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EMPIRICAL METHOD FOR CALCULATING PIPE INTERSECTIONS CONTAINING FISSILE SOLUTIONS

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The authors also express appreciation to Howard W. King and Grover Tuck of Nuclear Safety for a comprehensive review and comments.

EMPIRICAL METHOD FOR CALCULATING PIPE INTERSECTIONS CONTAINING FISSILE SOLUTIONS

Bruce B. Ernst and C. L. Schuske

Abstract. An empirical method has been developed for calculating safe nuclear criticality parameters for complex arrays of intersecting cylinders (pipes or arms) containing enriched uranyl-nitrate solutions.

The critical parameters defined by this method include cylinder diameters, angles of intersection, cylinder spacings, and the total number of intersecting cylinders involved in arrays.

Discussed also are applications to typical problems encountered in fissile processing plants.

INTRODUCTION

Frequently, the designer of fissile processing plants and process equipment for such plants is confronted with the problem of complex piping systems. In the past because of lack of critical data, the criticality specialist circumvented such situations whenever possible, or made use of conservative approximations to pipe intersections.

A model has been developed by means of curve-fitting methods applied to the critical data reported recently by B. Ernst.[1] The critical data were obtained on intersecting cylindrical geometries and utilized aqueous solutions of uranyl nitrate at about 93 percent of uranium 235 (^{235}U) isotopic content. The aqueous solution had a density of 450.8 grams of ^{235}U per liter. The purpose of the model is to facilitate rapid analysis of intersection problems commonly found in the fissile process plant. In the formulation of the model, sufficient (but not over) conservatism is included to prevent penalizing designers of such equipment.

Two examples of use of the model are illustrated, together with experimental data as obtained.

[1]Bruce B. Ernst. *Critical Parameters of Bare Intersecting Pipes Containing Uranyl Nitrate Solution.* RFP-1196. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. (In Press.)

Definitions:

CENTRAL COLUMN – The main column or cylinder from which branching of arms occurs.

ARMS – Any pipe or cylinder intersecting the central column.

CONTACT AREA – The area subtended by an arm and another arm or an arm and the central column. (See Figure 1, where D = diameter; angles are theta (θ) and cosecant θ; and A = area.)

QUADRANT – Quadrant is a sector of a cylinder 18 inches long; where alpha (α) equals 90°. The quadrant is shown by the shaded area in Figure 2.

EXPERIMENTAL DATA

The critical parameters of aqueous uranyl nitrate filled cylindrical geometries reported by Ernst[2] are given in Tables I, II, and III. (Data shown have not been corrected for experimental error.)

Because of the complex nature of these geometries (arrays), a column of each Table identifies a specific illustration of that geometry in the text. For example, in Table I, note Figures 3 and 4; in Table II, Figures 4, 5, 6, 7, and 8; and in Table III, Figures 9, 10, and 11. The approach was used in place of providing a lengthy description of each array. In all arrays, the central column was made of a ⅛-inch thick stainless steel pipe of square cross section. The internal dimensions of the square column were 7.0 by 7.0 inches.

All experiments are considered to have minimal reflection because they were performed at least 4 feet above the concrete floor of the critical facility, and at least 10 feet from the nearest wall. No other reflecting surfaces of consequence were near, with the exception of the actual vessel walls. The

[2]*Ibid.*

TABLE I. Critical Parameters for Arrays of Arms Intersecting the Central Column [Theta (θ) = 90°].

Critical Vertical Edge-to-Edge Spacing of Arms along Central Column (inches)	Critical Number of Arms in the Array	Critical Solution Height (H_c) along Column and above Top Arm in Array (inches)	Identifying the Experimental Array (Figure No.)	*Value of (a) (inches)
(Inner Diameter Arms, 6.40 Inches; Wall Thickness 0.11 Inches.)				
0.00	5.8	Central Column Full	3	Not Applicable
5.19	8	45.94	3	Not Applicable
3.50	8	0.708	3	Not Applicable
4.00	8	1.97	3	Not Applicable
4.50	8	4.26	3	Not Applicable
6.63	12	Central Column Full	3	Not Applicable
6.63	∞	Central Column Full	4	6.63
(Inner Diameter Arms, 5.35 Inches; Wall Thickness 0.11 Inches.)				
0.00	7.95	Central Column Full	3	Not Applicable
0.25	8	49.59	3	Not Applicable
0.00	8	4.37	3	Not Applicable
2.00	12	3.02	3	Not Applicable
2.13	12	24.55	3	Not Applicable
3.00	16	31.34	3	Not Applicable
2.75	16	Central Column Full	4	4.19
1.75	12	Central Column Full	4	3.44
1.00	12	0.44	4	3.19
(Inner Diameter Arms, 4.34 Inches; Wall Thickness 0.078 Inches.)				
0.00	16.65	Central Column Full	3	Not Applicable

*D = Outer diameter of arms.
S = Surface to surface distance.

$a = \frac{(D + S)}{2}$; (See Figure 4).

FIGURE 1. Surface Area in Contact with Central Column.

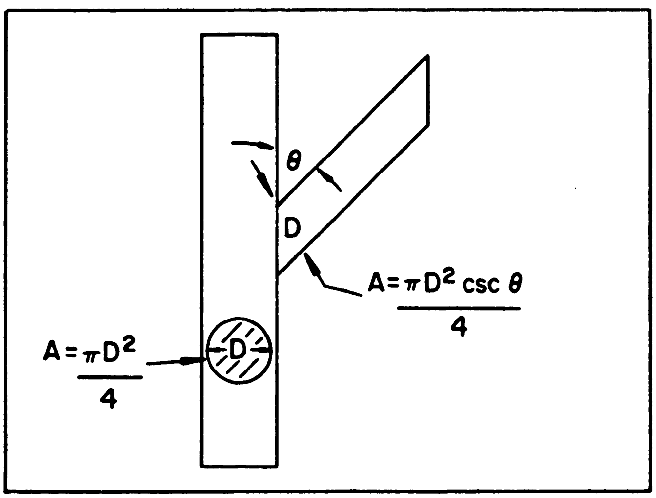


FIGURE 2. Typical Quadrant.

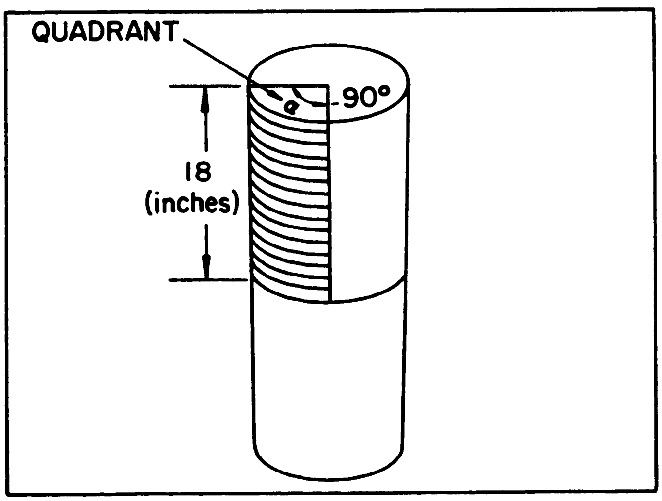


TABLE II. Critical Parameters for Arrays of Arms Intersecting the Central Column [Theta (θ) = 45°].

Critical Vertical Edge-to-Edge Spacing of Arms along Central Column (inches)	Critical Number of Arms in the Array	Critical Solution Height (H_c) along Column and above Top Arm in Array (inches)	Identifying the Experimental Array (Figure No.)
(Square Arms, 7.0 Inches; Wall Thickness, 0.125 Inches.)			
Not Applicable	2	Subcritical with Arms and Central Column Filled	5
Not Applicable	2	4.82	6
(Inner Diameter Arms, 6.40 Inches; Wall Thickness, 0.11 Inches.)			
9.46	6	Central Column Was Filled	7
9.37	6	11.10	7
(Inner Diameter Arms, 5.35 Inches; Wall Thickness, 0.11 Inches.)			
6.16	8	Center Column Was Filled	8
5.81	8	11.97	8
4.81	8	4.78	8

TABLE III. Critical Parameters for Clusters of Arms Intersecting the Central Column [Theta (θ) = 90°].

(Inner Diameter Arms, 6.40 Inches; Wall Thickness, 0.11 Inches.)

