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LOS ALAMOS SCIENTIFIC LABORATORY

OF THE UNIVERSITY OF CALIFORNIA ◦ LOS ALAMOS NEW MEXICO

CRITICAL MASSES OF ORALLOY LATTICES IMMERSED IN WATER

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LOS ALAMOS SCIENTIFIC LABORATORY
OF THE UNIVERSITY OF CALIFORNIA LOS ALAMOS NEW MEXICO

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CRITICAL MASSES OF ORALLOY LATTICES IMMERSED IN WATER

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Contract W-7405-ENG. 36 with the U. S. Atomic Energy Commission

ABSTRACT

A solid cube of oralloy (94.3% enrichment) becomes critical at 24 kg when immersed in an infinite water reflector. Various critical lattices have been obtained by dividing this solid shape into small units and uniformly dispersing them (in water) at various mean densities. For a given size of oralloy unit, there is a mean density at which the critical mass is minimum. The hydrogen-uranium atomic ratio (optimum) of the cores with minimum critical masses are:

Oralloy Unit Dimension	H:U (opt)	m_c (min) (kg)
1 in. cube	5.4	22.3
1/2 in. cube	15	14
1/8 by 12 in. rod	53	6.5

The optimum hydrogen-uranium atomic ratio for lattices with oralloy units of intermediate size can be reasonably predicted by interpolation. Measurements with nonuniformly dispersed oralloy do not indicate a critical mass below the minimum observed with a uniform lattice. Multiplication measurements with Au, Ag, and Cd rods inserted in the oralloy

matrix have yielded the following effective cross-section ratios:

$$\sigma_a(Ag)/\sigma_a(Au) = 0.86, \text{ and } \sigma_a(Cd)/\sigma_a(Au) = 1.58.$$

These values are independent of position and lattice spacing for ranges examined.

CONTENTS

1. Introduction

Certain oralloy (Oy) lattices were immersed in infinite water for determining critical masses at various low-Oy densities. Three Oy unit sizes were used. The choice of Oy unit sizes (1 in. and 1/2 in. cubes, and 1/8 in. diameter by 12 in. long rods) was due mainly to availability of material or convenience of assembly. Unless otherwise indicated Oy cubes were arrayed in cubic lattices, and the rods as square cylindrical matrices.

The critical masses of these assemblies help bridge the gap between critical mass data for solid units of Oy (water tamped) and for immersed Oy lathe turnings[(1)] or solutions with molecular units.[(2)] In a broad sense, the experiment shows safe and unsafe conditions for handling heterogeneous Oy-water mixtures.

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(1) J. D. McLendon and J. W. Morfitt, Critical Mass Tests on Oralloy Machine Turnings, Carbide and Carbon Chemicals Company Report Y-A2-71, Feb. 29, 1952.

(2) C. K. Beck *et al.*, Critical Mass Studies, Part III, Carbide and Carbon Chemicals Company Report K-343, April 19, 1949.

The neutron multiplication data for determining all critical masses for each Oy unit size are given. Relations of critical mass (m_c) vs unit size, hydrogen-uranium atomic ratio (H:U) and density (ρ), and relations of reciprocal multiplication (1/M) vs H:U and ρ are plotted and/or tabulated. With the cubic arrays, core densities and/or H:U atomic ratios within the core are obtained from the composition of a cube with corners at the centers of eight grouped oralloy units. Similarly, a cylinder of square cross-section (corners at centers of four grouped rods) characterizes the average composition of the rod lattices. The cylinder length is taken to be 12 in. plus the center-to-center separation of adjacent rods.

In the rod lattices, effects of replacing Oy rods by nonfissionable materials were measured as changes in neutron multiplication. Results for Oy, Al, Ag, Au, Cd, and Tu (normal U), where significant, are interpreted as effective capture cross section ratios.

2. Equipment

2.1 Oralloy Material

The cubic units were constructed of Topsy Oy blocks[3] of the following groupings:

No. of Blocks	Dimension (in.)	Grams/Block	% U-235 (av.)	Total Wt. (kg)
366	1/2x1/2x1/2	38.36	94.52	14.039
208	1/2x1/2x1	76.75	94.07	15.965
112	1/2x1x1	153.40	93.99	17.181

These blocks had been machined to ±0.0005 in. tolerances.

Oralloy for the rods was drawn into 1/8 in. diameter wire from which 157 pieces 12 ± 1/8 in. were cut. The length variation resulted from an attempt to obtain uniform mass per rod. The average weight per rod was 44.561 ± 0.6 grams. (However, five rods had diameters of 0.123 in. and averaged 42.34 grams each.) The total 6.996 kg Oy was 93.614% U-235.

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[3] Originally made for the Topsy Assembly Machine for critical assemblies in tuballoy tamper. R. H. White: Topsy - A Remotely Controlled Machine for the Study of Critical Assemblies, Los Alamos Scientific Laboratory Report LA-1579, June 1953.

2.2 Replacement Materials

For replacement measurements, the following 1/8 in. diameter x 12 in. long rods were used:

No. of rods	Total grams
4 Al	26.025
4 Ag	102.114
2 Au	94.244
4 Cd	77.183
4 Tu	178.244

2.3 Neutron Sources

A neutron source (mock-fission) was centered in each configuration. Source No. 17 was used with the 1/2 in. cubic units. The strength of this source (initially $\sim 5.5 \times 10^6$) diminished from 4.85×10^6 to 4.25×10^6 neutrons per second during its use. Source No. 18 was used for the other measurements - diminishing in strength from 3.95×10^6 to 6.35×10^5 neutrons per second.

2.4 Neutron Counters

For measuring neutron multiplication, four GE neutron counting tubes, lined with boron enriched in B^{10}, were set up near the assembly with model No. 503 pre-amplifiers and amplifiers. A regulated variable electronic voltage supply furnished the proper collection potential for these tubes.

Pulses were fed through coaxial cables to model No. 750 scaler recording units in the control room that is 1200 ft from the assembly laboratory. The counting tubes were placed, initially, on the A-frame of the assembly machine (Fig. 1). Later, they were placed in brass tubes at the ends of the cross-bars for immersion into the water reflector (Fig. 2).

2.5 Assembly Machine

The Comet was used as the basic assembly machine. This multipurpose machine had been built to operate under the prevailing safety regulations for Pajarito Site. It consists of a hydraulic lift (ram) which rises vertically, and an air piston which extends downward from an A-frame superstructure over the ram. The ram and piston are operated remotely from the control room. Electrical interconnections incorporate "fail safe" features to drop the ram by gravitation and raise the piston by spring force if power failure occurs, if radiation exceeds a pre-set level, or if manual scrams are operated.

The ram supported a 28 in. high x 35-1/2 in. diameter steel tank filled to within 5 in. from the top with water. The hydraulic pressure was adjustable to maintain a set rate of lift and to obtain a minimum scram time without spilling water or damaging the hydraulic system. The air piston supported the Oy lattice (Figs. 3 and 4). From crossed bars attached to this piston hung four 1/2 x 24 in. long brass rods. Lucite plates to support the Oy were spaced on these rods by means of aluminum cylinders and clamps. A lucite box with variable side dimensions was fastened to the bottom support plate to hold the solid cubic configurations. Countersunk holes or grooves permitted reproducible spacing of cubic Oy units on the plates, whereas drilled holes positioned the Oy rods.

