517	6.230	7.024	7.436	8.088
	Uranium Dioxide; H/U = 20; 1.138 g U/cm^3; 100 wt % ^{233}U					
64	6.695	7.455	8.454	9.161	9.511	10.310
125	6.250	6.950	7.910	8.590	9.000	9.790
216	5.941	6.600	7.511	8.200	8.615	9.419
343	5.628	6.300	7.175	7.875	8.260	9.100
512	5.412	6.073	6.859	7.640	8.094	8.850
729	5.220	5.850	6.678	7.371	7.830	8.622
1000	5.040	5.664	6.478	7.133	7.614	8.312

Verification by Monte Carlo calculation of oxides having ^{235}U content below U(93.2) was limited to examination of only one composition. The $U(50)O_2$ at a hydrogen-to-uranium ratio of 3.0 was selected as an example and the arrays of even numbered units for each cell size were computed. In this case the KENO Monte Carlo code was programmed to determine the radius, r_c, that would result in a multiplication factor of unity within one standard deviation for 30×10^3 neutron histories. These radii are given in Table 3 along with the corresponding multiplication factors. The agreement with the entries in Table 2 is considered to be good. It is interesting to note the comparison between the corresponding radii of the 64-unit arrays in the two tables. Since both sets of data were determined by Monte Carlo calculation with about the same precision, their differences may be interpreted as a measure of the accuracy of the statistical process by which they were produced. The average ratio of the corresponding radii in Tables 2 and 3 is 0.9954 ± 0.0037.

The greatest projected usefulness of the criticality data presented in Fig. 2 is their possible adoption as a basic system of reference to provide guidance in the storage of fissile materials and in the evaluation of factors affecting array criticality. For each of the fissile materials considered, there is a sufficient number of critical systems defined in the table to produce the characteristic criticality display shown in Fig. 2. There remains the establishment of a scale for the selection of a degree of safety, or, equivalently, the degree of subcriticality acceptable to current or future safety practices. This is managed effectively by determining the behavior of the array multiplication factor as the radius of the units in a critical array is reduced from r_c. Thus, an observed change in reactivity may be associated with a given change in the mass of the units in an array. The relationship is explored in the following section.

Table 3. Monte Carlo Calculated Critical Radii in Centimeters and Their Multiplication Factors for $U(50)O_2$ Spheres at an H/U = 3 in Water-Reflected Cubic Arrays.

N	$a_n = 12.7$	k_{eff}	σ ($\times 10^3$)	$a_n = 15.24$	k_{eff}	σ ($\times 10^3$)
64	8.342	1.000	6.7	9.268	1.003	5.7
216	7.228	1.000	6.1	8.145	1.005	6.1
512	6.599	1.004	7.3	7.408	0.997	5.4
1000	6.096	0.996	6.6	6.869	0.996	6.4
N	$a_n = 19.05$	k_{eff}	σ ($\times 10^3$)	$a_n = 22.86$	k_{eff}	σ ($\times 10^3$)
64	10.404	1.000	6.5	11.505	1.003	5.4
216	9.278	1.004	6.5	10.242	1.005	5.1
512	8.505	1.000	7.2	9.377	0.998	5.8
1000	7.894	0.998	5.9	8.847	1.002	6.6
N	$a_n = 25.4$	k_{eff}	σ ($\times 10^3$)	$a_n = 30.48$	k_{eff}	σ ($\times 10^3$)
64	12.120	1.005	6.2	13.036	1.000	5.8
216	10.861	1.006	6.2	11.930	0.999	5.9
512	9.961	1.004	6.8	10.978	0.997	5.6
1000	9.325	1.001	6.5	10.411	1.001	6.0

III. THE RELATION BETWEEN ARRAY MULTIPLICATION FACTOR AND UNIT MASS

The ability to establish an upper limit for the multiplication factor of subcritical arrays is useful in circumventing detailed analyses in routine practices. The adoption of an acceptable margin of safety for a plant-wide storage system would allow the interchange of components among the storage arrays without violation of specified limits. Safety factors usually arbitrarily applied because of changes in arrays such as, array moderation, unit and cell shape, or array composition, may be expressed as a reduction in the allowable mass limit provided an estimate of their effect on array reactivity can be made.

Five reflected arrays from Table 2, selected to provide a range of neutron spectra and of unit sizes, were calculated to determine the decrease in array reactivity as the radius of the units in the arrays was reduced. An additional array of metal cylinders, having equal height and diameter, also was examined. These calculations are summarized in Fig. 3 where the array multiplication factor is shown as a function of the ratio of the radius, r, of the unit in the subcritical array to that of the radius, r_c, in the critical array. The critical conditions for the arrays are given in the table at the bottom of the figure.

It appears that the variation in k_{eff} is almost direct with the fractional reduction in r_c over the initial 10% in k_{eff}. Below a k_{eff} of $\sim$0.9, the direct relation, represented by the line shown, is most nearly maintained for units having predominately fast neutron leakage; but for units containing water, the reduction in k_{eff} is more rapid than is the decrease in r/r_c. This behavior parallels that of the multiplication factor for single units when the radius is reduced. Thus, an upper bound for the multiplication factor of water-reflected arrays comprised of spheres of reduced radius may be taken as the ratio r/r_c. It is possible, therefore, to specify a minimal margin of safety in terms of the array multiplication factor.

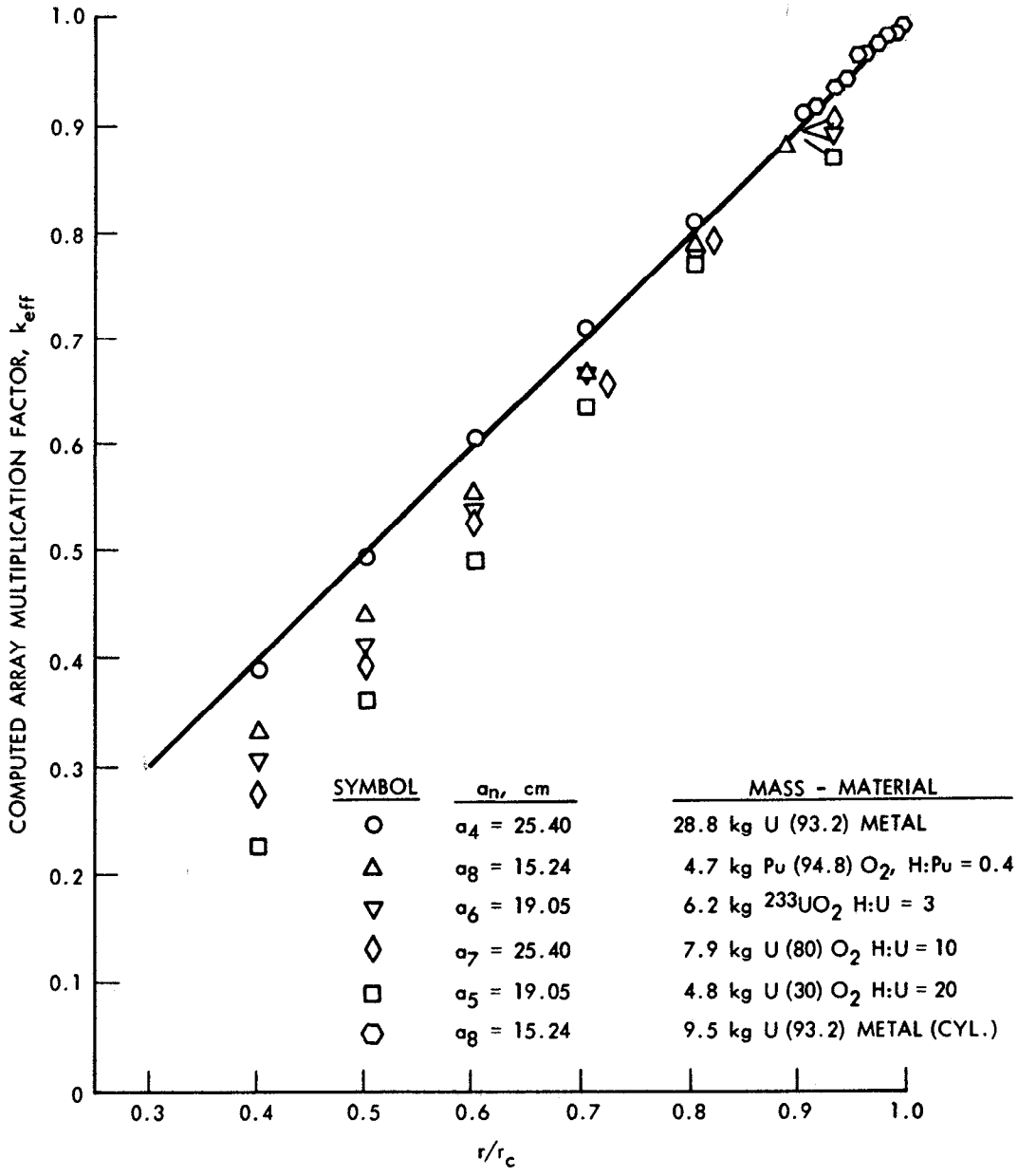

Fig. 3. Effect on the Computed Array Neutron Multiplication Factor Caused by a Reduction in the Spherical Radius Required for Criticality in Water-Reflected Arrays.