Critical Vertical Edge-to-Edge Spacing of Arms along Central Column (inches)	Critical Number of Arms with Central Column Filled	Identifying the Experimental Array (Figure No.)
0	∞	9
0	18.25	10
0	6	11

central column length when filled was essentially infinite. A reciprocal multiplication (1/M) plot of solution height in this column without intersecting arms indicated no measurable multiplication beyond 40 inches of solution height.

All arms used in the experiments were 54 inches long and were effectively infinite for all critical values reported in Tables I, II, and III.

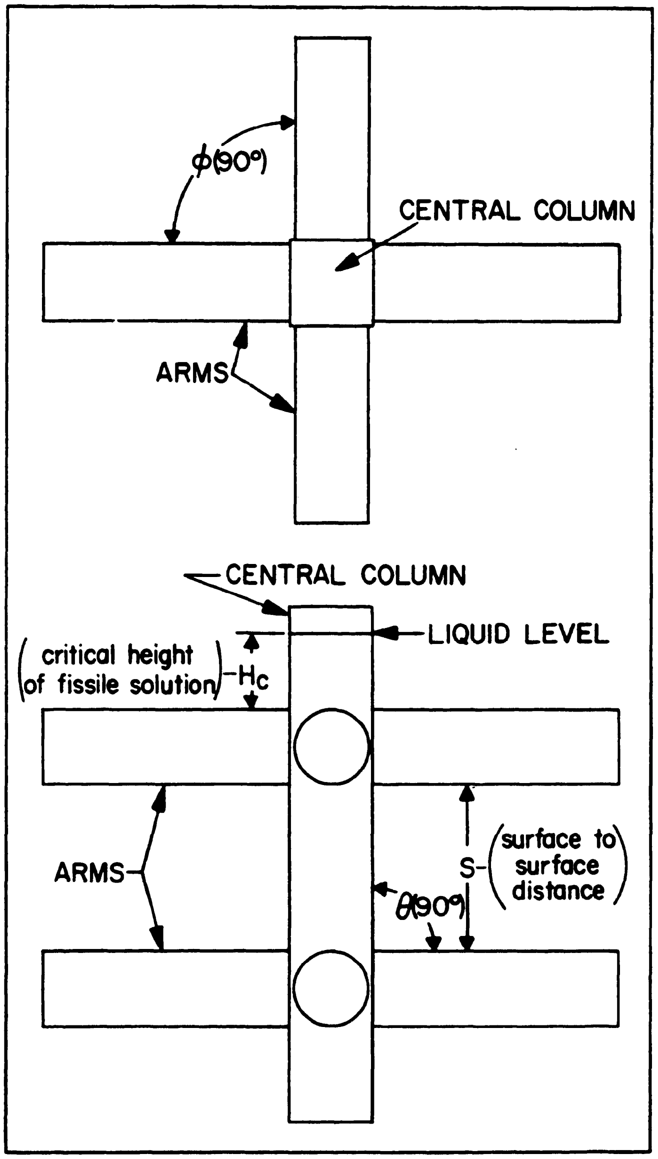
All experimental arrays contained 450.8 grams of ^{235}U per liter solutions. This is desirable, since the minimum critical volume occurs in this concentration region. Thus, these critical data can be considered the limiting cases and can be used conservatively for all concentrations.

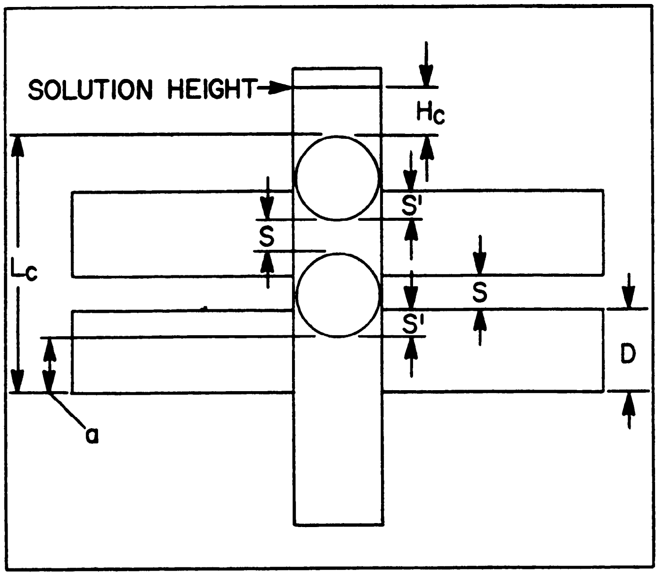
Empirical Analysis of Experimental Data:

In order to develop a calculational method for pipe intersections that will fit a wide range of cases, certain extrapolations of the experimental data were necessary. The method of Schuske and Morfitt[3] was used. The method permitted evaluation of arm edge-to-edge spacings of an infinite array of arms along a central column of infinite length.

[3]C. L. Schuske and J. W. Morfitt. "An Empirical Study of Some Critical Mass Data." Y-533. Union Carbide Corporation, Oak Ridge, Tennessee.

FIGURE 3. Typical Experimental Geometry.





Legend

S, S′ – Surface to surface distance.
L_c – Distance.
H_c – Critical height of fissile solution.
a – Diameter minus overlap.
D – Cylinder diameter.

FIGURE 4. Typical Offset Arm Geometry.

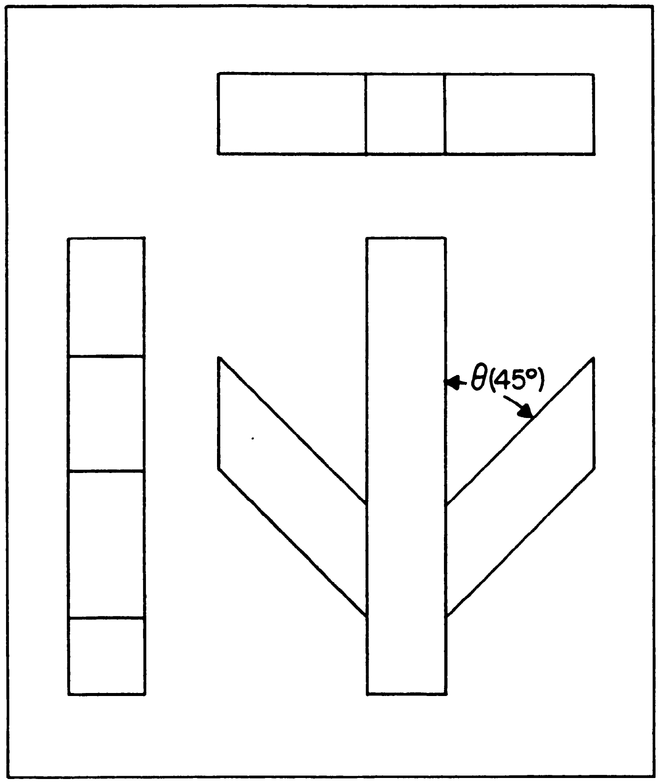


FIGURE 5. Square Arms Intersecting Central Column at Angle Theta (θ) Equal to 45° and Angle Phi (ϕ), 180°.

FIGURE 6. Square Arms Intersecting Central Column at Angle Theta (θ) Equal at 45° and Angle Phi (ϕ), 90°.