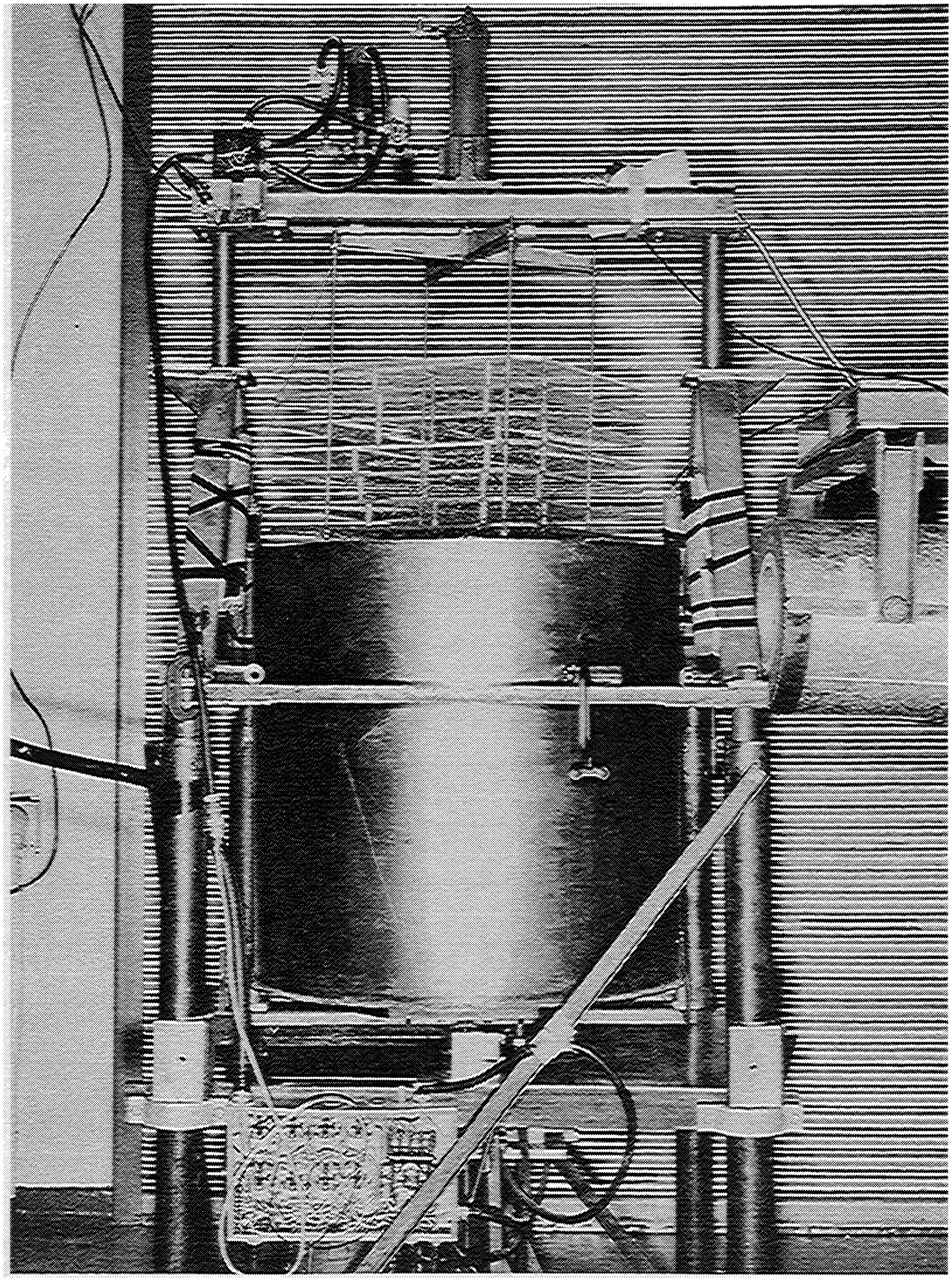

Fig. 1 Assembly machine with superstructure modified for cubic arrays.

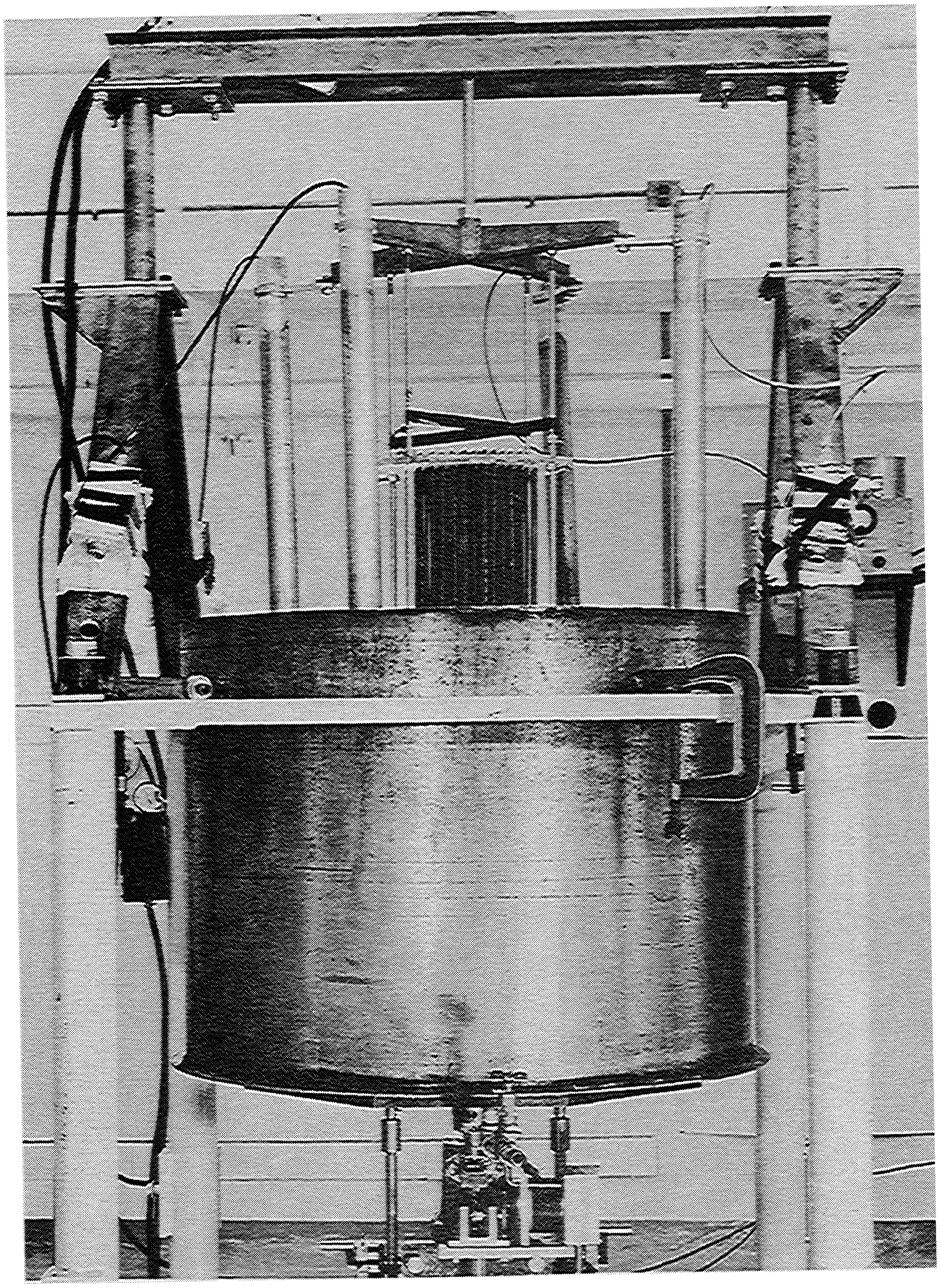

Fig. 2 Assembly machine with superstructure modified for 1/8 in. rod arrays.