IV. INFLUENCE OF CONCRETE ON NEUTRON INTERACTION BETWEEN ARRAYS

In nuclear safety practices, it is perhaps more common to consider arrays reflected by concrete rather than by water. The common use of water as a reference material against which reflector effects are evaluated in safety analyses is an outgrowth of the development of practices based upon experimental data. Water is a convenient and effective reflector to use in the experimental determination of minimum critical parameters of fissile materials. A significant advantage of water as a reflector about large arrays is its ability to provide neutron shielding; for example, very negligible neutron coupling will be realized between two arrays separated by a 30-cm thickness of water. Compared to water and dependent upon the thickness employed, concrete as a reflector exhibits an improved neutron ableDo. Concrete is less effective than water as a neutron reflector for thicknesses less than about 15 cm principally because of the transmission of neutrons through the concrete, i.e., a greater fraction of neutrons are lost from the enclosed fissile material. It follows that the reactivity observed in a system of coupled arrays separated by concrete will be greater than the same system when separated by water. Calculative results have detected the effect of neutron transmission through a layer of concrete between arrays as thick as 40 cm.

Although there are many isolated instances, experimental and calculated, that may be cited to demonstrate the points mentioned above, there is a need to establish a basis whereby such comparisons may be expressible in terms of a safety margin. In performing a systematic study of the influence of concrete on array interaction, it is reasonable to expect some effect on the observed neutron multiplication factor of coupled systems due to array size, the reactivity of an individual array, the concrete thickness present, and the number of arrays considered. Of these parameters, the one most likely to provide a measure and control of reactivity changes is the multiplication factor of an individual array. The array reactivity is essentially determined by the unit radius or mass, thus, the water-reflected critical arrays of

Table 2 and the reactivity-mass relationship depicted in Fig. 3 furnish a crude, but acceptable, index of a safety margin in terms of reactivity.

The study was made economically feasible by applying the differential albedo method used in the water-reflected array studies to concrete and by including the effect of neutrons transmitted through the concrete. The resultant albedo-transmission technique (A-T) is described and validated in Appendix A. Observed reductions in computing times compared to complete neutron tracking in concrete have been between 3 and 10 depending upon the size and number of arrays as well as the concrete thickness considered.

An assembly of arrays as described in the Monte Carlo calculation consisted of the following. Each array was cubic; the reflecting concrete was located at the outer cell boundaries of the arrays; the arrays were identical; and the same concrete thickness separated adjacent arrays as reflected the assembly of arrays.

Utilizing the calculated criticality data of Table 2, it is possible to assign a maximum subcritical multiplication factor to each array considered when reflected by water. This was accomplished by assigning to the spherical units in each cell a radius determined by the product of r_c, from the table, and a selected subcritical value for k_{eff}. In this manner each array in a system of arrays will have the same multiplication factor. This similarity among the individual arrays makes more meaningful any observed change in the reactivity of the systems when the parameters describing the system are varied.

The calculated multiplication factor for planar arrangements of arrays in concrete are given in Table 4. The array occupying each concrete enclosure in a system is described by the radius, r, of the spherical unit, its mass, m, the total mass in the array, N_m, the half-cell dimension, a_n, the neutron multiplication factor of the array with a water reflector, and the cubic array dimension. The concrete thickness was taken as 10.16, 20.32, 30.48, and 40.64 cm surrounding a single array and surrounding and separating the adjacent array arrangements.

Table 4. Calculated Multiplication Factors for Concrete-Reflected and -Separated Systems of Arrays.

Arrangement of Arrays	System Multiplication Factor for Concrete Thicknesses[a] of 10.16 cm	20.32 cm	30.48 cm	40.64 cm
	r = 4.111 cm, m = 5.460 kg U(93.2) metal, N_m = 1179 kg a_6 = 12.70 cm, k_{eff} (water-reflected array)$^m \simeq$ 0.85 Cubic array dimension = 152 cm (5 ft)			
1 x 2 x 1	0.867	1.007	1.007	0.992
2 x 2 x 1	0.946	1.055	1.030	1.006
1 x 1 x 1[b]	0.790	0.943	0.975	0.977
	r = 3.869 cm, m = 4.551 kg U(93.2) metal, N_m = 983 kg a_6 = 12.70 cm, k_{eff} (water-reflected array)$^m \simeq$ 0.80 Cubic array dimension = 152 cm (5 ft)			
1 x 2 x 1	0.793	0.937	0.931	0.936
2 x 2 x 1	0.892	0.993	0.922	0.947
1 x 1 x 1	0.733	0.874	0.915	0.916
	r = 3.627 cm, m = 3.749 kg U(93.2) metal, N_m = 810 kg a_6 = 12.70 cm, k_{eff} (water-reflected array)$^m \simeq$ 0.75 Cubic array dimension = 152 cm (5 ft)			
1 x 2 x 1	0.736	0.871	0.967	0.870
2 x 2 x 1	0.826	0.921	0.896	0.879
1 x 1 x 1	0.678	0.821	0.836	0.849
	r = 3.485 cm, m = 3.326 kg U(93.2) metal, N_m = 3326 kg a_{10} = 12.70 cm, k_{eff} (water-reflected array)$^m \simeq$ 0.85 Cubic array dimension = 254 cm (8.3 ft)			
1 x 2 x 1	0.828	0.996	1.008	0.995
2 x 2 x 1	0.945	1.066	1.026	1.007
1 x 1 x 1	0.763	0.936	0.956	0.982
	r = 3.280 cm, m = 2.773 kg U(93.2) metal, N_m = 2773 kg a_{10} = 12.70 cm, k_{eff} (water-reflected array)$^m \simeq$ 0.80 Cubic array dimension = 254 cm (8.3 ft)			
1 x 2 x 1	0.772	0.921	0.934	0.926
2 x 2 x 1	0.876	0.993	0.973	0.940
1 x 1 x 1	0.714	0.868	0.905	0.917

Table 4 (Cont'd)

Arrangement of Arrays	System Multiplication Factor for Concrete Thicknesses[a] of 10.16 cm	20.32 cm	30.48 cm	40.64 cm
	r = 4.998 cm, m = 9.811 kg U(93.2) metal, N_m = 2119 kg a_6 = 19.05 cm, k_{eff} (water-reflected array) $\simeq$ 0.85 Cubic array dimension = 228.6 cm (7.5 ft)			
1 x 2 x 1	0.849	0.954	0.946	0.944
2 x 2 x 1	0.922	0.994	0.976	0.947
2 x 2 x 2	1.019	1.052	1.004	0.973
	r = 5.666 cm, m = 14.294 kg U(93.2) metal, N_m = 3088 kg a_6 = 25.4 cm, k_{eff} (water-reflected array) $\simeq$ 0.85 Cubic array dimension = 304.8 cm (10 ft)			
1 x 2 x 1	0.861	0.931	0.933	0.934
2 x 2 x 1	0.908	0.965	0.951	0.934
2 x 2 x 2	0.983	1.013	0.971	0.948
	r = 6.069 cm, m = 17.566 kg U(93.2) metal, N_m = 3794 kg a_6 = 30.48 cm, k_{eff} (water-reflected array) $\simeq$ 0.85 Cubic array dimension = 365.8 cm (12 ft)			
1 x 2 x 1	0.868	0.933	0.924	0.914
2 x 2 x 1	0.909	0.954	0.921	0.925
2 x 2 x 2	0.972	0.986	0.943	0.922
	r = 8.105 cm, m = 18.694 kg U(30)(H/U = 0.4), N_m = 1196 kg a_4 = 12.70 cm, k_{eff} (water-reflected array) $\leq$ 0.85 Cubic array dimension = 101.6 cm (3.3 ft)			
1 x 2 x 1	0.851	1.010	1.025	1.011
2 x 2 x 1	0.965	1.086	1.058	1.025
1 x 1 x 1	0.764	0.948	0.986	1.004
	r = 7.629 cm, m = 15.590 kg U(30)(H/U = 0.4), N_m = 998 kg a_4 = 12.70 cm, k_{eff} (water-reflected array) $\leq$ 0.80 Cubic array dimension = 101.6 cm (3.3 ft)			
1 x 2 x 1	0.787	0.953	0.965	0.960
2 x 2 x 1	0.915	1.032	1.009	0.972
1 x 1 x 1	0.696	0.891	0.933	0.931

Table 4 (Cont'd)

Arrangement of Arrays	System Multiplication Factor for Concrete Thicknesses[a] of 10.16 cm	20.32 cm	30.48 cm	40.64 cm
	r = 7.152 cm, m = 12.485 kg U(30)(H/U = 0.4), N_m = 822 kg a_4 = 12.70 cm, k_{eff} (water-reflected array) ≤ 0.75 Cubic array dimension = 101.6 cm (3.3 ft)			
1 x 2 x 1	0.724	0.899	0.910	0.907
2 x 2 x 1	0.841	0.977	0.949	0.912
1 x 1 x 1	0.641	0.835	0.870	0.888
	r = 2.889 cm, m = 1.990 kg Pu metal, N_m = 430 kg a_6 = 12.70 cm, k_{eff} (water-reflected array) ≃ 0.85 Cubic array dimension = 152.4 cm (5 ft)			
1 x 2 x 1	0.856	0.936	0.933	0.938
2 x 2 x 1	0.903	0.975	0.966	0.938
1 x 1 x 1	0.789	0.909	0.918	0.914
	4 = 2.719 cm, m = 1.659 kg Pu metal, N_m = 358 kg a_6 = 12.70 cm, k_{eff} (water-reflected array) ≃ 0.80 Cubic array dimension = 152.4 cm (5 ft)			
1 x 2 x 1	0.782	0.873	0.871	0.872
2 x 2 x 1	0.852	0.923	0.893	0.879
1 x 1 x 1	0.746	0.832	0.861	0.863
	r = 2.549 cm, m = 1.367 kg Pu metal, N_m = 295 kg a_6 = 12.70, k_{eff} (water-reflected array) ≃ 0.75 Cubic array dimension = 152.4 cm (5 ft)			
1 x 2 x 1	0.730	0.802	0.808	0.808
2 x 2 x 1	0.783	0.843	0.821	0.812
1 x 1 x 1	0.685	0.775	0.796	0.796
	r = 3.267 cm, m = 2.901 kg Pu metal, N_m = 2901 kg a_{10} = 22.86 cm, k_{eff} (water-reflected array) ≃ 0.85 Cubic array dimension = 457.2 cm (15 ft)			
1 x 2 x 1	0.844	0.912	0.900	0.906
2 x 2 x 1	0.882	0.941	0.920	0.908
1 x 1 x 1	0.818	0.880	0.899	0.896