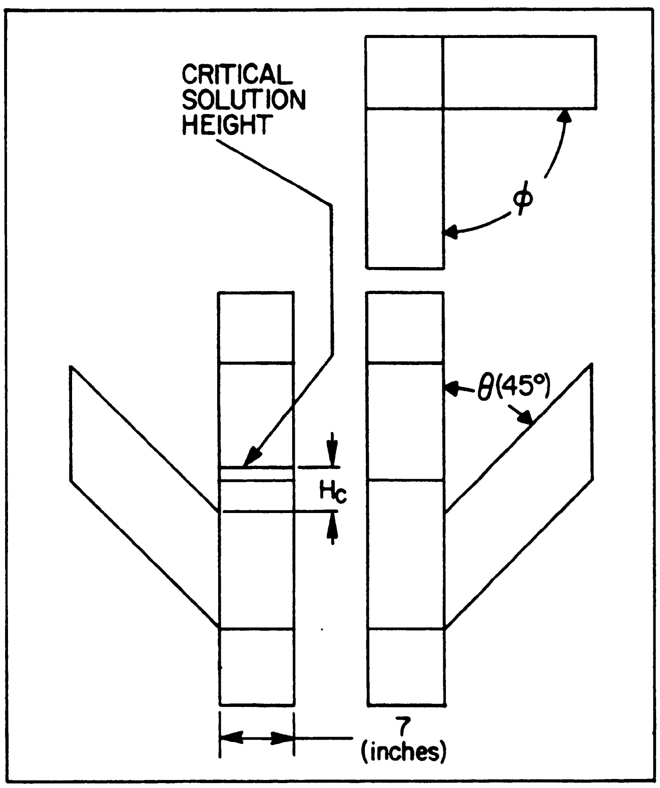
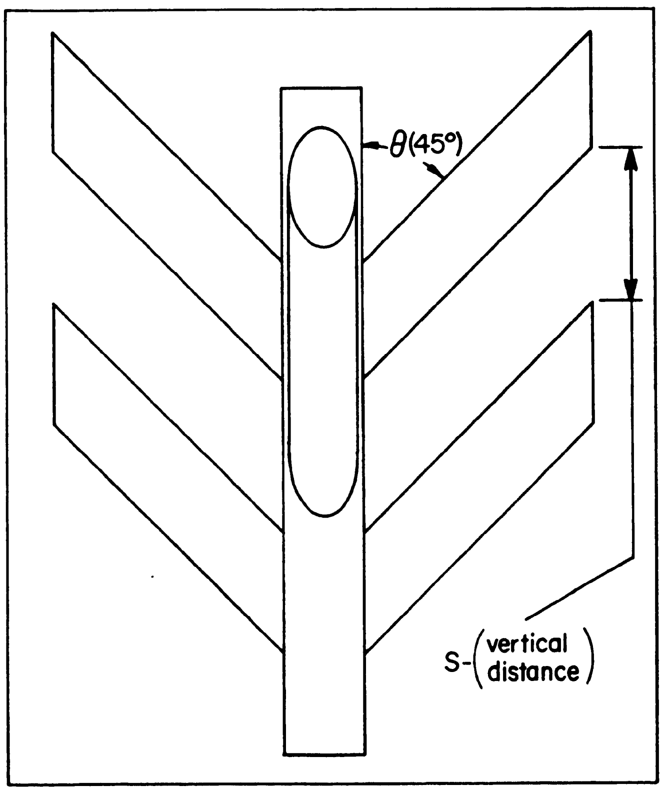


FIGURE 7. Typical Assembly for 6.40-Inch Inner Diameter Arms. Angle Theta (θ) is 45° and Phi (ϕ), 90°.



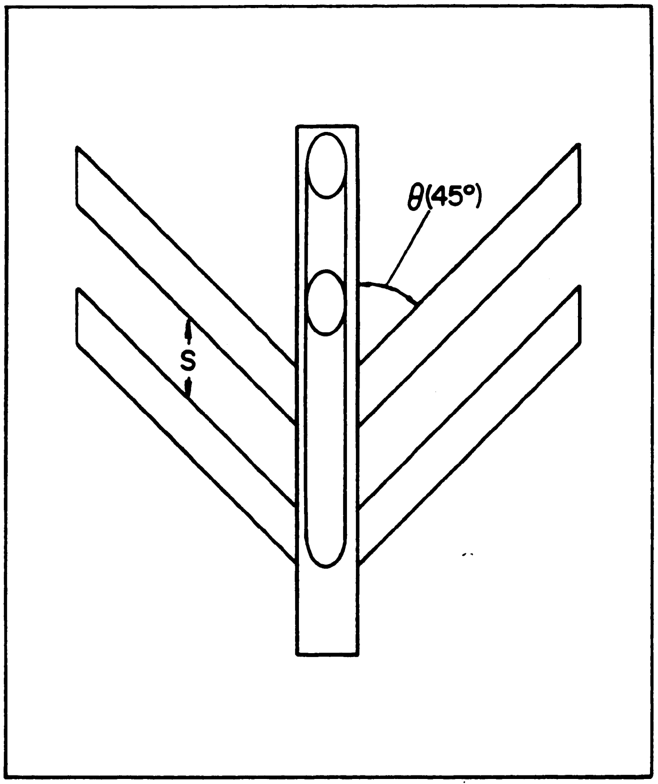


FIGURE 8. Typical Assembly for 5.35-Inch Inner Diameter Arms. Angle Theta (θ) is 45° and Angle (ϕ), 90°.

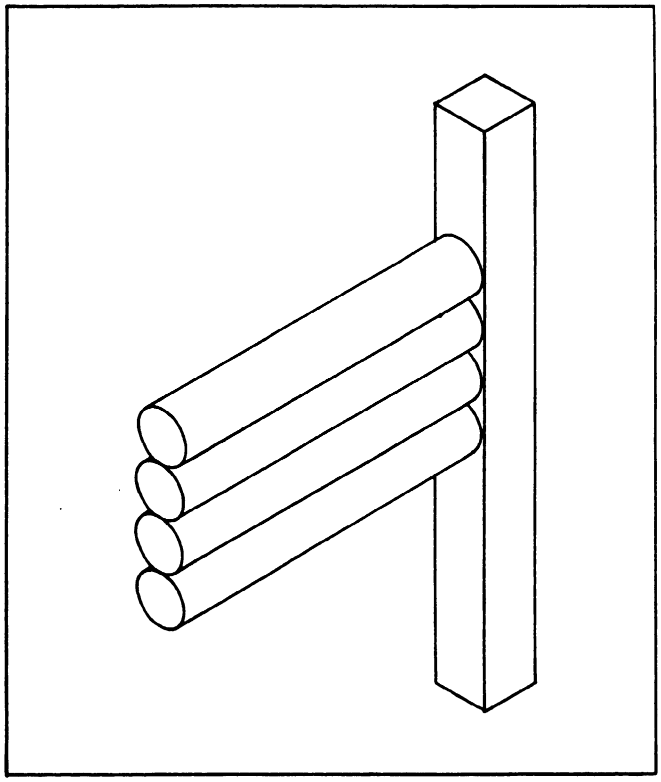
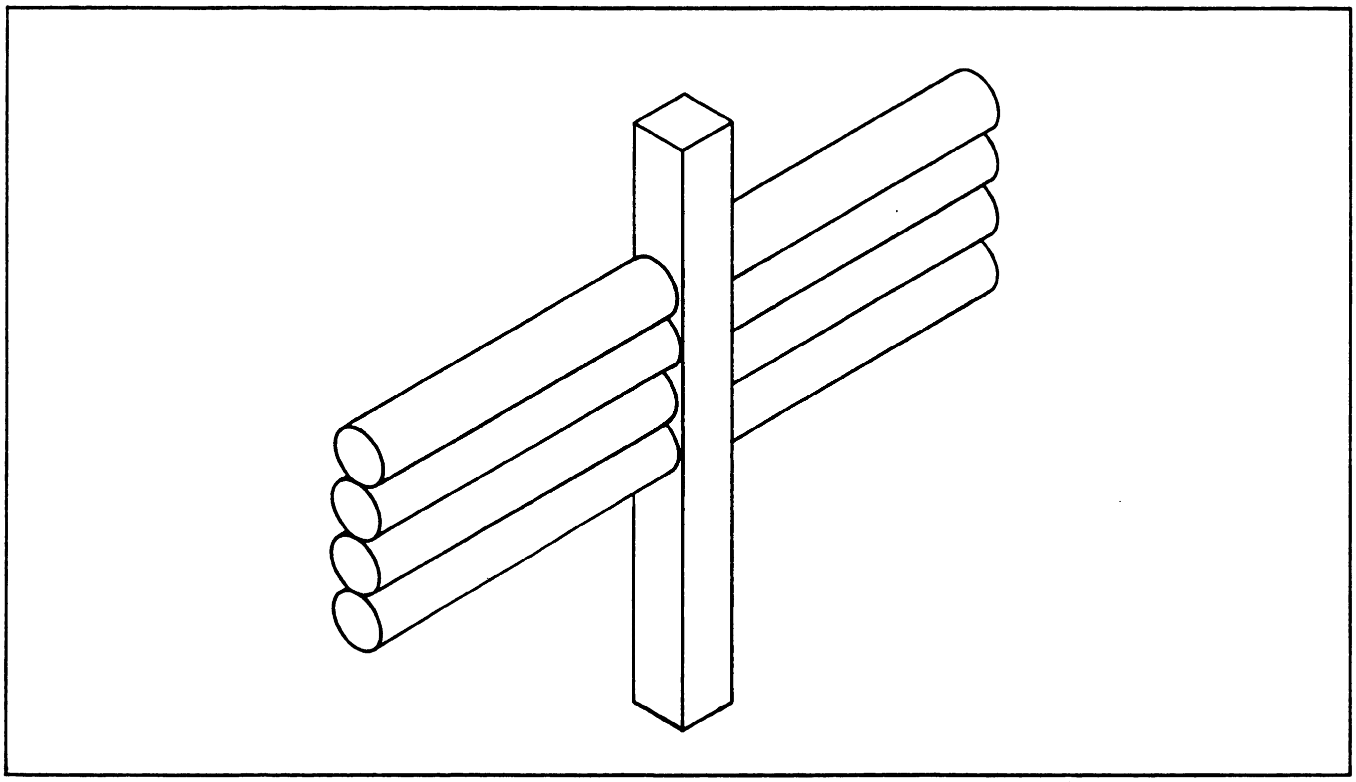


FIGURE 9. Planar Array.