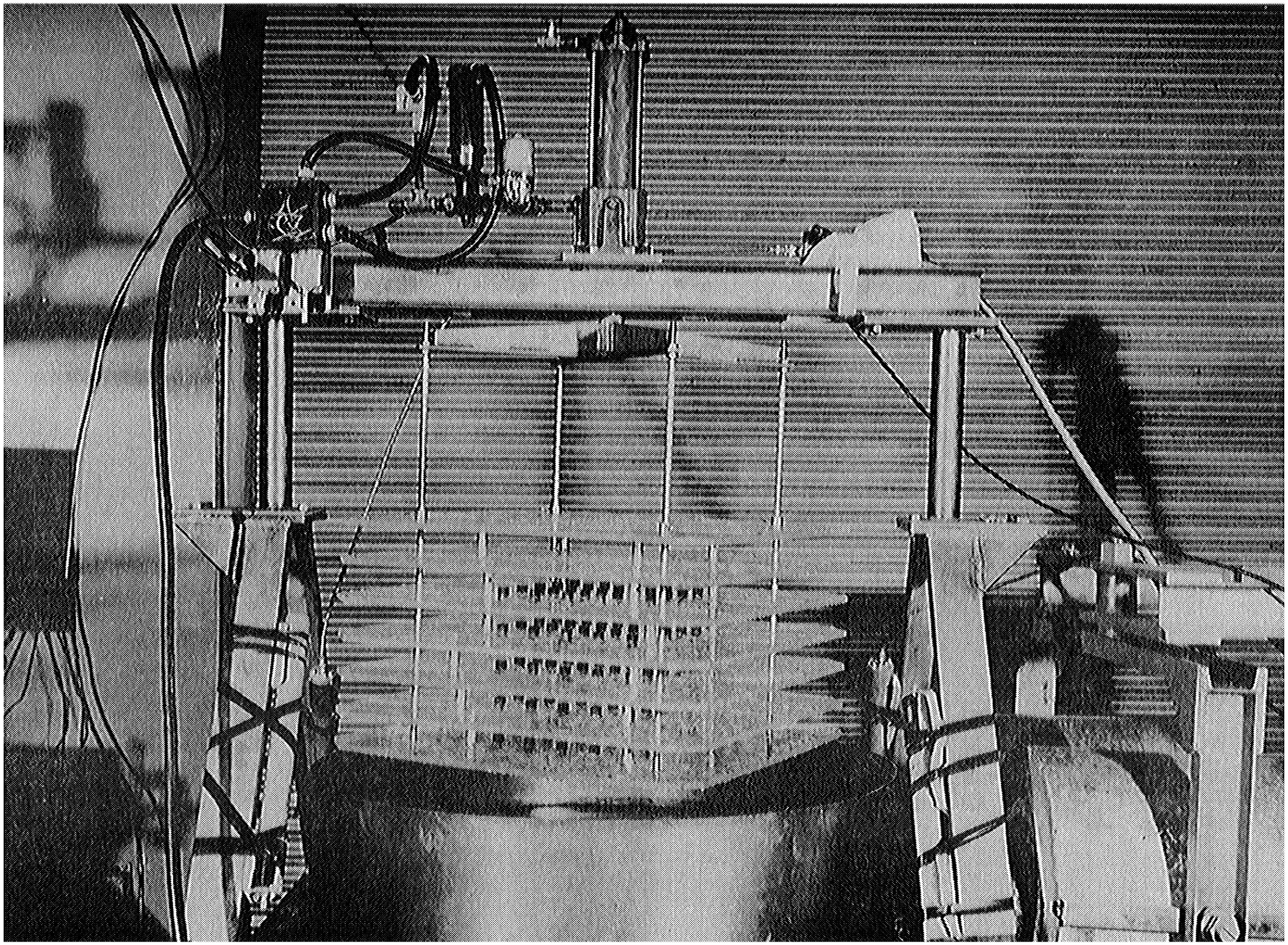

Fig. 3 An array of 1/2 in. cubes stacked at 2 in. center-to-center spacing.

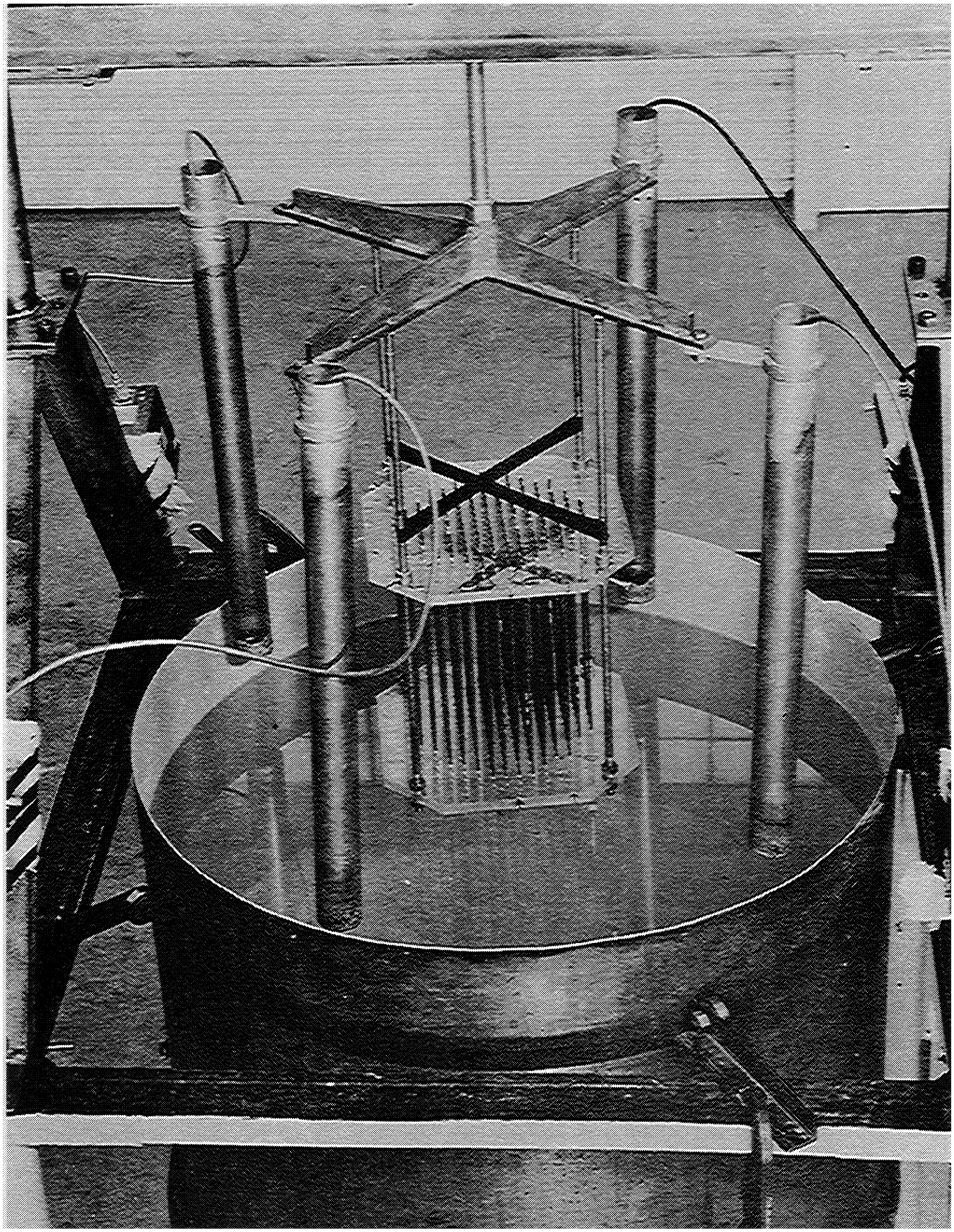

Fig. 4 An array of 1/8 in. rods at 7/8 in. center-to-center spacing.

3. Procedure

The standard procedure for obtaining a critical mass value is to measure neutron multiplications of configurations with progressively increasing masses of Oy. Reciprocal neutron multiplication vs Oy mass then extrapolates to zero at the critical mass.