Table 4 (Cont'd)

Arrangement of Arrays	System Multiplication Factor for Concrete Thicknesses[a] of 10.16 cm	20.32 cm	30.48 cm	40.64 cm
	r = 3.075 cm, m = 2.399 kg Pu metal, N_m = 2399 kg a_{10} = 22.86 cm, k_{eff} (water-reflected array) ≃ 0.80 Cubic array dimension = 457.2 cm (15 ft)			
1 x 2 x 1	0.792	0.855	0.854	0.843
2 x 2 x 1	0.834	0.882	0.867	0.856
1 x 1 x 1	0.760	0.825	0.839	0.837
	r = 2.883 cm, m = 1.977 kg Pu metal, N_m = 1977 kg a_{10} = 22.86 cm, k_{eff} (water-reflected array) ≃ 0.75 Cubic array dimension = 457.2 cm (15 ft)			
1 x 2 x 1	0.738	0.770	0.781	0.778
2 x 2 x 1	0.776	0.822	0.812	0.796
1 x 1 x 1	0.713	0.773	0.780	0.793

a. Maximum standard deviation on computed multiplication factor is ± 0.006.

b. Estimated by subtracting fissions due to transmitted neutrons in 1 x 2 x 1 array arrangement.

Considering the results from the single array calculations of Table 4, one may conclude that concrete 12 to 15 cm thick is equivalent to a 20-cm-thick water reflector about an array. This is indicated for U(93.2) and ^{239}Pu in Fig. 4 where the average of the difference between the values of k_{eff} with concrete and with water is given as a function of the concrete thickness. Although the tabulated $U(30)O_2$ arrays exhibit this effect, to a greater degree than U(93.2) metal, it is believed that the results for the oxide may be biased high because the cubic array dimension of 101.6 cm represents a dimension in the A-T calculation which is known to cause a larger k_{eff} (about 2% in this case) than would be determined by completely tracking neutron histories.

It may be seen from the data for the planar array arrangements that reductions in the individual array multiplication factors lead to at least an equal reduction in the system multiplication factors. These results are sufficiently consistent to allow prediction of a change in the reactivity of a system produced by a specified change in the reactivities of the component arrays. It is interesting to observe that the composition of the arrays constituting these various systems are widely different but that the reactivity of corresponding systems are not significantly different. There is, however, the evidential suggestion that a larger gain in coupling reactivity may be associated with arrays having smaller units (necessarily higher fissile material density arrays), hence, larger numbers of neutrons being exchanged per unit surface area between the arrays in a system. The observed reactivity changes, however, do not exhibit a strong dependence on the array dimensions.

Displayed in the data of Table 4 is the apparent peaking of reactivity corresponding to a concrete thickness near 20 cm. With greater concrete thicknesses, the reactivity contribution by transmitted neutrons decreases and the resultant increase is due primarily to the increased neutron reflection by the thicker concrete.

The importance of transmitted neutrons between coupled arrays separated by less than 20-cm-thick concrete is suggested in Table 4 by the three entries for the a_6 arrays of U(93.2). These eight-array,

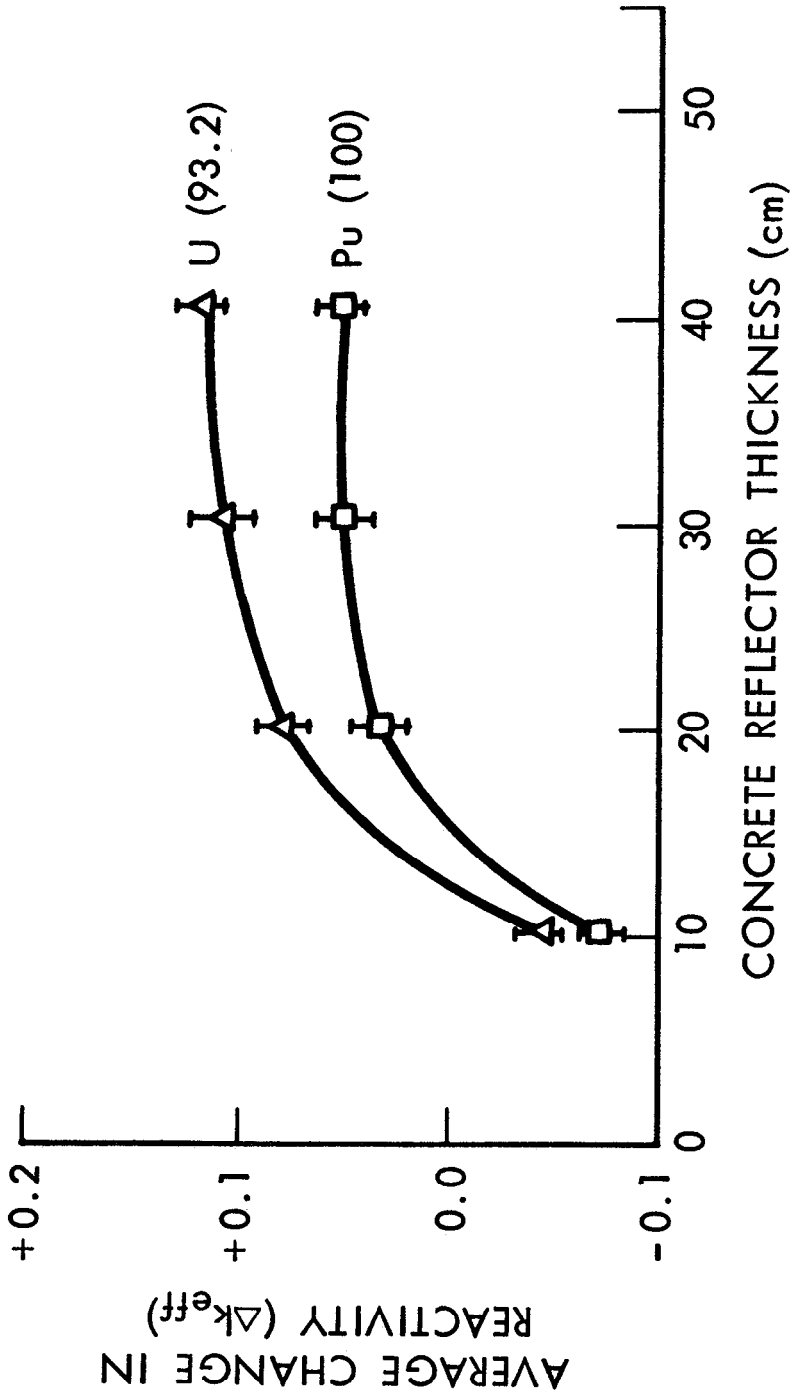

Fig. 4. Observed Change in the Computed Array Neutron Multiplication Factor When Concrete of Various Thicknesses Replaces a 20-cm-thick Water Reflector.

three-dimensional systems (2 x 2 x 2) point out that the contribution to reactivity due to the transmitted neutrons can exceed that from reflection by thick concrete and also evidence the presence of appreciable neutron transmission. Further evidence of these effects is recorded in Table 5 for systems of larger numbers of coupled arrays. The latter data are graphically summarized in Fig. 5 for comprehensive assimulation. The lines joining points of the same arrays are for clarity and identification and are not intended to display a functional behavior.

Some measure of the importance of the concrete separating arrays in a system is contained in the calculations reported in Table 6. The U(93.2) metal arrays selected from Table 4 for these calculations are identified by $a_6 = 12.70$ cm, $a_4 = 19.05$ cm, and $a_8 = 30.48$ cm. The arrays comprising the planar systems separated by a 20.32-cm thickness of concrete show that the maximum change in reactivities of the systems, brought about by a removal of the concrete separating arrays, is less than 3%, not always in the same direction. Larger reactivity changes are observed in calculations of systems containing a 40.64-cm thickness of concrete. An additional check of the A-T treatment was made in the calculations for the first array in Table 6 by repeating the calculation with complete neutron tracking in the concrete. The agreement of the results are considered to be satisfactory. The results suggest that a slight adjustment in array reactivity would permit two arrays to be stored in a larger single concrete enclosure without compromising a selected safety margin.

The response of the reactivity of a system of arrays to additional reflection external to the concrete was examined. The results are presented in Table 7 where it is observed that the maximum increase in reactivity occurs for the thinnest concrete, as expected. As the concrete thickness is increased, the effect diminishes and is almost not discernible at 40.64 cm.

Finally, there was exploration of the effect on the reactivity of two-array systems as the distance between the two arrays was increased. These calculations are presented in Table 8 for 10.16-cm-thick concrete

Table 5. Calculated Multiplication Factors for Concrete-Reflected and Separated Planar and Cubic Array Arrangements.