FIGURE 10. Planar Array. Angle Phi (ϕ) is 180°.



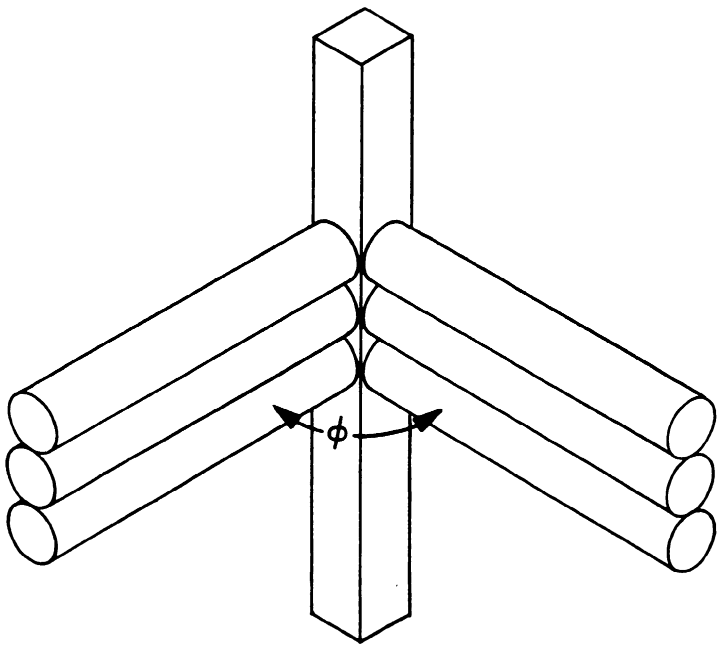


FIGURE 11. Intersecting Planar Array. Angle Theta (θ) is 90° and Phi (ϕ), 90°.

These arrays were previously described in Figure 3, Page 3. The spacings were found to be 7.8 and 5.3 inches, corresponding to arm diameters 6.40 and 5.35 inches, respectively. The technique was also used to determine the arm diameter for an infinite number of arms for zero-inch, edge-to-edge spacings. The value of this limiting cylinder diameter is 3.7 inches.

Refer to Figures 12 and 13 for a graphical description of these data.

Corrections to Experimental Data:

EDGE-TO-EDGE SPACING OF ARMS ALONG CENTRAL COLUMN – A measurement error of ±0.25 inches of edge-to-edge spacing must be applied to all data appearing in Tables I, II, and III. Therefore, increase all edge-to-edge spacings by 0.25 inches.

GAP BETWEEN ARMS AND COLUMN – A maximum gap of 0.125 inches is possible between the intersecting arms and the central column. (Each arm was a completely enclosed vessel to facilitate edge-to-edge spacing changes along the column.)

The gap was converted to a correction on each arm diameter. This correction was evaluated experimentally and is discussed in Appendix A. The magnitude of this correction is 0.28 inches. Therefore, all arms should be reduced by 0.28 inches; *i.e.*, 6.40 inches becomes 6.12 inches, etc.

CENTRAL COLUMN – The dimensions of the central column had an accuracy of ±0.062 inches.

FILL LINES TO THE ARMS – Each arm was connected to the central column by a 0.50-inch fill line. These fill lines supplied some reactivity to the overall system and thus are ignored for reasons of conservatism.

ANGLE THETA (θ) OF ARMS INTERSECTING THE CENTRAL COLUMN – A ±5-degree tolerance was used, therefore for all angles of theta (θ) greater than or less than 90°, increase the contact area by the amount of this tolerance.

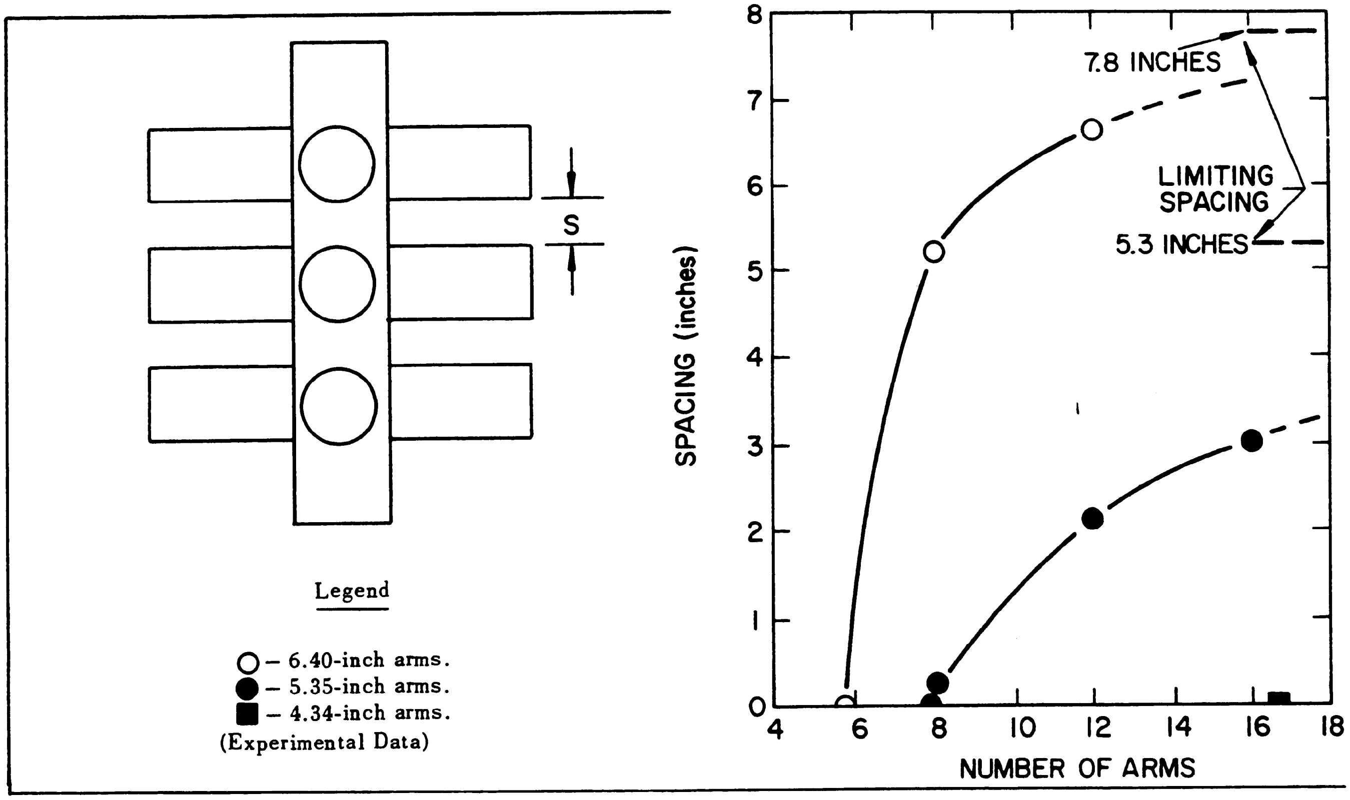
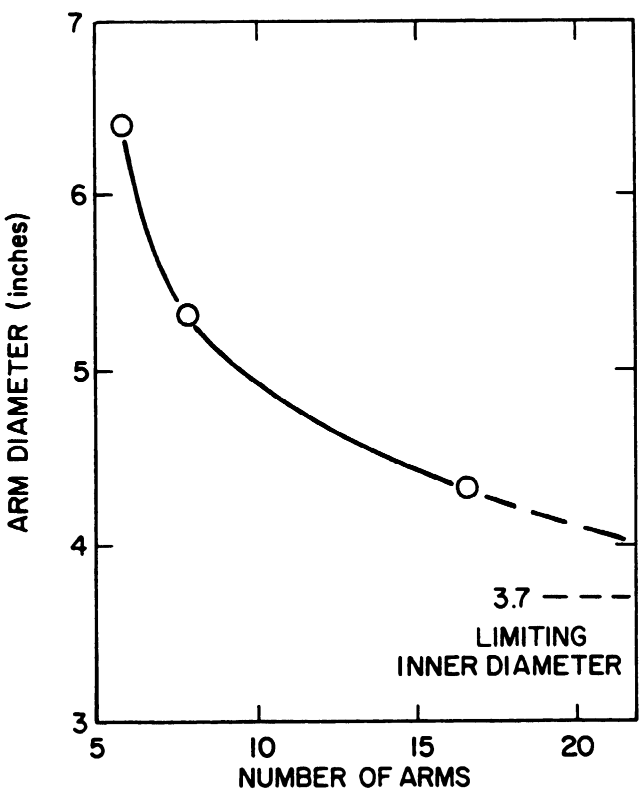