Neutron counting rate data were taken for each of two conditions of an assembled configuration -- without Oy in the configuration (unmultiplied count) and with Oy present (multiplied count). Neutron multiplication (M) is the ratio of multiplied count to unmultiplied count. The initial multiplied count was obtained with a known-safe mass of Oy. The configuration was then removed from the water for the addition of more Oy. After reimmersion, a second multiplied count was taken, and so on. Reciprocal multiplication (1/M) vs Oy mass was plotted at each step. The process was continued until a near critical configuration was attained, so that the data could be reliably extrapolated to 1/M = 0, to give the critical mass (m_c).

Critical masses were determined for Oy in the following forms, with effectively infinite water reflector:

1. A solid cubic configuration.
2. One inch cubes uniformly dispersed as cubic lattices at 1.25, 1.50, 1.75, and 2.00 in. center-to-center spacings; cores approximately cubic.
3. One-half inch cubes uniformly dispersed as cubic lattices at 0.75, 1.00, 1.17, 1.50, and 2.25 in. center-to-center spacings; cores approximately cubic.
4. One-eighth inch diameter rods uniformly dispersed as square matrices at 0.500, 0.625, 0.750, 0.875, and 1.000 in. center-to-center spacing of the rod axes; cores approximately circular cylinders.
5. One-eighth inch diameter rods in nonuniform configurations; cores approximately circular cylinders.

For each size of Oy unit, there was determined a minimum critical mass [m_c(min)], and the core H:U atomic ratio ($H{:}U_{opt}$) at which this minimum occurred. The relation, m_c(min) vs $H{:}U_{opt}$, was then plotted, including other known data for heterogeneous and homogeneous Oy-water mixtures.

In computing H:U values, the hydrogen density of the lucite (relatively small volume) was taken as equal to that of water.

For determining reactivity contributions of various materials, 1/M values were obtained with and without Oy rods in a given radial position, and with rods of other materials in place of the Oy. Initial runs were made with the replacement of only one rod; later, four rods at equivalent positions were replaced.

4. Data

4.1 Solid Oy Cube

A m_c of $24._0$ kg was determined for a solid Oy cube immersed in infinite water reflector. This m_c is less than 1% greater than that reported for a spherical Oy ball in water,[4] after correction for density and U-235 enrichment differences.[5]

Two sets of data were obtained to make this determination. For one set the single unmultiplied count was obtained with a bare neutron source in the water. For the other set the unmultiplied counts were obtained with the source centered within water-tamped tuballoy (Tu) cubes. The multiplied counts, in both instances, were obtained with the source centered within Oy cubes. The final configuration assembled, a 4.0 x 4.0 x 4.5 in. square-based block of Oy

(4) E. C. Mallary, Oralloy Cylindrical Shape Factor and Critical Mass Measurements in Graphite, Paraffin, and Water Tampers, Los Alamos Scientific Laboratory Report LA-1305, Oct. 27, 1951.

(5) H. C. Paxton, Critical Masses of Fissionable Metals as Basic Nuclear Safety Data, Los Alamos Scientific Laboratory Report LA-1958, Jan. 1955.

weighing 22.09 kg, gave a multiplication of 110.5. Figure 5 shows 1/M vs Oy mass and the extrapolation to critical.

4.2 One Inch Oy Cubic Units

Measurements with 1 in. Oy cubic units in variously spaced arrays determined a m_c(min) of 22.3 kg at H:U_{opt} of 5.4 (Fig. 6).

Table I summarizes results for each of the four series of measurements made to determine critical masses. Figure 7 shows the 1/M vs Oy mass plots with extrapolations to critical. The unmultiplied count was taken with a bare neutron source in the water at the central position of the Oy lattices.

4.3 One-half Inch Oy Cubic Units

Measurements with 1/2 in. Oy units, in regular cubic arrays at various spacings, determined a m_c(min) of $14._0$ kg at H:U_{opt} of 15 (Fig. 8).

Table II summarizes results for each of the five series of measurements made to determine the critical masses. Also, results for a series of measurements with a body-centered cubic array are given, and a data point for a displaced body-centered array. Figure 9 shows the 1/M vs Oy mass plots and the extrapolations to critical.

TABLE I.

CRITICAL MASS DATA SUMMARY FOR LATTICES OF 1 IN. OY CUBES IMMERSED IN WATER

(Oy Unit = 306.8 gm, 16.387 cm^3, 18.72 gm/cm^3, 94.3% U-235)

	Largest Measured Array					m_c (extrap.)	
Spacing (in.)	No. Units	kg Oy	Mult.	Average Oy Density	Average H:U Core	No. Units	kg Oy
1.25	76	23.317	100.5	9.59	1.33	83.44	$25._6$
1.50	64	19.635	57.5	5.55	3.31	74.97	$23._0$
1.75	64	19.635	80.6	3.49	6.08	73.01	$22._4$
2.00	68	20.862	73.6	2.34	9.76	79.86	$24._5$
Optimum (1.70)				3.84	5.4	72.69	$22._3$

TABLE II.

CRITICAL MASS DATA SUMMARY FOR LATTICES OF 1/2 IN. OY UNITS IMMERSED IN WATER

(Oy Unit = 38.35 gm, 2.048 cm^3, 18.72 gm/cm^3, 94.52% U-235)

	Largest Measured Array					m_c (extrap.)	
Spacing (in.)	No. Units	kg Oy	Mult.	Average Oy Density	Average H:U Core	No. Units	kg Oy
0.75	343	13.154	21.77	5.55	3.31	469.36	18.0
1.00	343	13.154	90.19	2.34	9.76	378.10	14.5
1.17	343	13.154	123.6	1.45	16.48	371.58	14.25
1.30*	341	13.077	142.7	1.39	18.83	367.67	14.1
1.50	343	13.154	34.36	0.69	36.26	521.51	20.0
2.25	343	13.154	6.88	0.21	125.70	>1434	>55
1.50H 0.75V	341	13.077	95.0	1.39**	18.83**	375.49	14.4
Optimum (1.138)				1.59	15	365.06	14.0

*Body Centered.

**Displaced Body Centered, assumed identical to Body Centered.

4.4 One-eighth Inch Diameter by 12 Inch Oy Rod Units

4.4.1 Uniform Core Densities

Measurements with uniform arrays of 1/8 in. diameter by 12 in. long Oy rods determined a m_c(min) of 6.5 kg at $H:U_{opt}$ of 53 (Fig. 10).

Table III summarizes results for each of the five series of measurements for determining the critical masses. Figure 11 shows the 1/M vs Oy mass plots and the extrapolations to critical.

TABLE III.

CRITICAL DATA SUMMARY FOR UNIFORM ARRAYS OF 1/8 IN. DIAM. BY 12 IN. LONG OY RODS IMMERSED IN WATER

(Oy Unit = 44.561 gm, 2.380 cm^3, 18.72 gm/cm^3, 93.614% U-235)

	Largest Measured Array					m_c (extrap.)	
Spacing (in.)	No. Units	kg Oy	Mult.	Average Oy Density	Average H:U Core	No. Units	kg Oy
0.500	157	6.996	85.6	0.870	28.61	171	7.62
0.625	140	6.238	125.3	0.551	45.96	149	6.64
0.750	144	6.421	169.5	0.379	67.47	152	6.76
0.875	157	6.996	139.1	0.276	93.25	173	7.70
1.000	157	6.996	51.6	0.209	123.43	>203	>9.0
Optimum (0.668)				0.480	53	146	6.5

4.4.2 Nonuniform Core Densities

A pair of lucite plates having three sets of holes drilled at 0.5, 0.75, and 1.0 in. center-to-center spacing permitted arrays of rods with different average Oy densities in various radial zones.