Arrangement of Arrays	Multiplication Factor of Systems Concrete Thicknesses of 10.16	20.32	30.48	40.64
	r = 4.111 cm, m = 5.460 kg U(93.2) metal, N_m = 1179 kg a_6 = 12.70 cm, k_{eff} (water-reflected) ≃ 0.85 Cubic array dimension = 152.4 cm (5 ft)			
1 x 2 x 1	0.867	1.007	1.007	0.992
1 x 3 x 1	0.893	1.018	1.018	0.999
1 x 4 x 1	0.912	1.033	1.031	1.003
2 x 2 x 1	0.946	1.055	1.030	1.006
3 x 3 x 1	1.050	1.113	1.065	1.015
4 x 4 x 1	1.084	1.143	1.069	1.024
2 x 2 x 2	1.075	1.135	1.070	1.018
3 x 3 x 3	1.276	1.222	1.114	1.036
4 x 4 x 4[b]	1.415	1.264	1.127	1.048
1 x 1 x 1[b]	0.790	0.943	0.975	0.977
	r = 5.508 cm, m = 13.131 kg U(93.2) metal, N_m = 840 kg a_4 = 30.48 cm, k_{eff} (water-reflected array) ≃ 0.85 Cubic array dimension = 152.4 cm (5 ft)			
1 x 2 x 1	0.874	0.945	0.949	0.941
1 x 3 x 1	0.886	0.958	0.952	0.939
1 x 4 x 1	0.902	0.979	0.957	0.949
2 x 2 x 1	0.921	0.991	0.968	0.942
3 x 3 x 1	0.971	1.014	0.979	0.964
4 x 4 x 1	1.020	1.037	0.991	0.962
2 x 2 x 2	1.006	1.029	0.978	0.964
3 x 3 x 3	1.161	1.089	1.017	0.967
4 x 4 x 4[b]	1.264	1.128	1.018	0.968
1 x 1 x 1[b]	0.819	0.899	0.928	0.931
	r = 5.738 cm, m = 14.846 kg U(93.2) metal, N_m = 7601 kg a_8 = 30.48 cm, k_{eff} (water-reflected array) ≃ 0.85 Cubic array dimension = 487.7 cm (16 ft)			
1 x 2 x 1	0.862	0.931	0.925	0.934
1 x 3 x 1	0.882	0.948	0.946	0.924
1 x 4 x 1	0.891	0.949	0.948	0.940
2 x 2 x 1	0.917	0.962	0.951	0.936
3 x 3 x 1	0.964	0.998	0.969	0.937
4 x 4 x 1	1.004	1.009	0.968	0.932
2 x 2 x 2	0.993	1.012	0.968	0.932
3 x 3 x 3	1.109	1.058	0.989	0.950
4 x 4 x 4[b]	1.215	1.087	0.989	0.958
1 x 1 x 1[b]	0.815	0.891	0.905	0.924

a. Maximum standard deviation on computed multiplication factor is ±0.006.
b. Estimated by subtracting fissions due to transmitted neutrons in the 1 x 2 x 1 array arrangement.

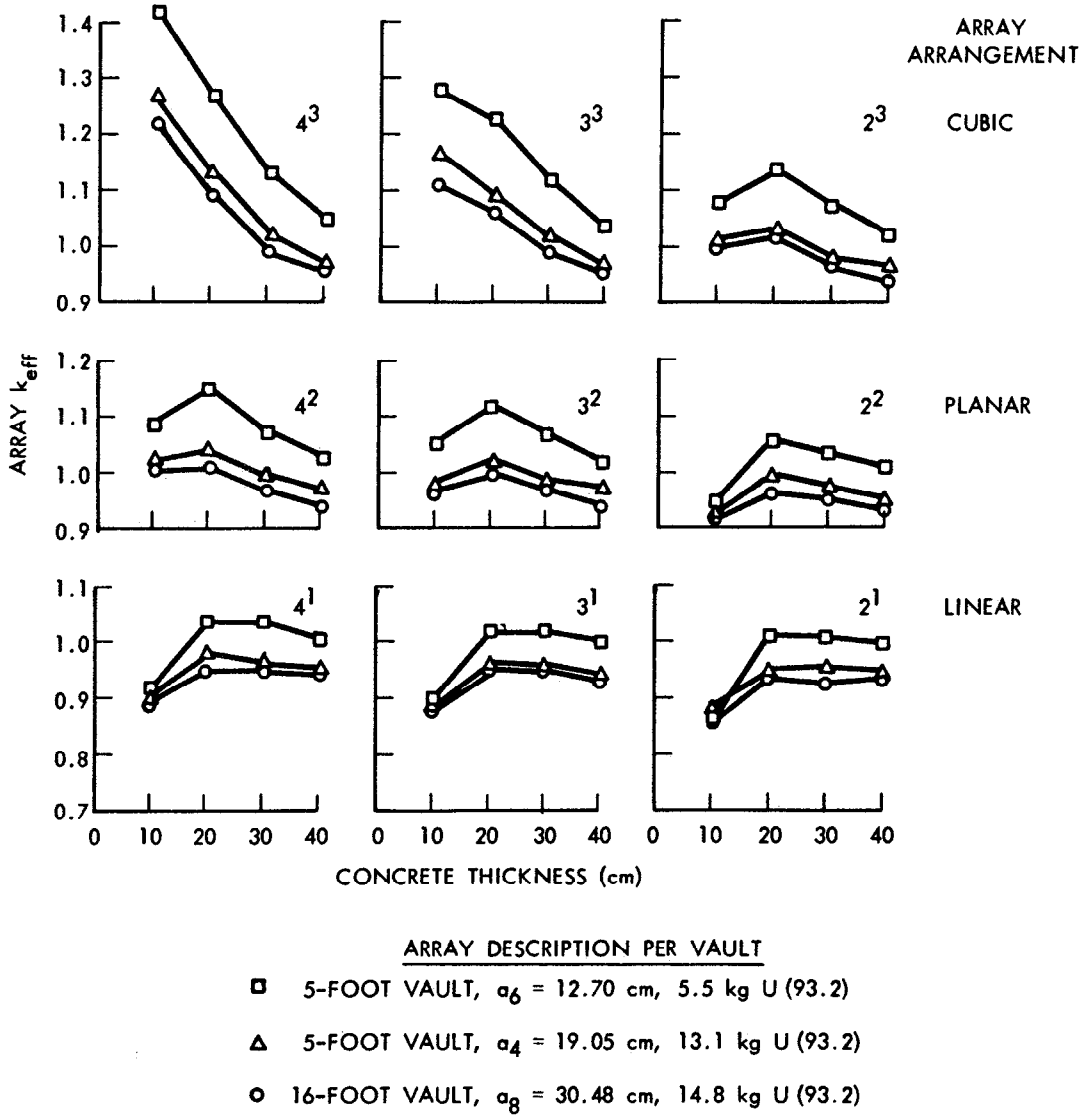

Fig. 5. Computed Neutron Multiplication Factors for Arrays Neutron-Coupled Through Concrete as a Function of the Concrete Thickness for Linear, Planar, and Cubic Arrangements of Arrays.

Table 6. Calculated Multiplication Factors for Concrete-Reflected Systems of Arrays Separated by Concrete.

Array Description	Separating Concrete[a]	Multiplication Factor[b] of Systems 1 x 2 x 1	2 x 2 x 1
r = 4.111 cm, m = 5.460 kg U(93.2)	20.32 cm	1.007	1.055
a_6 = 12.70 cm, k_{eff} (water-reflected array) ≃ 0.85	None	0.986	1.064
	None	0.983[c]	1.052[c]
r = 5.508 cm, m = 13.131 kg U(93.2)	20.32 cm	0.949	0.968
a_4 = 19.05 cm, k_{eff} (water-reflected array) ≃ 0.85	None	0.947	0.997
r = 5.738 cm, m = 14.846 kg U(93.2)	20.32 cm	0.931	0.962
a_8 = 30.48 cm, k_{eff} (water-reflected array) ≃ 0.85	None	0.946	0.987
	40.64 cm	0.934	0.936
	None	0.957	1.013

a. Thickness of system reflector same as quoted separator thickness.
b. Maximum standard deviation for computed multiplication factors is ± 0.006.
c. Value determined with neutron tracking performed in concrete.

Table 7. Comparative Calculated Multiplication Factors for Concrete-Reflected and -Separated Arrays when Externally Reflected.

r = 5.666 cm, m = 14.294 kg U(93.2), N_m = 3088 kg
a_6 = 25.4 cm, k_{eff} (water-reflected array) ≃ 0.85
Cubic array dimension = 304.8 cm (10 ft)

	Multiplication Factor[a] of System for Concrete Thicknesses of			
	10.16 cm	20.32 cm	30.48 cm	40.64 cm
No external reflector	0.908	0.965	0.951	0.934
30-cm-thick water	1.019	0.985	0.951	0.938

a. Maximum standard deviation of computed multiplication factor is ± 0.006.

Table 8. Calculated Multiplication Factors for Two 10.16-cm-thick Concrete-Reflected Arrays as a Function of Their Separation.