FIGURE 12. Critical Arm Spacing Versus Number of Arms.

FIGURE 13. Arm Diameter Versus Number of Arms (Spacing Equals Zero).



Corrections to Empirical Data:

Because of a possible extrapolation error in arriving at the limiting cases of spacing, increase the value 7.8 and 5.3 inches by 10 percent, to obtain 8.58 and 5.83 inches, respectively.

The empirically derived diameter of 3.7 inches must be decreased by the gap correction and 10 percent due to possible extrapolation error. Therefore, the acceptable diameter is 3.0 inches.

SAFE DATA USED IN FORMULATING ENGINEERING MODEL

In addition to corrections to the experimental and empirical data given, an additional 10-percent correction is imposed on all experimentally and empirically determined data. The correction includes reduction of all arm dimensions by 10 percent, and a 10-percent increase in all edge-to-edge spacings.

These safe dimensions provide the limiting values that appear in the section on *Rules and Criteria* which begin on Page 8.

In addition to these corrections, the square cross-sectioned central column was converted to a circular cross section by a constant buckling relation. This was considered desirable since most process piping is circular in cross section and the *Rules and Criteria* section make use of circular cross sections. The reported limiting value for the central column is 6.5 inches. The simple buckling conversion method is presented in Appendix B.

RULES AND CRITERIA

The following data should be used to solve pipe intersection problems.

Pipe Intersections for Minimal Reflection:

1. Maximum central column diameter is 6.5 inches.

2. Maximum contact area in each single quadrant of the central column is 23.75 inches^2.

   The contact area must be distributed in such a manner that it is impossible to find a quadrant that can contain more than 23.75 inches^2 (see Figure 14).

3. Maximum number of arms intersecting a single quadrant is 4.

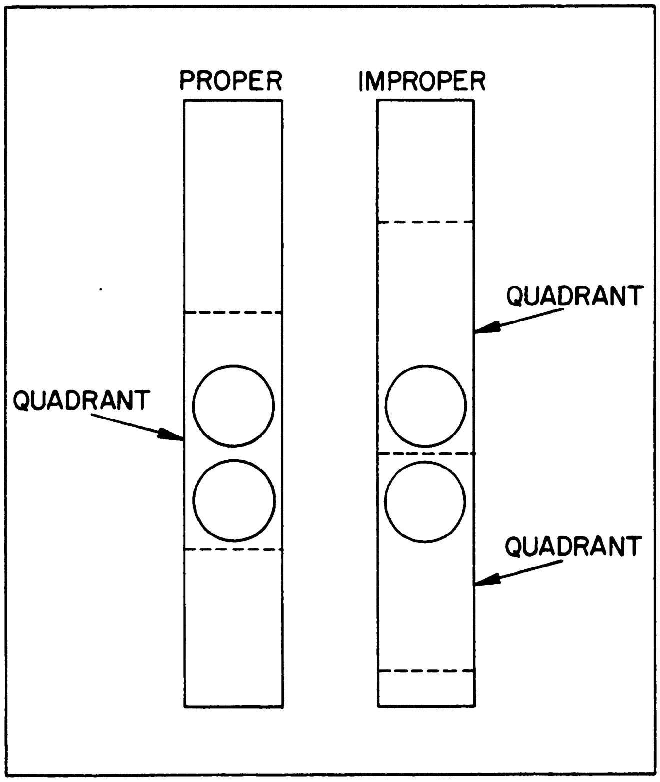


FIGURE 14. Quadrant Selection.

Pipe Intersections for Nominal Reflection:

1. Maximum central column diameter is 5.5 inches.

2. Maximum contact area in each single quadrant of the central column is 16.0 inches^2. The contact area must be distributed in such a manner that it is impossible to find a quadrant containing more than 16.0 inches^2.

3. Maximum number of arms intersecting a single quadrant is 4.

Most process plant applications involve a reflector condition described as *nominal*. The amount or thickness of the reflector which fits this condition is assumed to be 0.5 inches of water. Appendix C contains a graph (C-1, Page 16) of reflector savings as a function of reflector thickness. The reflector savings for 0.5 inches of water reflector is 0.5 inches. The above criteria was thus obtained by reducing the central column diameter by 1.0 inch, thus giving an acceptable value of 5.5 inches. The limiting acceptable contact area was likewise reduced to 16.0 inches^2.

The reader may find that the general criteria are too restrictive. In this event, application of the experimental data is recommended, making use of all corrections and the reflector savings which more closely approximate the reflector conditions of the problem. Also, the reader must keep in mind that corrected experimental data are critical data and thus would need additional corrections to insure safety.

The techniques for problem solving shown under the *Problems* section (No. I and II) are also applicable for this section.

Pipe Intersections for Full Reflection:

1. The maximum central column diameter is 4.1 inches.

2. Maximum contact area in each single quadrant of the central column is 9.6 inches^2. The contact area must be distributed in such a manner that it is impossible to select a quadrant containing more than 9.6 inches^2.

3. Maximum number of arms intersecting a single quadrant is 4.

As expressed earlier, direct use of the corrected experimental data with a safety factor is recommended where the general criteria given in this section are too restrictive.

All arm and central column dimensions should be reduced by the ratio below with an additional safety factor commensurate with the conditions of the problem. For a discussion of the ratio, refer to Appendix C.

$$\frac{\text{Full-reflected infinite cylinder diameter}}{\text{Unreflected infinite cylinder diameter}} = 0.635 \text{ inches}$$

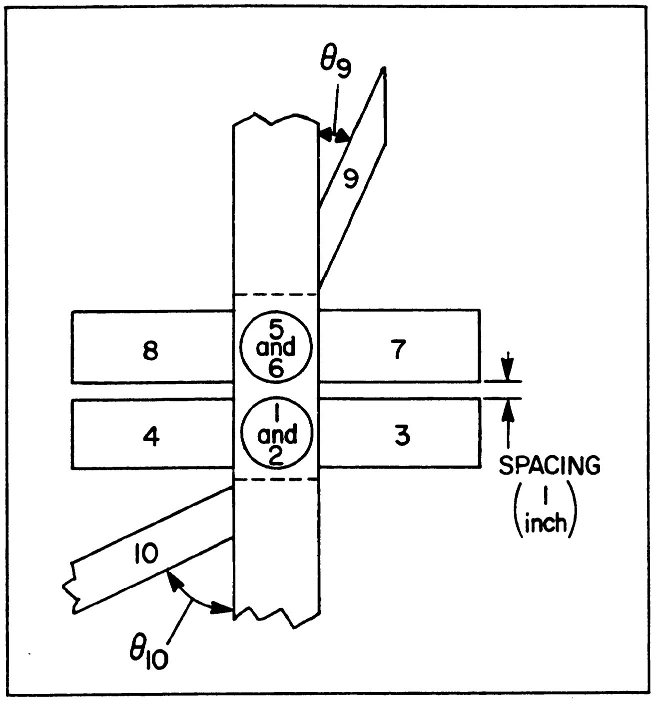


FIGURE 15. Typical Intersection Problem. Arms 1 through 10 with Angle Theta (θ_9), 30° and Theta (θ'_{10}), 45°.