First, 80 rods (3.565 kg) were arranged as a central region at $\rho_{Oy} = 0.870$. Surrounding this central region was an approximate annulus of Oy rods in positions such that $\rho_{Oy} = 0.379$. In this fashion, configuration A, Fig. 12, was attained. Standard procedures were used for obtaining critical mass by plotting 1/M vs Oy mass and extrapolating. Next, configurations B and C were obtained by reducing the central region to 64 rods (2.450 kg) and 16 rods (0.712 kg) respectively, and extending the region at $\rho_{Oy} = 0.379$ inward. Then configuration C' was obtained by changing 14 rods (0.624 kg) at the perimeter of configuration C from $\rho_{Oy} = 0.379$ to $\rho_{Oy} = 0.209$. Configuration D had 16 central rods at $\rho_{Oy} = 0.870$, next 55 rods (2.451 kg) at $\rho_{Oy} = 0.379$, and on the outside, 60 rods (2.674 kg) at $\rho_{Oy} = 0.209$. Only one data point was obtained for this configuration. In configuration E there were 65 central rods (2.896 kg) at $\rho_{Oy} = 0.870$, surrounded by rods at $\rho_{Oy} = 0.209$. Central regions similar to the one in E were used in configurations F, G, H, and J to construct arrays to give more extreme density variations.

For configuration J, from the center (ρ_{Oy} = 0.870), rods were systematically removed or added at positions that would give an over-all decrease in density as the perimeter of the cylinder was approached.

In the cross-sectional diagrams of these cylindrical configurations, Fig. 12, (configuration C' not shown) the open triangles designate the central region and/or the initial run. The open circles represent the rods that were added as the configuration expanded, and the closed triangles designate proposed extrapolation of the configuration to critical for estimating critical density, ρ_c.

Table IV summarizes the critical data for all the nonuniform configurations. For comparison, interpolated critical masses of uniform lattices at the average oralloy densities of the nonuniform configuration are listed in the final column. In each of five cases (configurations C, C', D, E, and J) the critical mass of the nonuniform array is below that of the uniform lattice. Even in these cases, however, the critical masses are no less than the minimum for uniform lattices.

Figures 13 and 14 show the 1/M vs Oy mass plots for nonuniform arrays, with extrapolations to critical.

TABLE IV.

DATA SUMMARY FOR NONUNIFORM ARRAYS OF 1/8 IN. BY 12 IN. OY RODS

Configuration	kg Oy (max)	1/M (min)	m_c (kg Oy)	Av. Oy density at critical, ρ_{Oy}(Av.)	Percent m_c, (kg Oy) in region at: ρ_{Oy}=0.870	ρ_{Oy}=0.379	ρ_{Oy}=0.209	m_c(Oy) of uniform array at ρ_{Oy}(Av.)
A	6.417	0.0057	6.77	0.536	52.63	47.37	--	6.60
B	6.149	0.0075	6.68	0.457	42.67	57.33	--	6.52
C	6.060	0.0078	6.62	0.399	10.76	89.24	--	6.68
C'	6.060	0.0073	6.58	0.353	10.83	71.77	17.40	6.89
D	5.837	0.0094	6.49	0.285	11.11	38.19	50.69	7.48
E	6.417	0.0097	7.12	0.290	45.14	--	54.86	7.42
F	6.417	0.0064	6.85	0.425	44.81 37.01	--	11.02	6.58
G	6.283	0.0157	7.31	0.696	41.82 58.18	--	--	7.03
H	6.283	0.0087	--	--	--	--	--	6.76
J	6.283	0.0064	6.64	0.381	--	--	--	6.75

4.5 Replacements

Removal of Oy rods from several positions in a configuration gave measurable differences in 1/M. Consequently, there were systematic measurements of $\Delta(1/M)$ as Oy rods were removed from a series of radial positions (Fig. 15). Similarly, changes in 1/M were determined as rods of Au, Ag, Cd, Al, and Tu were placed in vacant Oy positions. As displacement of water by nearly inert Al or Tu resulted in negligible 1/M change, $\Delta 1/M_x$ (where x represents the material inserted) can be used to approximate the material effectiveness: $1/M_x - 1/M_{air}$. All such values of $\Delta(1/M_x)$ are given in Tables A-1 to A-5 in the Appendix. In these measurements, some values were obtained for one rod, some for two rods, and some for four rods in equivalent positions. Each $\Delta(1/M_x)$ value per rod of x was normalized by multiplying by the total number of Oy rods in the configuration. Illustrative values were then plotted against percent of core radius, Figs. 16 and 17, and radius, Fig. 18.

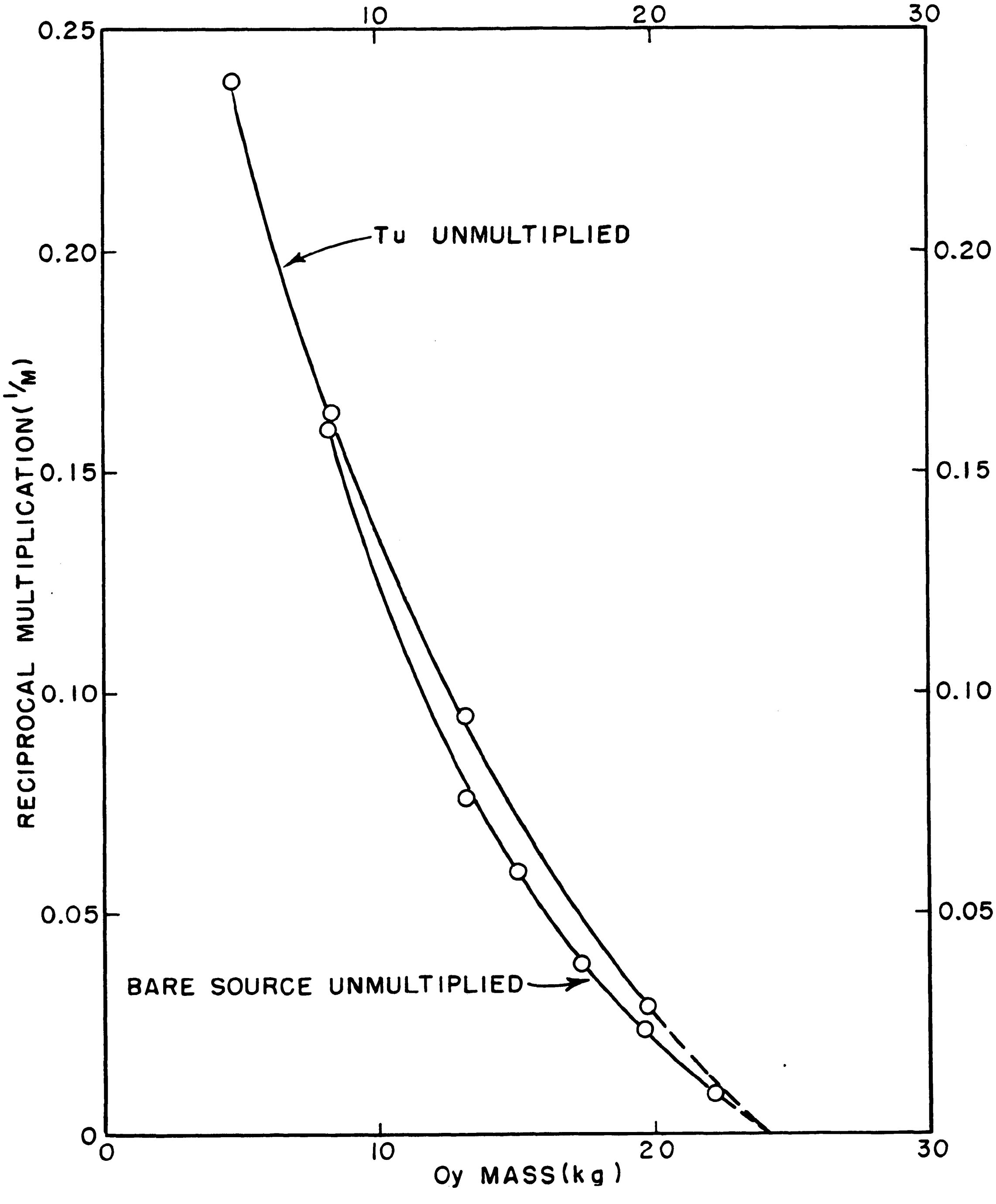

Fig. 5 Determination of m_c for solid Oy cube immersed in water.