Concrete Surface Separation Between Arrays (cm)	r, cm : m, kg U(93.2): N_m, kg : a_n, cm : Cubic Array Dimension, cm:	4.111 5.460 1179 ≃ 0.85 102.4 cm (5 ft)	3.869 4.551 983 ≃ 0.80 102.4 cm (5 ft)	5.738 14.846 7601 ≃ 0.85 487.7 cm (16 ft)
		Calculated Multiplication Factor		
0		0.854	0.785	0.868
4		0.846	0.787	0.875
10		0.839	0.781	0.869
20		0.836	0.769	0.870
50		0.815	0.746	0.865
100		0.803	0.744	0.855
200		0.796	0.741	0.839
400		--	--	0.846
500		--	--	0.843
600		--	--	0.838
∞		0.791	0.733	0.815

a. Maximum standard deviation on computed multiplication factor is ± 0.006.

surrounding each array. Beginning with the two reflected arrays in contact their separation was increased to about the edge dimension of the array. The reactivity for a single reflected arrays is included and represents an infinite separation. It is apparent that very little neutron coupling occurs when the separation of arrays is about equal to the edge dimension of the arrays. It is apparent that if the concrete thickness surrounding each of the arrays is increased and the effect of concrete as a reflector about a single array is compensated, then the reactivity of the coupled arrays will be less than that observed in Table 8. It is to be noted in the a_6 array that a reduction of 5% in the array reactivity resulted in a loss of about 6% in the reactivity of the pair of arrays, consistent with the result observed in Table 4. One can establish, therefore, a safety margin for a single concrete enclosed storage volume and can properly compensate for the possible reactivity contributed from a second storage volume and by any other fissile material in the vicinity.

In the absence of a reflector, the two arrays, a_6 and a_8, show very little neutron coupling as a function of their separation. Within a Δk_{eff} of $\pm 1\%$, there was no evidence of coupling at one meter separation. The unreflected multiplication factors for these individual arrays were 0.584 and 0.731, respectively, while both had a k_{eff} of $\sim$0.85 when water reflected.

REMARKS

Except for the units of intermediate ^{235}U enriched uranium, the results presented in Table 2 for all reflected arrays having an even number of units have been determined by Monte Carlo calculations. The calculations reported represent critical radii of spherical units and serve as a point of departure for nuclear safety applications. It is recommended that a minimum reduction of 5% in k_{eff} (corresponding to a 5% reduction in the radius of the unit) be utilized as a margin of safety to assure subcriticality. There is no reason to believe that a similar reduction for the intermediate ^{235}U-enriched-uranium arrays will not also result in subcriticality with this margin. It should be clear that each entry of Table 2 defines many other arrays of units of that mass by means of Eq. (3) for either larger or smaller values of n subject to the condition that $n \geq 4$. Application to values of $n < 4$ probably results in conservative spacing but this is an area in need of further investigation.

Using the Criticality Index Rule described in Ref. 1, any of the units in any of the cells of the arrays described in Table 2 may be combined into a reflected array of mixed units.

The simple correlation between the unit radius and computed array multiplication factor provides a consistent method of assigning a maximum expected array k_{eff}. The consistency is demonstrated in the numerous tabulations of calculated results for the effect of concrete in arrays. The totality of these correlations indicates that the array multiplication factor is principally controlled by the mass of the unit and that one may compensate for a known or estimated increase in array reactivity, resulting from a change in some basic storage condition, by an appropriate reduction in the mass of the units.

The calculated data for concrete as a reflector about arrays indicated that a thickness between 12.7 and 15.2 cm is equivalent to a 20-cm-thick water reflector. The calculated data also suggest that the effect of concrete as a reflector of plutonium systems is less than on comparable uranium systems.

Coupled arrays separated and reflected by concrete cannot be considered as isolated even for thicknesses as great as 40 cm. The reactivity contribution of the neutron coupling of several arrays is independent of the reactivity of the individual arrays in the system. The multiplication factor for a system responds to a change in the reactivity of the constituent arrays as the single arrays respond to a change in the reactivity of the constituent units. The magnitude of the gain in reactivity due to neutron coupling is dependent upon the concrete thickness and the number of arrays in a system.

APPENDIX A
MONTE CARLO CALCULATIONS

There is a need to establish the credibility of the criticality data generated by any calculational method if they are to have practical application. Confidnece in the Monte Carlo method used in this study is established in this appendix by comparison of calculations with critical experiments data. In general, two basic requirements should be met before computed results are acceptable. The first concerns the mechanics of the code. Verification that the code correctly specifies the intended mathematical operations and that the computer properly executes the code is usually accomplished by checks which are an integral part of the program or by appropriate test problems. The second requirement is for authenticated nuclear data as computer input. Nuclear cross-section sets should be checked for consistency with differential cross-sections and with integral data from representative well-defined experiments. Having once met these two requirements, the application of the code and its input data may be augmented by additional information, less well defined information, if so doing introduces greater similarity between the referenced experiments and the problem to be solved. These further explorations may establish biases and better define the areas of applicability of the method. It is believed that the KENO Monte Carlo code and the Hansen-Roach 16-group neutron cross-section sets constitute a validated computational method for the study of uniform arrays.

There follows a comparison of the computed and experimental results from a variety of assemblies in which different forms of both fissile and nonfissile materials were utilized. It is to be realized that many of the fissile materials reported in the text do not appear in this collection because simple critical experiments unencumbered by structures and diluents have not been performed.

Stratton[6,7] has correlated the results of critical experiments and of their calculations by neutron transport theory using the Carlsen[8,9] codes and the Hansen-Roach[3] 16-group neutron cross-section sets. The result of this effort is an excellent recording of the biases of the cross sections when used to compute systems with the three principal fissile materials in a variety of configurations and other materials.

The benchmark[10] problem of Godiva I, which was a U(93.8) metal sphere having a mass of 52.28 ± 0.16 kg at a density 18.75 g U/cm^3, was calculated by the KENO code. The experiment was described in the code as an unreflected sphere of U(93.8) at a density of 18.74 g U/cm^3 having a radius of 8.71 cm and resulted in a neutron multiplication factor of 0.99525 ± 0.00072. The standard deviation resulted from the tracking of 3.29 x 10^6 neutrons.

Given in Table A-1 are the computed neutron multiplication factors for a series of experiments[11] with subcritical cylinders of U(93.2) metal. The cylinders were spaced in three dimensions having an equal number along each edge of a near cubic array. The description of an average unit in the array is given, followed by the measured spacing required for criticality and the calculative result. The arrays were constructed with and without a 15.2-cm-thick paraffin reflector. Shown in Table A-2 is a collection of data for experimental arrangements with an unequal number of cylinders along each of the three dimensions of an array and their computed results.

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6. William R. Stratton, "Correlations of Experiments and Calculations," Proceedings Nuclear Criticality Safety, Las Vegas, Nevada, December 1966, SC-DC-67-1305 (1967).
7. William R. Stratton, "Criticality Data and Factors Affecting Criticality of Single Homogenous Units," LA-3612, Los Alamos Scientific Laboratory (1967).
8. B. Carlson, C. Lee, and J. Worlton, "The DSN and TDC Neutron Transport Codes," LAMS-2346, LosAlamos Scientific Laboratory (1959).
9. Bengt G. Carlson, "Numerical Solution of Transient and Steady State Neutron Transport Problems," LAMS-2260, Los Alamos Scientific Laboratory (1959).
10. Argonne Code Center: Benchmark Problem Book, ANL-7416, Argonne National Laboratory (1968).
11. J. T. Thomas, "Critical Three-Dimensional Arrays of Neutron Interacting Units Part II - U(93.2) Metal," ORNL-TM-868, Oak Ridge National Laboratory (1964).

Table A-1. Description of Critical Experimental Cuboidal Arrays of U(93.2) Metal Cylinders at a Density of 18.76 g U/cm^3 and the KENO Computed Multiplication Factors.

Unit Description			Unreflected Arrays				Reflected Arrays[c]			
Radius r (cm)	Height h (cm)	Mass (kg U)	Half Cell Dimensions[a] a_x, a_y (cm)	a_z (cm)	$\bar{\rho}$ (g U/cm^3)	KENO Computed[b] k_{eff}	Half Cell Dimensions[a] a_x, a_y (cm)	a_z (cm)	$\bar{\rho}$ (g U/cm^3)	KENO Computed[b] k_{eff}
					$n_x = n_y = n_z = 2$					
5.747	8.077	15.694	6.198	4.490	11.374	0.990	9.658	7.950	2.645	0.999
5.732	10.765	20.805	6.841	6.491	8.562	0.992	--	--	--	--
5.753	10.765	20.960	6.877	6.507	8.514	0.995	11.746	11.376	1.669	0.999
5.753	10.765	20.960	6.752	6.752	8.513	0.984	--	--	--	--
5.755	13.459	26.218	7.526	8.501	6.806	1.000	13.944	14.919	1.130	0.992
5.755	13.459	26.218	7.889	7.889	6.675	0.996	--	--	--	--
5.753	5.382	10.480	--	--	--	--	7.601	4.539	4.995	1.003
4.558	8.641	10.507	--	--	--	--	6.712	6.475	4.503	1.000
					$n_x = n_y = n_z = 3$					
5.755	5.382	10.484	6.758	3.695	7.767	0.985	10.099	7.036	1.826	1.000
4.558	8.641	10.489	5.776	5.539	7.096	0.993	9.275	9.038	1.686	0.999
5.745	8.077	15.683	7.847	6.141	5.185	0.999	12.842	11.136	1.067	0.992
5.742	10.765	20.877	8.924	8.564	3.827	1.002	15.316	14.956	0.744	0.999
5.742	10.765	20.877	8.801	8.801	3.828	0.988	--	--	--	--
5.743	13.459	26.113	9.990	10.977	2.980	0.983	18.239	19.225	0.510	0.998
					$n_x = n_y = n_z = 4$					
5.7405	5.382	10.434	7.7165	4.667	4.6933	0.986	11.9205	8.871	1.035	1.007

a. Experimental errors of dimensions are $\pm$ 0.013 cm for unreflected arrays and + 0.026 cm for reflected arrays.
b. Calculations for 30 x 10^3 neutrons, giving standard deviation less than $\pm$ 0.006 for all values.
c. Paraffin reflector thickness is 15.2 cm having a density of 0.93 g/cm^3.