PROBLEMS

Intersection Problem No. I:

GIVEN – The geometry shown in Figure 15. Assume minimal reflection. The central column diameter is 6.5 inches and Arms 1 through 8 have equal diameters.

PROBLEM – The problem is to maximize all of the arm diameters and minimize the spacings of Arms 9 and 10.

CALCULATIONS –

1. Select the quadrants as defined in the *Rules and Criteria,* Minimal Reflection.

2. Calculate the potential maximum area in contact with the central column.

The maximum surface area allowed per quadrant is 23.75 inches^2.

The largest diameter arms allowed for Arms 1 through 8 may be found from:

$$A = \pi r^2$$

However, due to the close spacing of these arms, there will be two per quadrant. Therefore,

$$\frac{23.75}{2} = \pi r^2$$

$$r^2 = 3.78 \text{ inches}$$

$$r = 1.944 \text{ inches, or}$$

$D = 3.89$ inches, the maximum diameter (D) that Arms 1 through 8 may have.

The maximum diameter for Arm 9 is given by:

$$23.75 = \pi r^2 \csc 30°$$

$$r^2 = \frac{23.75}{\pi \csc 30°} = 3.78 \text{ inches, or}$$

$$D = 3.89 \text{ inches}$$

The maximum diameter for Arm 10 is:

$$r^2 = \frac{23.75}{\pi \csc 45^\circ} = 5.34 \text{ inches}^2$$

$$r = 2.31 \text{ inches}$$

$$D = 4.62 \text{ inches}$$

Since the quadrant is placed around the central grouping of arms, this quadrant has the maximum allowable amount of surface area. The two arms where (theta) $\theta < 90^\circ$ will not be allowed in this quadrant. By centering the quadrant around the two central arms, the closest spacing that would be allowed for the top arm is: 9.00 - 3.89 = 5.11 inches and similarly for the bottom arm.

The above calculations are checked to determine if a quadrant exists that has more than 23.75 inches^2 in it. To do this, select a quadrant as shown in Figure 16.

The length of Arm 9 that is in contact with the central column is (3.89) (2.00) = 7.78 inches. Therefore, the total length occupied by the upper two cylinders is (7.78 + 5.11 + 3.89) = 16.78 inches. Thus, a quadrant has been found that has more than 23.75 inches^2. Therefore, the above calculation was nonconservative. To preclude this, it is necessary to respace the arms.

Begin with the upper quadrant at the top of the upper arm and place the maximum surface area in this quadrant. Therefore, (7.78 + S) = 18.0, where S = the arm separation. Therefore, S = 10.22 inches. This is the minimum separation for the upper arms from the two central arms.

To calculate the spacing of the bottom arm from the central two arms, note the length occupied by the bottom cylinder is: (1.414) (4.62) = 6.53 inches. Therefore, S = (18.0 - 6.53) = 11.47 inches.

The problem has been solved within the rules and criteria. The correct spacing and selection of quadrants is shown in Figure 17.

Intersection Problem No. II:

GIVEN – The geometry shown in Figure 18.

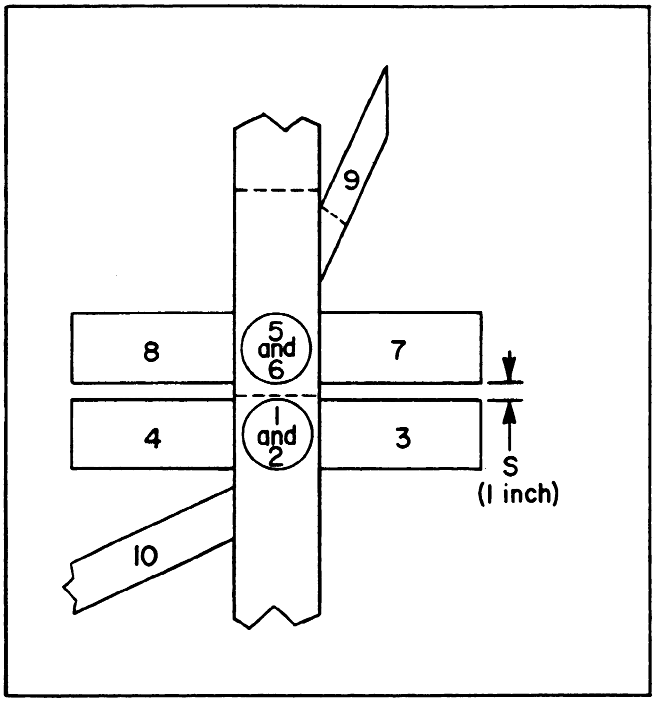
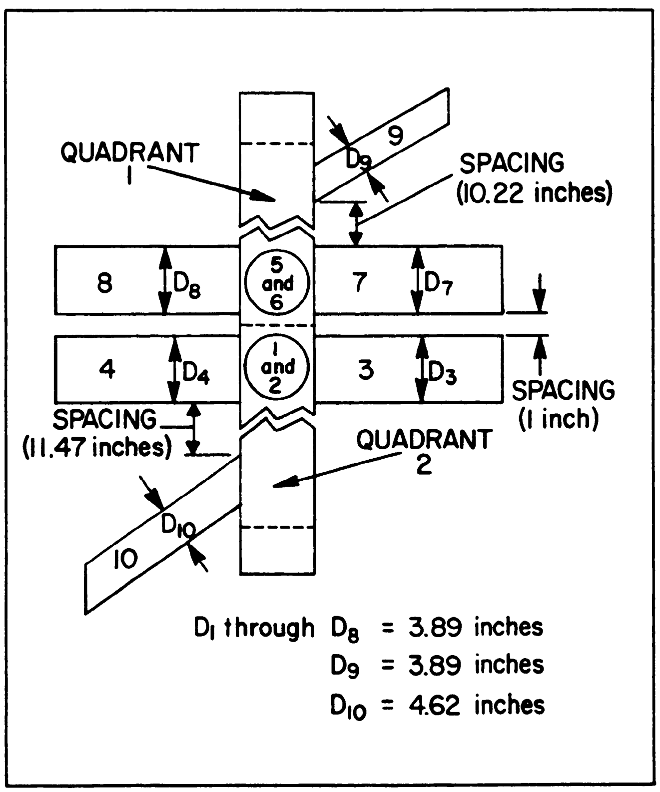
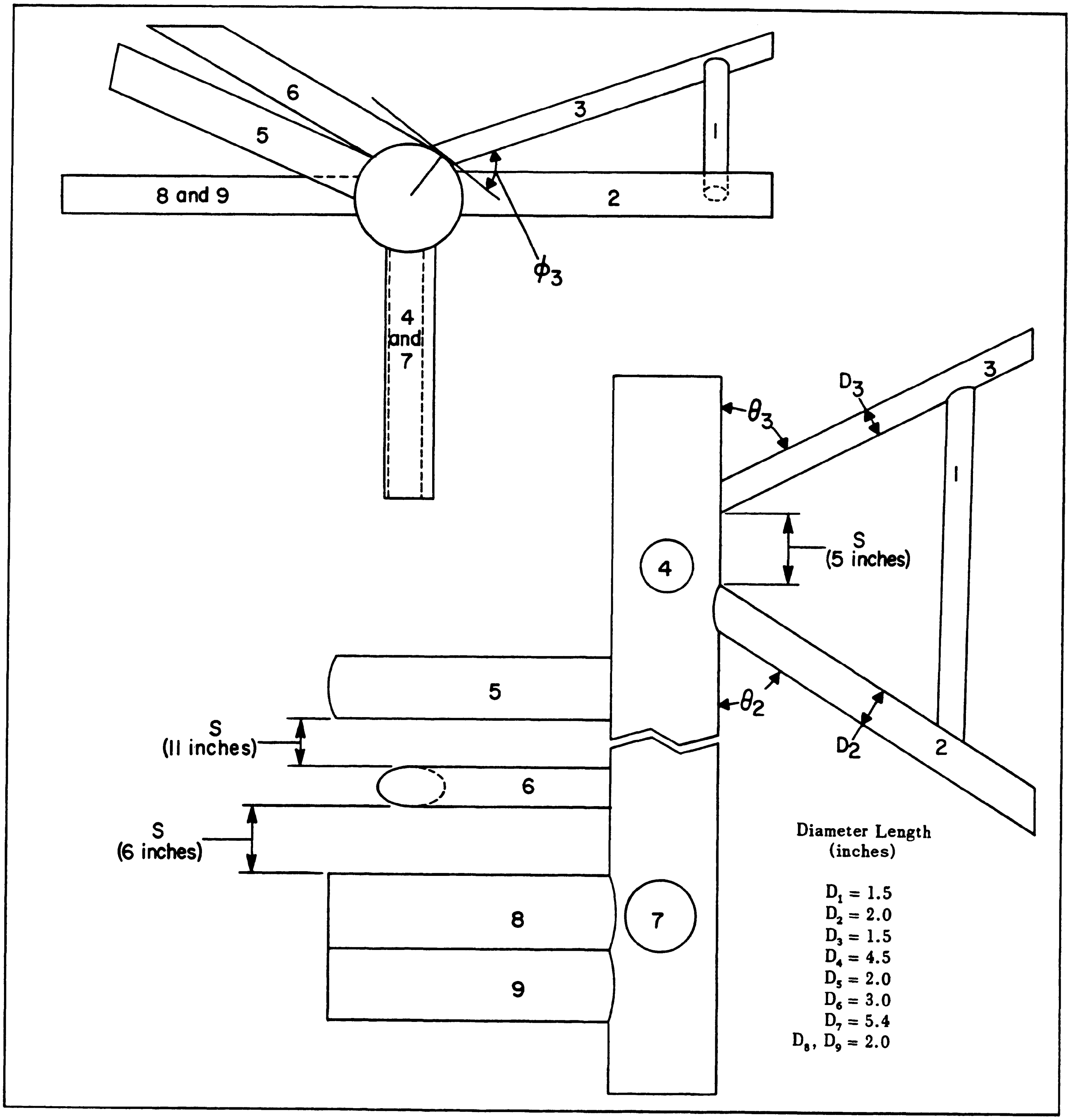