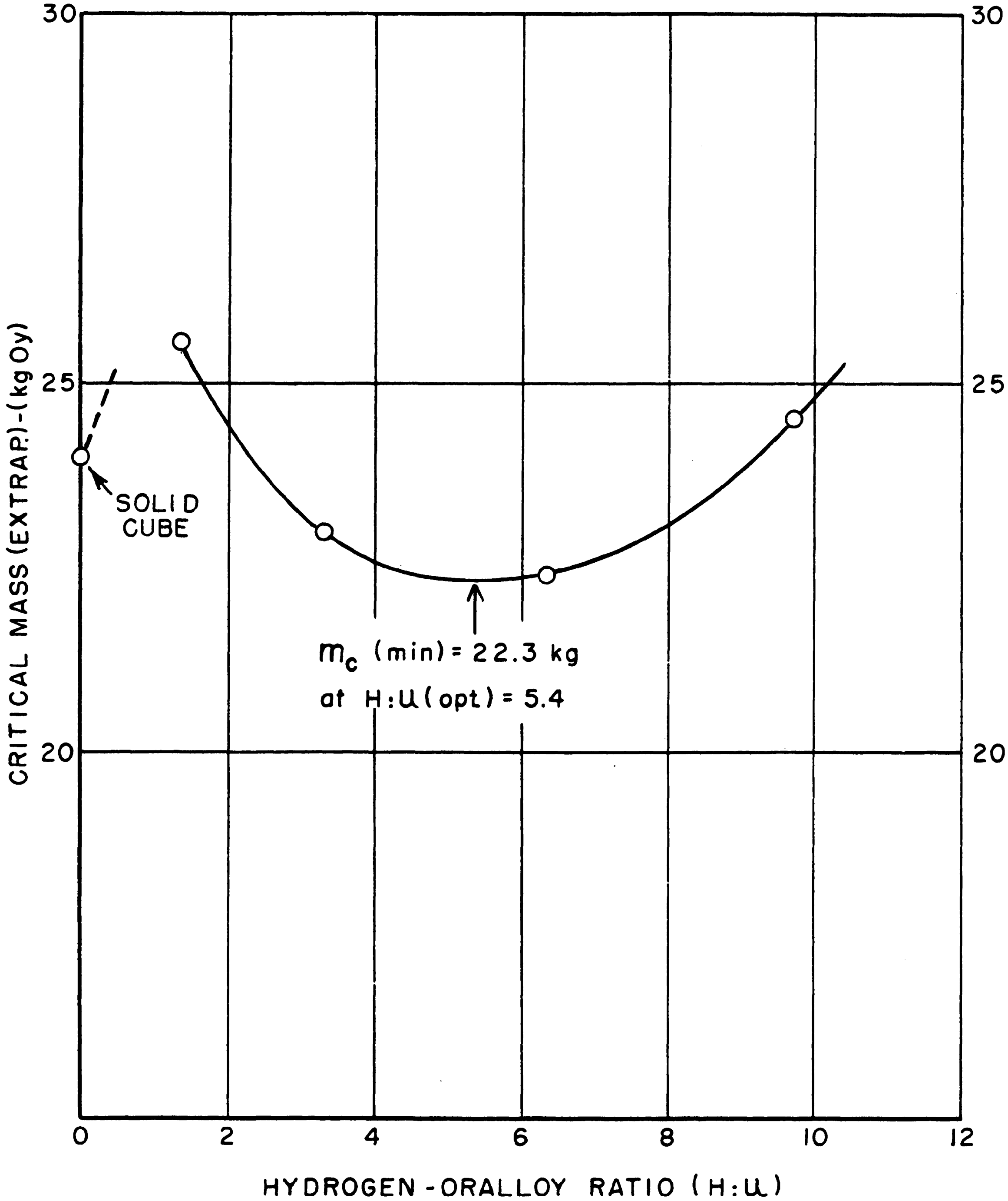

Fig. 6 Determination of m_c(min) for lattices of 1 in. Oy cubes immersed in water.

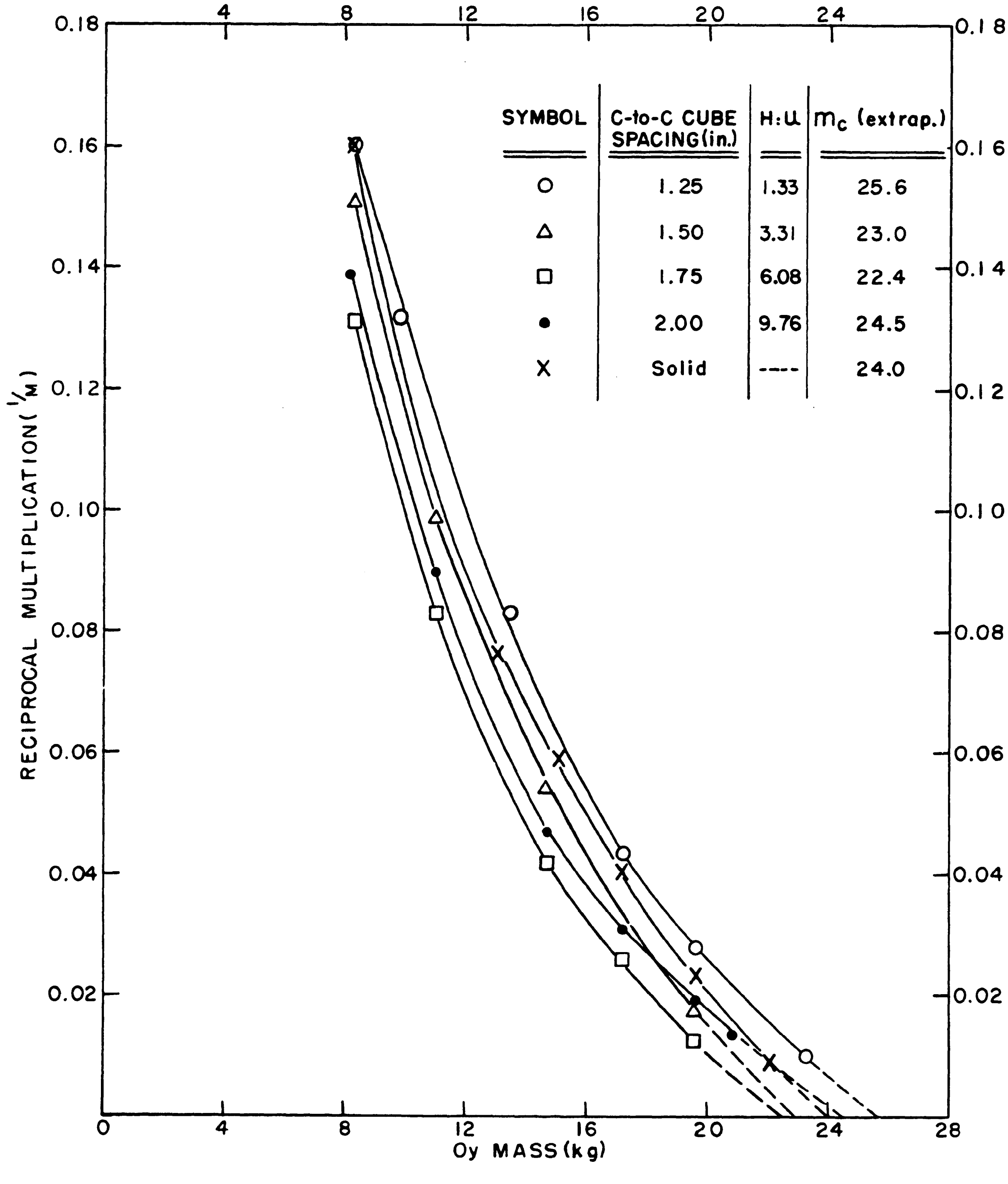

Fig. 7 Determinations of m_c for lattices of 1 in. Oy cubes immersed in water.