Table A-2. Description of Unreflected Noncuboidal Critical Experimental Arrays of U(93.2) Metal Cylinders at a Density of 18.76 g U/cm^3 and the KENO Computed Multiplication Factors.

Unit Description			Arrangement	Half Cell Dimensions[a]		$\bar{\rho}$	KENO Computed[b]
r (cm)	h (cm)	Mass (kg U)	$n_x n_y n_z$	a_x, a_y (cm)	a_z (cm)	(g U/cm^3)	k_{eff}
5.744	10.765	20.896	4 4 1	6.502	6.141	10.059	0.992
5.744	10.765	20.896	3 3 1	6.073	5.712	12.400	0.991
5.744	10.765	20.896	3 3 2	8.065	7.703	5.212	0.995
5.744	10.765	20.896	2 2 4	7.698	7.336	6.008	0.992
5.744	10.765	20.896	2 4 2	7.690	7.328	6.027	0.998
5.753	5.382	10.480	2 2 4	6.428	3.365	9.422	0.990
5.747	5.382	10.458	3 3 5	7.468	4.412	5.313	0.994

a. Experimental error for cell dimensions ± 0.013 cm.
b. 30 x 10^3 neutrons gave a standard deviation of ± 0.005

In another series of experiments,[12] uranium metal cylinders having an average diameter of 11.494 cm, a height of 8.077 cm, and a mass of 15.692 kg were each placed in a graphite block and arranged in an 8-unit cuboidal array. The measured criticality parameters and the calculated k_{eff} are reported in Table A-3. The graphite blocks (ρ = 1.766 g/cm^3) were stacked in contact as a 2 x 2 x 2 arrangement on top of a low-density aluminum framework. The experimental k_{eff} has been corrected for the support structure. The 15.2-cm-thick polyethylene reflector, the last two entries, was located at the boundaries defined by the reported half-cell dimensions.

The calculations of a series of experiments[13] utilizing ^{233}U aqueous nitrate solution in reflected and unreflected arrays is summarized in Table A-4. The nitrate solution contained 333 g U/liter, had a specific gravity of 1.468, and corresponded to an H:^{233}U atomic ratio of 73. The isotopic content of the uranium was 97.54 wt % ^{233}U; 6.47

12. E. C. Crume and J. T. Thomas, Trans. Am. Nucl. Soc. 12, 36 (1969).
13. J. T. Thomas, Trans. Am. Nucl. Soc. 10, 538 (1967).

Table A-3. Experimental and Calculated Criticality Conditions for Eight-Unit Arrays of U(93.2) Metal Cylinders in Graphite.

Half-Cell Dimension[a] (cm)		Polyethylene Reflector Thickness (cm)	k_{eff}	
$a_{x,y}$	a_z		Experimental[b]	KENO k_{eff}
18.54	15.50	0	0.996 ± 0.001	0.997 ± 0.003
16.00	12.95	0	0.993 ± 0.001	0.993 ± 0.005
13.46	10.43	0	0.990 ± 0.001	0.994 ± 0.005
20.13	17.08	15.2[c]	0.9995 ± 0.0004	1.006 ± 0.005
17.59	14.56	15.2[d]	0.9995 ± 0.0004	1.002 ± 0.005

a. Errors on measured dimensions are ± 0.01 cm for unreflected arrays and ± 0.02 cm for the reflected arrays.
b. Based on an assumed β of 0.007.
c. Graphite blocks have dimensions given in second entry of table.
d. Graphite blocks have dimensions given in third entry of table.

Table A-4. Experimental and Calculated Criticality of Unreflected and Reflected Cuboidal Arrays of ^{233}U Uranyl Nitrate Solution.

Number of Units in Array	Polyethylene Reflector Thickness (cm)	Half-Cell Dimensions (cm)		Average Uranium Density (g/cm^3)	KENO k_{eff}
		$a_{x,y}$	a_z		
8	0	10.22	9.56	0.179	0.981 ± 0.006
27	0	12.86	12.29	0.088	0.987 ± 0.006
8	15.2	15.98	15.18	0.046	1.000 ± 0.006
27	15.2	20.52	19.29	0.022	0.995 ± 0.006

ppm ^{232}U; and 1.047, 0.026, 0.001, and 1.386 wt % for the ^{234}U, ^{235}U, ^{236}U, and ^{238}U isotopes, respectively. The solution comprising a unit was contained in a 0.25-mm-thick stainless steel cylinder having an outside diameter of 18.28 cm and an external height of 17.67 cm. The containers had a capacity of 4.63 liters but contained only 4.30 liters of solution determined by weight.

Critical experiments with plutonium metal cylinders arranged in three dimensional arrays were conducted at the Lawrence Radiation Laboratory.[14,15] Two of the assemblies are described in Table A-5 along with their calculated neutron multiplication factors. The nominal 3 kg Pu cylinders were contained in aluminum cans. The details of the assembly are given in Ref. 15. The support structure was considered in the calculation.

Table A-5. Experimental and Calculated Criticality of Plutonium Metal[a] Arrays.

Number of Units in Array	Half-Cell Dimensions (cm) a_x, a_y	a_z	KENO k_{eff}
2^3	2.70	3.65	0.996 ± 0.003
2^{3b}	2.87	3.82	0.992 ± 0.004

a. Plutonium Unit: 3.026 kg Pu at 19.54 g/cm^3, radius and height are 3.265 and 2.315 cm, respectively. The isotopic content of the plutonium was 93.56 wt % ^{239}Pu, 5.87 wt % ^{240}Pu, 0.46 wt % ^{241}Pu, and 0.01 wt % ^{242}Pu. See Ref. 15 for assembly details.

b. One side of the array was reflected by a polyethylene slab 45.0 cm high by 34.3 cm wide by 20.2 cm thick and was not placed at the cell boundary but was spaced 0.43 cm from the surface of the plutonium.

Calculations of Systems Containing Concrete

Although the albedo of concrete is greater than that of water, it also has a greater transmission for neutrons. This latter property is of particular concern since it can result in greater neutron coupling between adjacent arrays when separated by structural thicknesses of concrete than by water.

14. H. F. Finn and N. L. Pruvost, "Livermore Plutonium Array Program," Proceedings of the Livermore Array Symposium, CONF-680909, p. 108 (1968).

15. J. R. Morten III, et al., "Summary Report of Critical Experiments Plutonium Array Studies, Phase I," UCRL-50175, Lawrence Radiation Laboratory (1966).

The number of critical experiments utilizing concrete as a material is not very large. It is unfortunate that none of the experiments provides a sensitive measure of the influence of transmitted neutrons on the multiplication factor of the assemblies, although its superiority as a neutron reflector is clearly demonstrated. Experimental data are necessary to establish the reliability of the calculative methods. It does not follow that if the multiplication factor observed experimentally is reproduced by the calculation, the neutronics in the calculation have been properly handled. However, the reliability of relative comparisons that may be made among a set of the calculations will be augmented. An additional difficulty, considering the applicability of such calculations to nuclear criticality safety, is the difference in compositions of concrete encountered in locales. It would be a tremendous task to describe the various concretes used throughout the country and more difficult still to explore and catergorize those properties significant to criticality calculations. To circumvent these difficulties, "Oak Ridge concrete" was adopted as the reference material used in the calculations. This "concrete" has been extensively used in shielding experiments and their calculations and is adequately described in the literature.[16]

The ability of the KENO Monte Carlo code, utilizing the Hansen-Roach 16-group neutron cross-section sets, to reproduce the multiplication factor of experimental assemblies having concrete as a material is summarized in Tables A-6 and A-7. One series of experiments conducted at the Los Alamos Scientific Laboratory,[17] used U(93.2) metal discs of 26.67-cm radius in contact with and separated by 20.3-cm-thick concrete 68.5 cm in radius. The calculations are summarized in Table A-6. Interpreting the calculative results at 95% confidence level and that the calculation employed properties of Oak Ridge concrete in place of those of the experimental concrete result in a favorable comparison.

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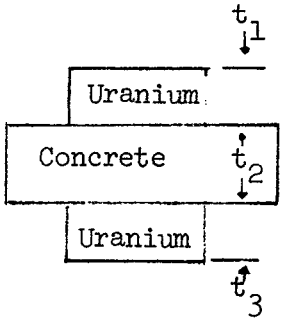
16. R. E. Maerker and F. J. Muckenthaler, Nucl. Sci. Eng. 26, 340 (1966).
17. T. G. McCreless et al., Trans. Am. Nucl. Soc. 8, 441 (1965).

Table A-6. KENO Monte Carlo Calculated Critical Experiments of U(93.2) Metal Discs of 26.67 cm Radius Separated by Concrete.

	Thickness of Material (cm)			KENO Computed[b]
	t_1	t_2[a]	t_3	k_{eff}
	0.00	20.3	4.58	0.979 ± 0.005
	2.40	20.3	4.43	0.993 ± 0.006
	2.70	20.3	4.40	0.988 ± 0.005

a. Concrete was 68.5 cm radius at a material density of 2.13 g/cm^3; unspecified composition, from Ref. 16.
b. Concrete assigned properties of Oak Ridge Concrete.