FIGURE 16. Typical Intersection Problem.

FIGURE 17. Final Safe Geometry.





Legend

Assume all Theta (θ) and Phi (ϕ)
Angles are 90°, except those below

$\phi_3 = 45°$ $\theta_2 = 45°$
$\phi_5 = 28°$ $\theta_3 = 60°$
$\phi_6 = 30°$

FIGURE 18. Complex Pipe Intersection Problem.

PROBLEM – Is this geometry safe?

SOLUTION –

1. Determine angle Phi (ϕ) between the central column and the intersecting arms. To do this, a line is drawn from the center of the column to the point of intersection of the line going down the center of the arm. A line tangent to the circle is drawn at the point of intersection. The angle between the tangent line and the side of the arm is equal to Phi (ϕ). (See Figure 19.)

The Phi angles are:

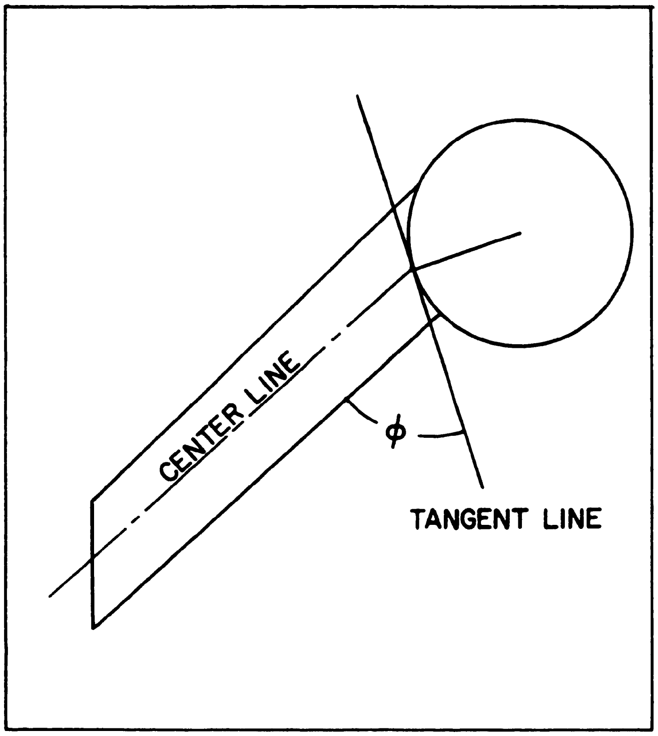
$$\phi_3 = 45°, \phi_5 = 28°, \text{ and } \phi_6 = 30°$$

All of the other angles are:

$$\phi = 90°$$

2. Next, determine the quadrants so they meet the criteria. This is done in the following manner:

FIGURE 19. Determination of Angle Phi (ϕ).



a. First note that if the angle between arms is less then 90°, then these arms may be placed in the same 90° sector. If these arms are spaced at a distance greater than 18 inches, it will not be possible to place them in the same quadrant. If the spacing is less than 18 inches, they may be placed in the same quadrant and the total surface area in contact with the central column for these arms is limited to 23.75 inches².

b. Thus, the following sets of arms may be placed in a quadrant:

Arms 1, 2, and 3
Arms 5 and 6
Arms 6, 8, and 9
Arms 7 and 4 are in quadrants of their own.

3. The surface area in contact with the central column can now be computed.

a. Arms 1, 2, and 3:

The intersection of Arm 1 with Arms 2 and 3 must be handled in the manner presented by Schuske[4] in which an effective diameter for Arms 2 and 3 are calculated from:

$$D_{eff(2)} = [(D_1)^2 + (D_2)^2]^{1/2}, \text{ and}$$

$$D_{eff(3)} = [(D_1)^2 + (D_3)^2]^{1/2}$$

Arm 1 must be separated from the central column by a minimum distance equal to five times the diameter of the largest arm it intersects. The total area (A) in contact with the central column is given by:

$$A = \frac{\pi}{4}[D^2_{eff(3)} \csc \theta_3 \csc \phi_3$$

$$+ D^2_{eff(2)} \csc \phi_2 \csc \theta_2]$$

[4]C. L. Schuske. "An Empirical Method for Calculating Subcritical Pipe Intersections." Interim Report. Rocky Flats Division, The Dow Chemical Company, Golden, Colorado. July 17, 1956.

Therefore:

$$D_{eff(2)} = [4.0 + 2.25]^{1/2} = 2.5 \text{ inches}$$

$$D_{eff(3)} = [2.25 + 2.25]^{1/2} = 2.12 \text{ inches}$$

$$A = \frac{\pi}{4} [(2.12)^2 (1.55) (1.414) + (2.5)^2 (1.0) (1.414)]$$

$$= 12.71 \text{ inches}^2$$

Thus, this quadrant is safe.

b. Arms 5 and 6:

The total area in contact with the central column is:

$$A = \frac{\pi}{4} [(D_6)^2 \csc \phi_6] + [(D_5)^2 \text{scs } \phi_5]$$

$$= \frac{\pi}{4} [3^2 (2.0) + 2^2 (2.130)] = 20.83 \text{ inches}^2$$

Thus, it is safe to place these two arms in the same quadrant.

c. Arms 6, 8, and 9:

It can be seen that Arm 6 can also be placed in a quadrant with Arms 8 and 9. Therefore, this quadrant must be calculated to see if it is safe.

The total area in contact with the central column is:

$$A = \frac{\pi}{4} [(D_6)^2 \csc \phi_6] + [(D_8)^2 + (D_9)^2]$$

$$= \frac{\pi}{4} [3^2 (2.0) + (2^2 + 2^2)]$$

$$= 20.42 \text{ inches}^2$$

Therefore, it is safe to place these three arms in the same quadrant.

d. Arm 7:

Arm 7 is unable to be placed in a quadrant with another arm. Therefore the only criteria that this arm must satisfy is to have its total intersection area equal to less than 23.75 inches².

$$A = \frac{\pi}{4} (5.4)^2 = 22.90 \text{ inches}^2$$

Thus, this arm may be placed on the column.

e. Arm 4:

Arm 4 can only be placed in a quadrant by itself since there are no other arms that are close enough to Arm 4 to be placed in the same quadrant. Therefore, the only criteria that Arm 4 need satisfy is that its total intersection area be less than 23.75 inches².

$$A = \frac{\pi}{4} (4.5)^2 = 15.90 \text{ inches}^2$$

Thus, this arm is safe when placed on the central column.

The above calculations show that the geometry shown in Figure 18 is safe for minimal reflection.