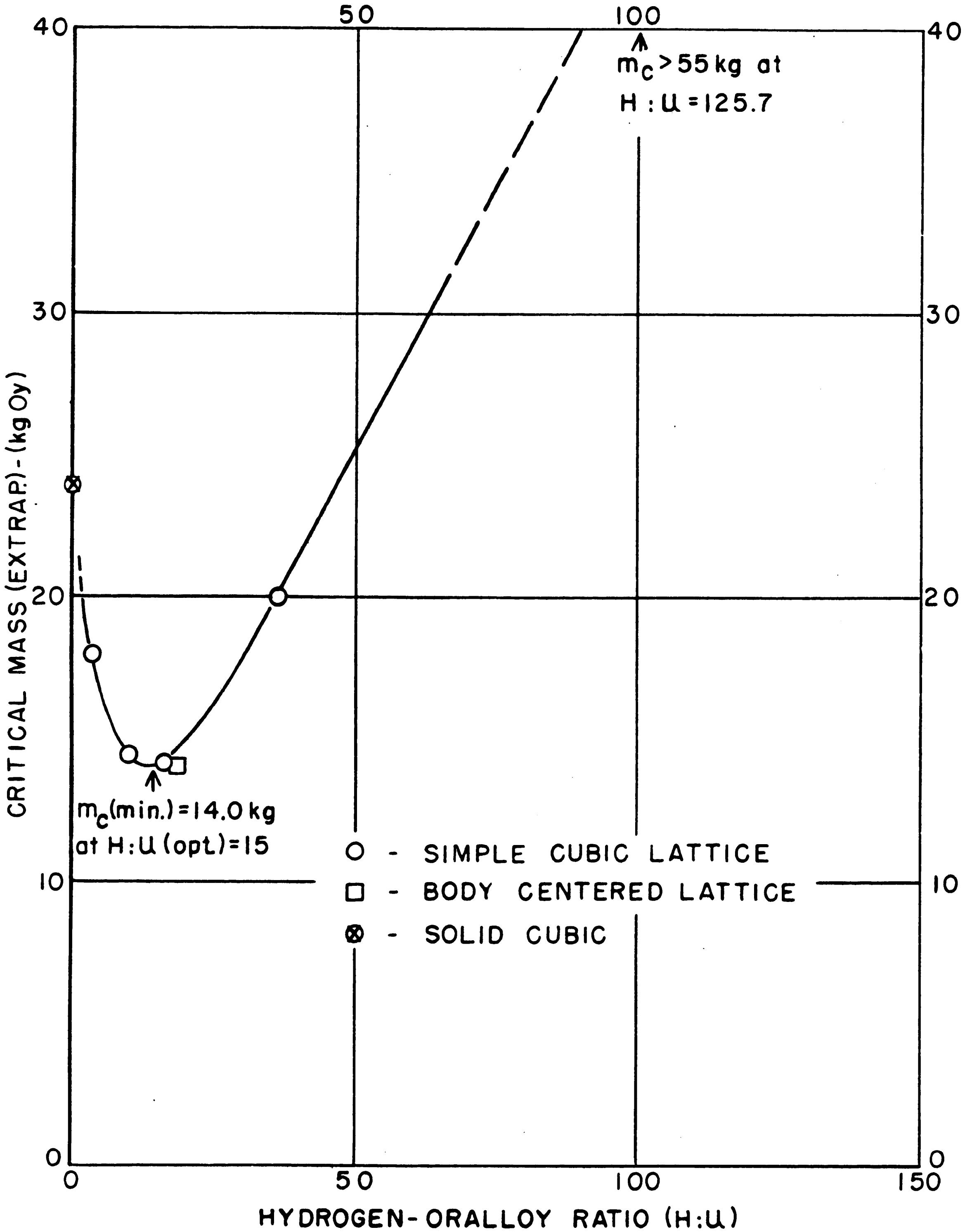

Fig. 8 Determination of m_c(min) for lattices of 1/2 in. cubes immersed in water.

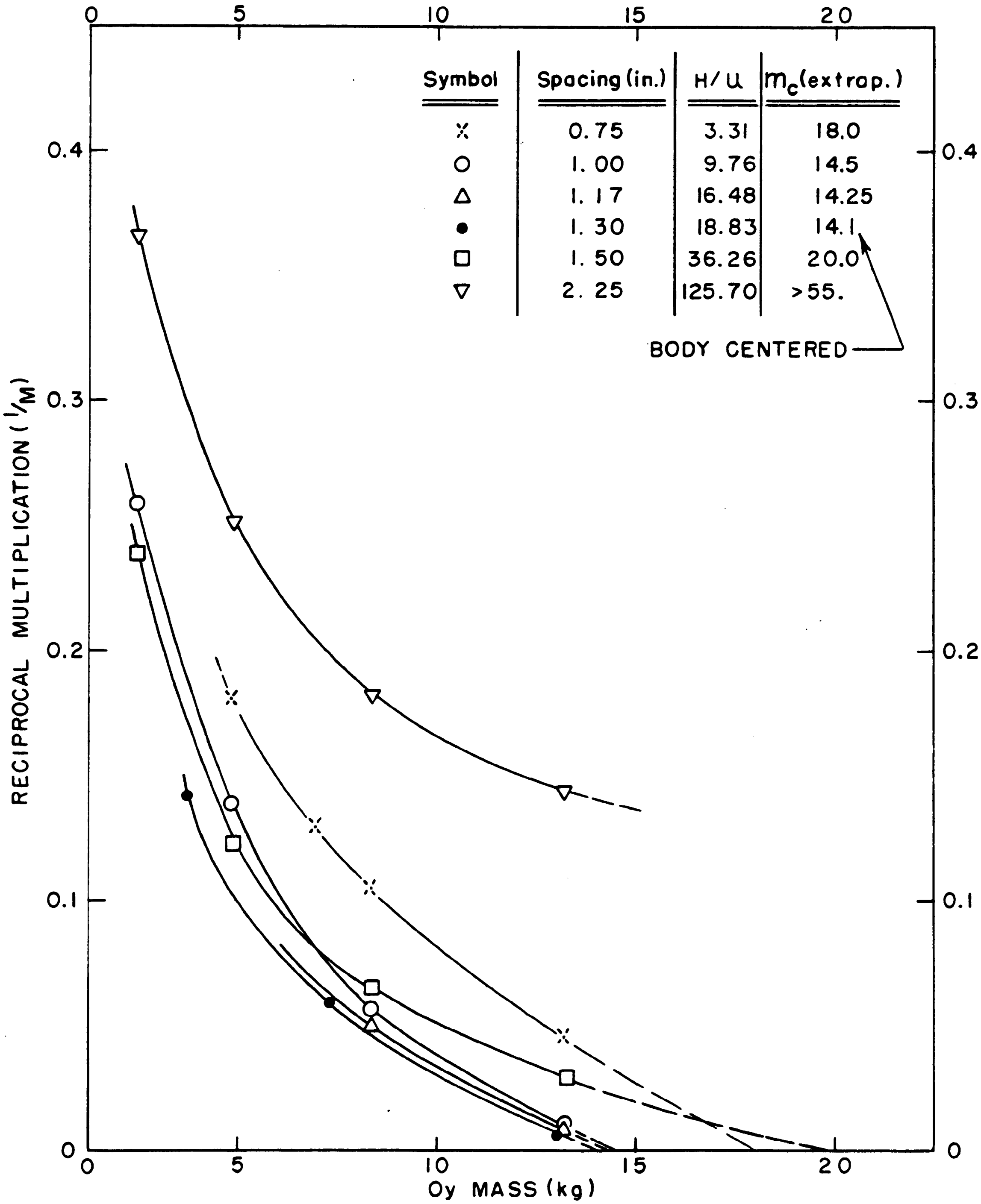

Fig. 9 Determinations of m_c for lattices of 1/2 in. Oy cubes immersed in water.