Table A-7. Experimental and Calculated Criticality of an Aqueous Plutonium Nitrate Sphere Reflected by Concrete.[18]

	Computed k_{eff}			
	KENO		ANISN	
Cross-Section Sets	Oak Ridge Concrete	Portland Concrete	Oak Ridge Concrete	Portland Concrete
Pu-293, Pu-240	1.0010 ± 0.0057	1.0096 ± 0.0053	1.0177	1.0176
Pu-239-13, Pu-240-16	1.0222 ± 0.0061	1.0074 ± 0.0060	1.0196	1.0196

Material Atomic Number Densities (x 10^{24})

	25.4 cm Thick Concrete		29.6 g Pu/cm³ as $Pu(NO_3)_4$ Sphere of 14.698 cm Radius	
Element	Oak Ridge 2.13 g/cm³	Portland Common 2.35 g/cm³	Element	N
H	8.51-3	1.386-2	H	6.3264-2
C	2.021-2	--	N	1.1467-3
O	3.560-2	4.608-2	O	3.4770-2
Na	--	1.747-3	^{239}Pu	7.120-5
Mg	1.880-3	--	^{240}Pu	3.400-6
Al	6.000-4	1.715-3	0.112-cm-thick stainless-steel shell	
Si	1.680-4	1.663-2	Cr	1.674-2
Cu	1.112-2	1.521-3	Fe	6.408-2
Cr	--	3.700-4	Ni	6.590-3
Fe	1.700-4	--		

18. R. C. Lloyd et al., Nucl. Sci. Eng. 25, 165 (1966).

An experiment conducted at Battelle Northwest Laboratory used concrete as a reflector about a sphere of plutonium nitrate solution. Portland common concrete at a density of 2.30 g/cm^3 having a thickness of 25.4 cm was used. The experiment, materials, and calculations are described in Table A-7. The calculation was performed by the ANISN code as well as KENO and for two cross-section sets. The results are considered to be in good agreement.

Accepting the combination of the Monte Carlo code and the Hansen-Roach neutron cross-section sets as producing valid multiplication factors representative of criticality, there remains a need to reduce the computing time characteristically encountered when hydrogenous materials are present in a computation. The differential albedo technique developed and first applied to water reflectors in criticality calculations[19] was extended to finite thicknesses of concrete. It was first necessary to determine the information required to give proper consideration to the fraction of neutrons transmitted through various thicknesses of concrete. The combination of neutron albedo and transmission effects resulted in an albedo-transmission (A-T) approach that treats as rigorously as possible the neutron transport through a moderator without actually performing the neutron tracking each time a neutron enters the moderator. There are two assumptions which can influence the accuracy of the calculation. The first assumes that the moderator be a rectangular parallelepipedel shell at the exterior of the cell. The second, that the thickness of the shell is sufficiently small with respect to its other dimensions to assure that the radial dispersal of a neutron, either being reflected or traveling through the shell, will be small. If these two conditions are met, other necessary assumptions do not significantly affect the results.

The information needed to carry out the A-T calculation is generated with a one-dimensional S_n-type calculation. The "fixed source" option is used with a source neutron incident at the left boundary of a slab of the moderator material at given polar angle and with a given energy.

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19. G. E. Whitesides and J. T. Thomas, Trans. Am. Nucl. Soc. 12, 889 (1969).

The source is normalized to one source neutron incident on the slab and the code produces the angular and energy distribution of neutrons which return from the slab and the similar distributions of those traversing and exiting at the right boundary of the slab. Since the input source is one neutron, the integral over the outward angles and energy at the left boundary gives the probability that a neutron will return from the slab. A similar integral at the right boundary yields the probability that a neutron will pass through the slab. Information generated for neutrons entering at all angles and all energies forms a table of probabilities used in the A-T calculation.

A special version of KENO was written which allows the A-T treatment to be applied to arrays of fissionable systems in such a manner as to allow the calculation of arrays of subarrays with a specified thickness of a given interspersed moderator between the subarrays. The subarrays may consist of any number of units in each coordinate direction while the array may have any number of subarrays in each coordinate direction. The special version of KENO also permits the simulation of an external reflector around the array by means of the albedo portion of the A-T treatment. The external reflector need not be the same material as the interspersed moderator.

The validity of the A-T technique was established by a comparison of KENO calculations made with actual neutron tracking in concrete to those employing the A-T data. A 27-unit subarray of U(93.2) metal cylinders each having a radius of 5.76 cm, equal height and diameter, and a mass of 22.5 kg U was enclosed in 4-, 8-, and 12-in.-thick concrete. Each system of these subarrays was representative, then, of volumes separated and reflected by concrete of equal thickness. The arrangements considered and the results of the comparative calculations are presented in Table A-8. The A-T technique appears to be quite acceptable in reproducing the actual tracking results and is 3 to 10 times faster depending upon the concrete thickness employed.

Table A-8. Comparison of Computed Array Multiplication Factors Utilizing Actual Neutron Tracking in Concrete and the Albedo-Transmission Representation for Various Concrete Thicknesses.

Subarray: 27-unit array of 22.5 kg U(93.2) metal cylinders, $r = 5.76$ cm, $h = 2r$ centered in 42.66 cm cubic cells.

Subarray Arrangement	Effective Array Multiplication Factor			
	Albedo-Transmission		Actual Neutron Tracking	
	k_{eff}	$\sigma \times 10^3$	k_{eff}	$\sigma \times 10^3$
	4-in.-thick Concrete			
1 x 1 x 1	0.897	9	0.901	8
1 x 2 x 1	0.924	6	0.929	4
2 x 2 x 2	1.050	6	1.052	4
∞ x ∞ x ∞	1.530	6	1.560	8
2 x 2 x 2[a]	1.130	7	1.120	5
	8-in.-thick Concrete			
1 x 2 x 1	0.999	5	0.999	4
2 x 2 x 1	1.031	4	1.033	5
2 x 2 x 2	1.073	5	1.078	4
∞ x ∞ x ∞	1.220	6	1.223	4
2 x 2 x 2[a]	1.077	5	1.085	7
	12-in.-thick Concrete			
1 x 2 x 1	0.996	5	0.996	4
2 x 2 x 1	1.005	6	1.018	4
2 x 2 x 2	1.025	6	1.025	4
∞ x ∞ x ∞	1.092	7	1.082	4
2 x 2 x 2[a]	1.032	5	1.031	6

a. These arrays were additionally reflected by 15-cm-thick paraffin.

APPENDIX B

NB_N^2-METHOD AND APPLICATIONS

It has been observed[1,5,20] from both experiments and Monte Carlo calculations that cuboidal arrays, of identical units, having the same neutron multiplication factor may be related by the product of the number of units in an array, N, and the simple geometric buckling for cubic geometry, B_N^2.

The following simple heuristic reasoning based on a monoenergic neutron behavior suggested the constancy of product NB_N^2 for a given fissile material. In unreflected assemblies of subcritical units one is dealing entirely with leakage neutrons when adjusting spacing and number of units to criticality. The achievement of criticality establishes a balance between the fraction of neutrons absorbed in the fissile material and that fraction lost to the assembly by leakage. This balance must persist in an assembly of N-units as it does for a single unit of the fissile material having the same multiplication factor. Now, in a system with large N, the average fissile material density, ρ, is correspondingly small. Were such a large system homogenous, the neutron production factor per unit volume would be uniform and the multiplication factor would be expressible as the product of the production term and a term representing the neutron nonleakage fraction. A large homogeneous system at low density reasonably may be expected to have its neutron nonleakage fraction expressible as

$$\frac{1}{1 + M_N^2 B_N^2}$$

where B_N^2 is a geometric buckling and M_N^2 is a neutron migration area for the fissile material. A consequence of these assumptions is that two different systems having the same neutron leakage fraction would have

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20. J. T. Thomas, "Criticality of Large Systems of Subcritical U(93) Components," ORNL-CDC-1, Oak Ridge National Laboratory (1967).

$$M_N^2 B_N^2 = M_{N'}^2 B_{N'}^2 \quad .$$

The migration area can be shown to have a dependence on the density, expressible as an inverse square law. In systems where the fissile material is lumped into units, however, the examination of the dependence of N on the average fissile material density, ρ, for experimental and calculated arrays indicate that the density exponent, s, asymptotically approaches a value of minus 2 as a limit. If, finally, the assumption is made that the migration area has the same density dependence as does N, because of discrete units instead of homogeniety, then it follows that

$$M_N^2 = c\ N$$

where c is a dimensional constant that is taken as unity (since the intended application is relative rather than absolute). The resulting conclusion is that two different assemblies of the same units have

$$NB_N^2 = N'B_{N'}^2 \quad .$$

This relation is verified by the set of four unreflected critical experimental arrays utilizing five-liter volumes of $U(92.6)O_2(NO_3)_2$ solution.[21] Any two of the arrays determine the constants NB_N^2 and the result is a favorable correlation of the set of experiments. This set of experiments is sufficient also to demonstrate that the simple direct equating of geometric bucklings of the arrays does not relate the criticality of different numbers of units.

The constants necessary for the application of the NB_N^2 method have been determined from other experimental arrays[1,5,20,22] and used to describe still different arrays which, in turn, were computed by validated Monte Carlo codes.

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21. J. T. Thomas, "Critical Three-Dimensional Arrays of Neutron-Interacting Units," ORNL-TM-719, Oak Ridge National Laboratory (1963).
22. J. T. Thomas, "The Effect of Unit Shape on the Criticality of Arrays," ORNL-CDC-4, Oak Ridge National Laboratory (1967).

