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$$\sigma(m) = c_2 (m_0 - m) = \frac{nm}{d^2} (1 - c/\sqrt{N})^2 \quad . \tag{1}$$

In these relations, $\sigma(m)$ is a limiting surface density ($g - cm^{-2}$); m_0 is the unreflected critical mass in the geometry of the unit; n^3 is N; c is a constant characterizing the geometry of center spaced units and equals 0.55; c_2 is a constant dependent on the type of fissile material and is influenced by the unit shape, by the array shape, and by the array reflector material.

For cuboidal arrays of cubic cells, the mass m required for criticality satisfies the equation

$$\left(\frac{m_0}{m} - 1\right) = \left[\frac{11.924(\sqrt{N} - c)}{nd}\right]^2 (5R^{-0.672} - 1) \ , \tag{2}$$

where the array shape is represented by the parameter $R = \frac{N^{1/3}}{3} \sum_1^3 \frac{1}{n_i}$ and n_i is the number of units along the three directions of the reflected cuboidal array The value of R should not exceed 5.34. The definition of criticality by these equations has been determined as conservative below $m = 0.1\ m_0$ and as very good for greater values of m representing criticality to within 1% in k_{eff}.

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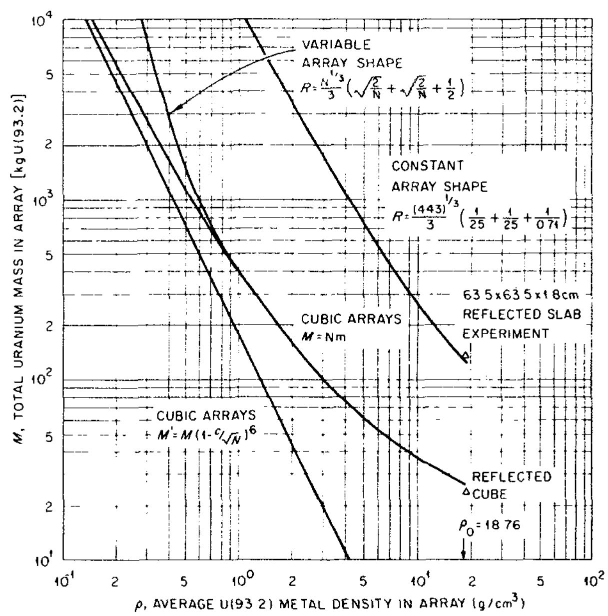

Fig. 1. Density analog representation of reflected array criticality for 20 kg U(93.2) metal spherical units.

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As fuel dissolves, heat is liberated in the [illegible] follows:

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For 5% enriched fuel at 15 g ^{235}U/liter concentration [illegible] heat release is ~28 cal/liter. Thus, for [illegible] operated at 80 to 85°C, cooling must be maintained [illegible] prevent boiling.

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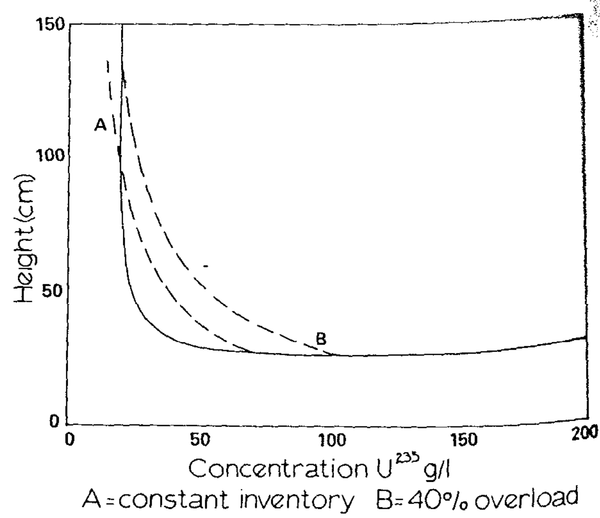

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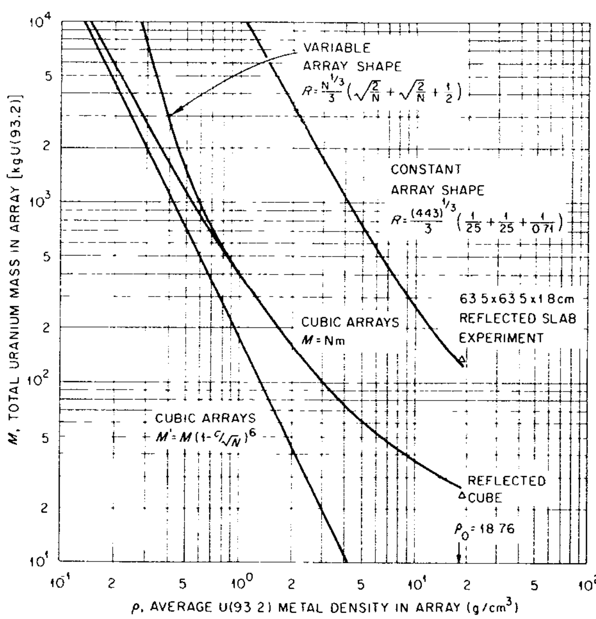

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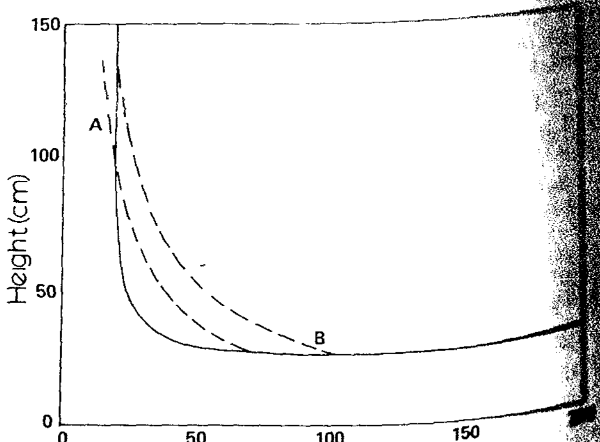

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