CONCLUSIONS

The purpose of the reported data is to make available to the design engineer, critical data describing complex pipe intersections in such a manner that problems involving intersections can be expeditiously solved.

In order to present a simplified engineering approach, a certain amount of conservatism was necessary. However, the reader may have a special problem which could be better analyzed by direct reference to the experimental data. For this reason, the experimental data and corrections to these data are presented.

The problem section describes in detail two intersection-type problems.

Since the model was determined for a uranium concentration at which minimum critical volume occurs, it is possible to extend this model for other concentrations which would permit larger pipe sizes but would require concentration control.

APPENDIX A. Gap Correction between Central Column and Intersecting Arms.

APPENDIX B. Constant Buckling Conversion.

APPENDIX C. Reflector Savings Correction.

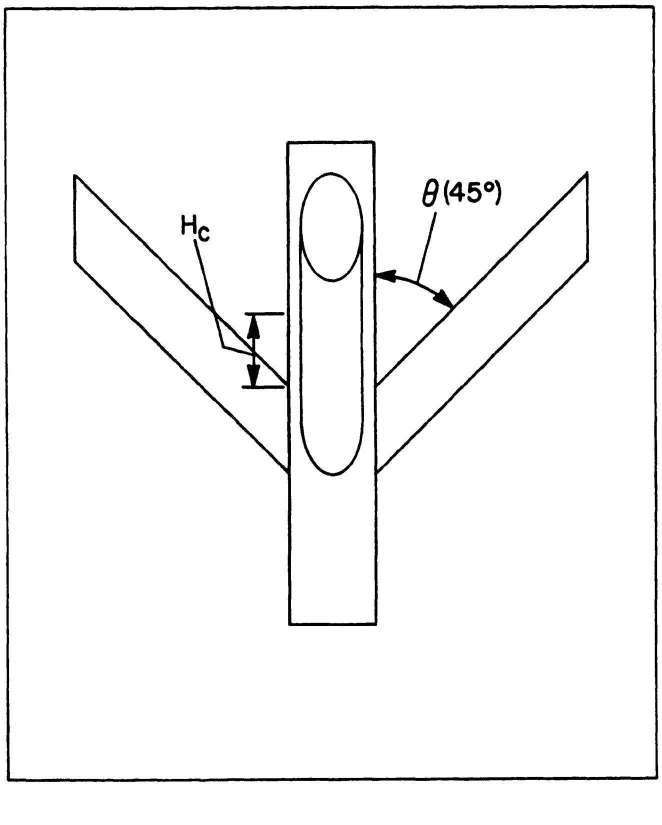
APPENDIX A. Gap Correction between Central Column and Intersecting Arms.

Since an air-gap and a stainless-steel interface exist between each arm and the central column, a correction to the arm diameters must be made.

To determine how much the diameter of an arm must be reduced to account for the gap, the configuration shown in Figure 1-A with four 6.40-inch, inner-diameter arms intersecting the central column at 45° was brought to the critical condition.

The central column was full and with equal air gaps between all the arms and the central column. The critical spacing for this geometry was ⅛ inches.

FIGURE 1-A. Typical Assembly to Determine Critical Surface Area in Contact with Central Column.



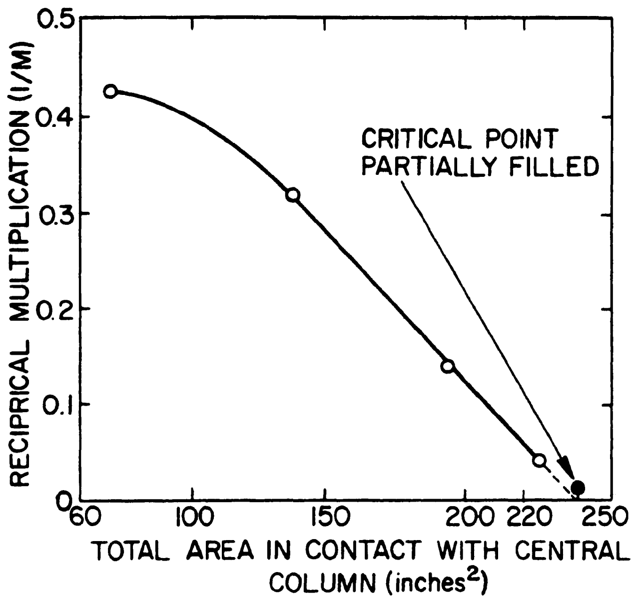


FIGURE 2-A. Arms Intersecting Central Column at a 45° Angle.

Another set of measurements was made on the same geometry, with no spacing between arms and central column. In these measurements, arm diameter was permitted to vary. These tests were done with one, two, three, and then three 6.40-inch, inner-diameter arms plus one 4.34-inch, inner-diameter arm.

The results are shown in Figure 2-A where the total area in contact with the central column versus the reciprocal multiplication (1/M) is plotted. This curve shows that 237 inches² is the critical area.

From the total critical contact area, it is possible to calculate the critical arm diameter when the experimental air gap, as well as the steel interface between the column and arms, is eliminated. The correction amounts to a reduction of 0.28 inches on each arm diameter.

APPENDIX B. Constant Buckling Conversion.

A Constant Buckling Conversion of an infinite cylinder of square cross section to an infinite cylinder of circular cross section is presented below:

Assumptions:

1. Assume equal buckling for a cylinder and a parallelepiped.
2. Use the buckling for minimum volume for these two geometries.
3. Where V_p = minimum volume[B-1] parallelepiped.

$$\frac{161}{B^3} = V_p$$

And where V_c = minimum volume of the cylinder,

$$\frac{148}{B^3} = V_c$$

Since the column is of infinite length we must use the volume per unit length:

$V_c' = \pi r^2$, r = radius of cylinder

$V_p' = W^2$, W = width of a side of the square column

$\frac{161}{148} = \frac{W^2}{\pi r^2}$ V_c' and V_p' = volume per unit length of the cylinder and parallelepiped respectively.

$r^2 = 14.34$
D = 7.57 inches

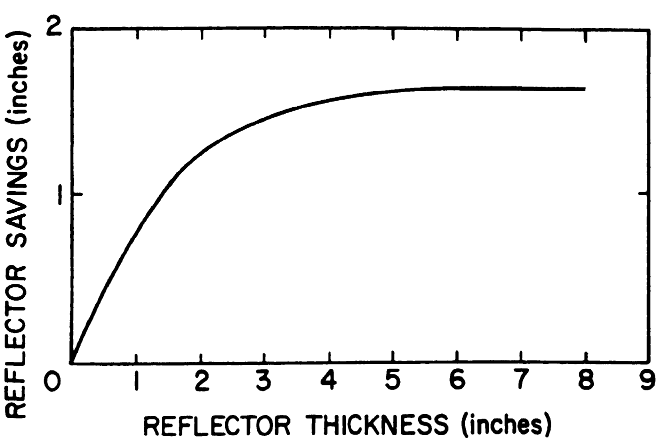
Here D is the equivalent diameter of a cylinder having the same reactivity as the 7 by 7-inch square column.

B-1 Harry Soodak and Edward C. Campbell. *Elementary Pile Theory.* John Wiley and Sons, Inc., New York. Chapman and Hall, Limited, London, England. 1950.

APPENDIX C. Reflector Savings Correction.

All experimental data presented are for systems with near minimal reflection. In order to extend the values of these data to normal plant conditions, a transport calculation of water reflector savings was done. Figure 1-C shows the reflector savings of the radius of an infinite cylinder as a function of the reflector thickness. All safe dimensions for minimal reflection must systematically be reduced by an amount 0.5 inches, which is due to nominal reflection of approximately 0.5-inch reflector equivalent around each arm and the central column.

FIGURE 1-C. Reflector Savings, 16-Group Transport Calculations.



Full-reflected cases would require reduction of all arms and central column diameters by an amount equal to:

$$\frac{\text{Full-reflected infinite cylinder diameter}}{\text{Unreflected infinite cylinder diameter}} = 0.635 \text{ inches}$$

The numerator of this ratio was reported by Schuske and Morfitt[C-1] as 5.4 inches. The denominator was derived from bare critical data (unpublished by C. L. Schuske) in the same manner as noted in Y-533.

C-1 C. L. Schuske and J. W. Morfitt. *An Empirical Study of Some Critical Mass Data.* Y-533. Union Carbide Corporation, Oak Ridge, Tennessee. December 6, 1949.