Water-Reflected Cubic Arrays

A number of interesting results derive from the constancy of NB_N^2 when fixed cell sizes are considered. Beginning with the expression for cubic arrays,

$$NB_N^2 = \frac{3\pi^2 n^3}{(2a_n n + 2\lambda)^2} , \qquad (B1)$$

which implicitly contains a complete physical description of the array configuration, the denominator is the square of a dimension of an extended edge of the array displayed as the number of cells, n, times the cell dimension $2a_n$ plus twice a parameter, λ, normally interpreted as an extrapolation distance of the array. Equation (B1), then, describes a volume occupied by n^3 units spaced on $2a_n$ centers. Unlike the typical problem: given units of radius r of a particular fissile material find the number required for criticality as a function of the spacing, the problem from the present viewpoint is to find the unique radius, r, of the unit that will result in criticality for the otherwise specified system. Clearly, the value of the coupled constants NB_N^2 and λ can have no dependence on the fissile material producing the desired multiplication factor. These constants are singularly geometry dependent subject to the correlating constraint of a chosen multiplication factor for the arrays, which, in turn, is a function only of the radius, r, of fissile material.

The constants, NB_N^2 and λ, are determined from any two systems of the same units having the same array multiplication factor, either experimental or calculated by a validated method. Other arrays of different N, then, will have spacings given by Eq. (B1) and the evaluated constants. The procedure may be carried out for units of various masses to characterize the fissile material at the chosen value of k_{eff}. In addition, the characterization of systems described by Eq. (B1) also is obtained, i.e., an implicit prescription of necessary adjustments to the neutron leakage fraction for corresponding changes in N and a_n. The fixed geometric pattern relating N and a_n may be exhibited by rewriting Eq. (B1) as

$$NB_N^2 = \frac{3\pi^2 n}{4\, a_n^2} \left[1 - \sqrt{\frac{4\,\lambda^2 NB_N^2}{3\pi^2 N}} \right]^2 . \qquad (B2)$$

Since there is no dependence on the type of fissile material, the quantity $\left\{ \frac{\lambda^2\, NB_N^2}{3\pi^2 N} \right\}^{\frac{1}{2}}$ can depend only on N. The implications, then, are that $\lambda^2 NB_N^2$ must be a constant or its influence in determining spacings is negligible. This disjunctive is probably not easily resolved because the data under consideration are derived by statistical processes.

It seems unnecessary but it will be remarked that Eq. (B2) is not an approximation to Eq. (B1).

Equation (B2) states that for a constant n, the parameter NB_N^2 varies as the inverse square of the half dimension of the cubic cell as is shown in Fig. 2 of the text for the data from Table 1. Presented in Table B-1 are data from Table 2 of the text giving the number of units and associated constants defining criticality for metal spheres of U(93.2), Pu(100)[a] and ^{233}U The values given for λ have been evaluated from Eq. (B2) and provide the average value for the product of λ^2 and NB_N^2 as

$$\lambda^2 NB_N^2 = 0.720 \pm 0.004 .$$

Equation (B2) may be written, therefore, as

$$NB_N^2 = \frac{3\pi^2 n}{4\, a_n^2} \left(1 - \frac{C}{\sqrt{N}} \right)^2 \qquad (B3)$$

where C is evaluated from

$$C = + \sqrt{\frac{4\lambda^2\, NB_N^2}{3\pi^2}} = 0.312 \pm 0.001 .$$

[a] Notation Pu(x) indicates the plutonium contains x wt % ^{239}Pu and (1 - x) wt % ^{240}Pu.

Table B-1. Array Constants NB^2_N and λ Determined for Water-Reflected Arrays Calculated by KENO Monte Carlo Code. The corresponding computed critical sphere radii for these three fissile materials are given.

Number of Units Along an Array Edge	Half-Cell Dimension a (cm)	Array Constants NB^2_N (cm^{-2})	Array Constants λ (cm)	U(93.2) ρ (g/cm^3): 18.76 r_o (cm): 8.77 Ratio of Radii of Critical Unit to Single Critical Sphere r_c/r_o	U-233 18.40 5.89	Pu-239 19.70 4.90
4	12.7	0.1695	2.07	0.606	0.684	0.749
6	12.7	0.2638	1.65	0.542	0.622	0.694
8	12.7	0.3571	1.42	0.499	0.580	0.648
10	12.7	0.4499	1.27	0.467	0.544	0.606
4	15.24	0.1177	2.48	0.658	0.742	0.800
6	15.24	0.1832	1.98	0.600	0.676	0.743
8	15.24	0.2480	1.70	0.549	0.637	0.700
10	15.24	0.3125	1.51	0.513	0.596	0.665
4	19.05	0.0754	3.07	0.738	0.804	0.854
6	19.05	0.1172	2.50	0.662	0.746	0.799
8	19.05	0.1587	2.13	0.619	0.699	0.760
10	19.05	0.2000	1.88	0.581	0.667	0.730
4	22.86	0.0523	3.73	0.784	0.851	0.901
6	22.86	0.0814	2.99	0.721	0.791	0.848
8	22.86	0.1102	2.57	0.670	0.759	0.813
10	22.86	0.1389	2.25	0.642	0.720	0.784
4	25.4	0.0424	4.13	0.816	0.879	0.910
6	25.4	0.0660	3.24	0.754	0.818	0.873
8	25.4	0.0893	2.85	0.712	0.796	0.841
10	25.4	0.1125	2.51	0.669	0.707	0.800
4	30.48	0.0294	5.02	0.857	0.916	0.935
6	30.48	0.0458	3.96	0.807	0.868	0.893
8	30.48	0.0620	3.40	0.763	0.825	0.874
10	30.48	0.0781	3.06	0.731	0.804	0.854
4	40	0.0171	6.45	0.906	--	--
4	60	0.0076	9.67	0.955	--	--
4	100	0.0027	16.57	0.980	--	--

The constant C in Eq. (B3) reproduces the lines shown with the data in Fig. 2 of the text. The constant is considered as being applicable to calculated arrays having spherical units and having neutron spectra representative of the arrays described in Table 2. The application of Eq. (B3) to other arrays and unit shapes or to other neutron spectra should be examined by Monte Carlo calculations to verify the result, i.e., the present demonstration does not, a priori, validate its application to other array configurations, unit shapes, or neutron spectra.

Although the method may be used with arrays having n = 2 or 3 to define the necessary constants, it is not considered prudent to infer parameters for arrays having high fissile-material density from values based on calculations for $n \geq 4$. The foregoing discussions have concerned water-reflected arrays with $n \geq 4$. Similar treatment of unreflected arrays may be made.

Neutron Multiplication Factor Constraint

A result of Eq. (B3) is the elimination of the need to determine the separate constants NB_N^2 and λ. A single calculated array of spheres of given properties serves to define other arrays of different numbers of the same units. Equation (B3) applied to two arrays of the same unit may be written as

$$a_n = a_{n_o} \frac{n_o}{n} \left(\frac{\sqrt{N} - C}{\sqrt{N_o} - C} \right) \qquad (B4)$$

where a_{n_o} is the half cell dimension of the known array. The use of this relation with any entry of Table 2, for example, is sufficient to define other water-reflected arrays of the specified unit.

Consideration has been given thus far to arrays having neutron multiplication factors of unity, except for the few cases shown in Fig. 3 that established the influence on the multiplication factor of reductions in the radius of the unit. The application of Eq. (B4) to subcritical arrays will now be demonstrated, thus removing the constraint requiring array criticality. As stated previously, the only constraint on Eq. (B1) and the subsequent results is that arrays have the same neutron

multiplication factor. The results for arbitrarily chosen subcritical arrays are summarized in Table B2. The reference arrays where selected from those given in Fig. 3 and the spacing a_n of the same units in arrays of different size were determined by Eq. (B4). These spacings and the corresponding KENO-calculated multiplication factors are given in the right hand column of the table. In view of the statistical results there is no apparent reason to restrict the application of the NB_N^2-method to arrays having multiplication factors of unity. The radius of the unit alone determines the array multiplication factor while Eq. (B4) appears to properly characterize the leakage fraction for arrays of the same units.

Table B-2. Validation of Neutron-Multiplication-Factor Constraint in NB_N^2 Method by Monte Carlo Calculations of Water-Reflected Arrays.

Fissile Material	Sphere Radius r (cm)	N_o, Reference Array a_{n_o} (cm)	$k_{eff} \pm \sigma \times 10^3$		N-Array from Eq. (B4) a_n (cm)	$k_{eff} \pm \sigma \times 10^3$	
$^{233}UO_2$ ($H/U \cong 3$)	4.883	$a_6 = 19.05$	0.6606	6.6	$a_{100} = 79.43$	0.6499	7.2
	6.241	$a_6 = 19.05$	0.8901	7.2	$a_{100} = 79.43$	0.8955	6.2
^{239}Pu Metal	3.724	$a_4 = 15.24$	0.9427	5.2	$a_6 = 19.01$	0.9344	5.2
					$a_8 = 22.12$	0.9439	5.0
					$a_{10} = 24.83$	0.9463	5.8
U(93.2) Metal	6.444	$a_4 = 25.4$	0.9075	5.7	$a_{50} = 93.37$	0.9048	5.0
	5.012	$a_4 = 25.4$	0.7079	5.2	$a_{50} = 93.37$	0.6991	4.9
$^{239}PuO_2$ ($H/Pu \cong 0.4$)	4.451	$a_8 = 15.24$	0.8787	6.5	$a_{100} = 54.62$	0.8559	6.3
	4.036	$a_8 = 15.24$	0.7819	6.0	$a_{100} = 54.62$	0.7613	5.1
	3.532	$a_8 = 15.24$	0.6633	4.9	$a_{50} = 38.60$	0.6689	5.4
	2.523	$a_8 = 15.24$	0.4414	5.0	$a_{50} = 38.60$	0.4346	4.9