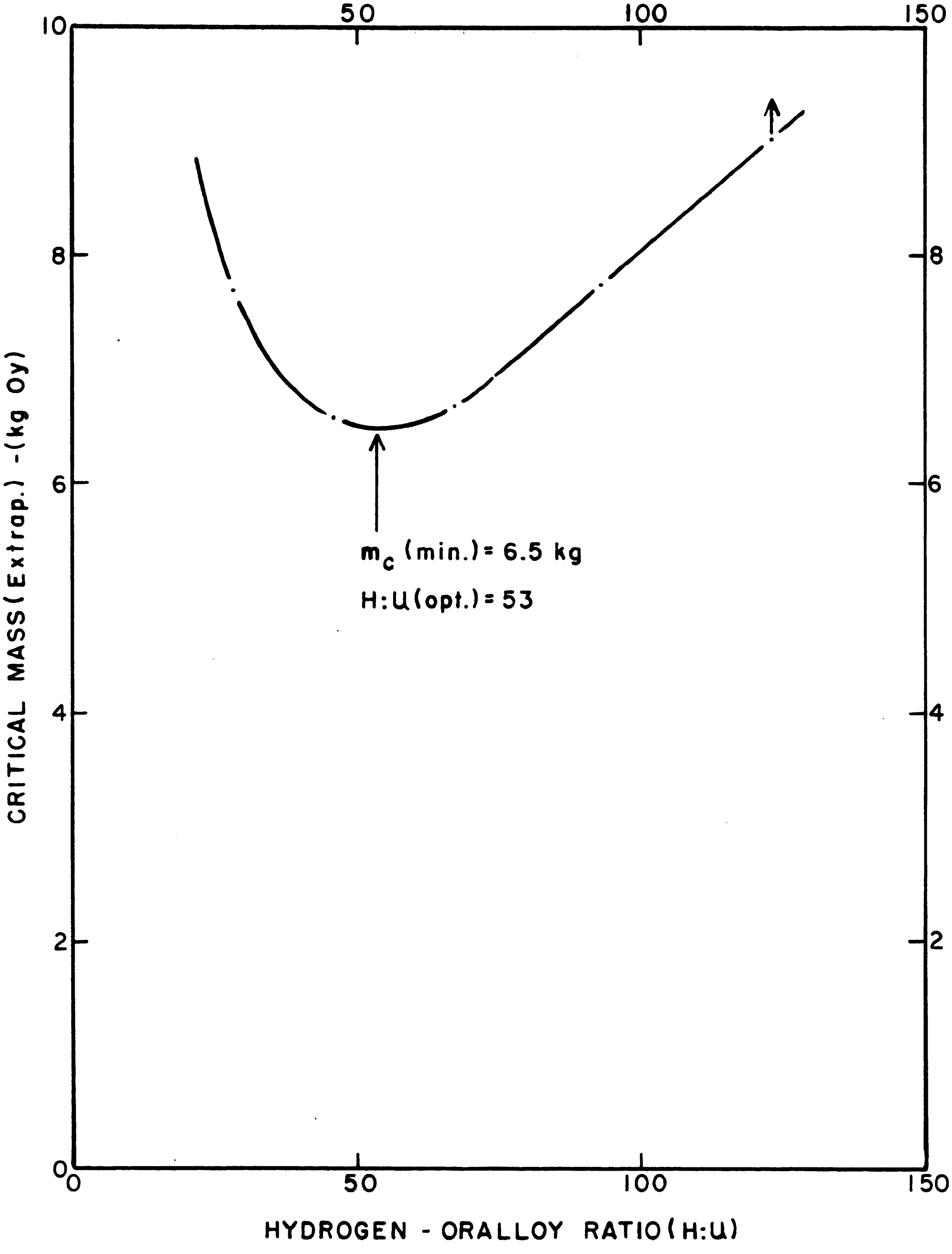

Fig. 10 Determination of m_c(min) for uniform lattices of 1/8 in. diameter Oy rod units immersed in water.

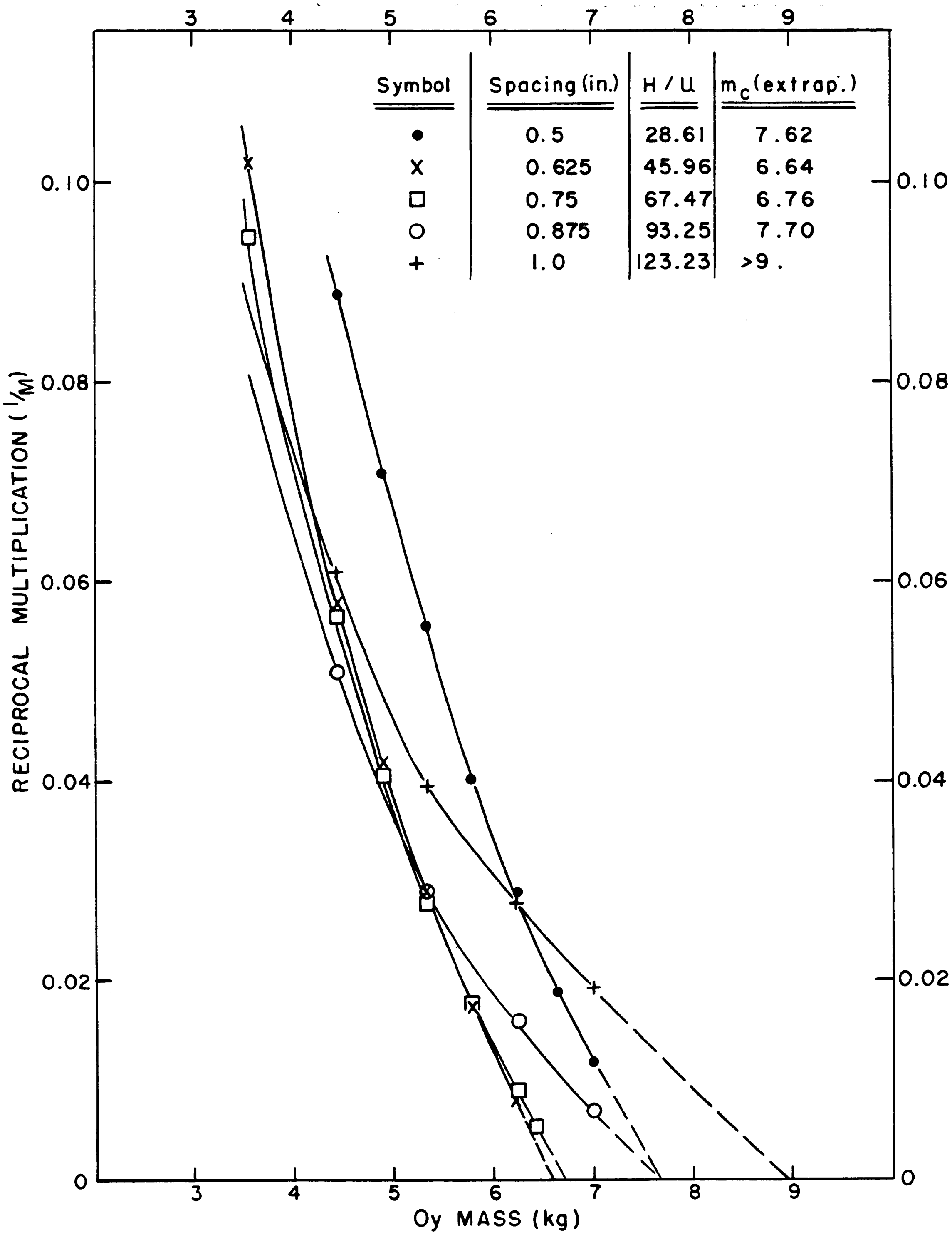

Fig. 11 Determinations of m_c with uniform lattices of 1/8 in. diameter Oy rod units immersed in water.

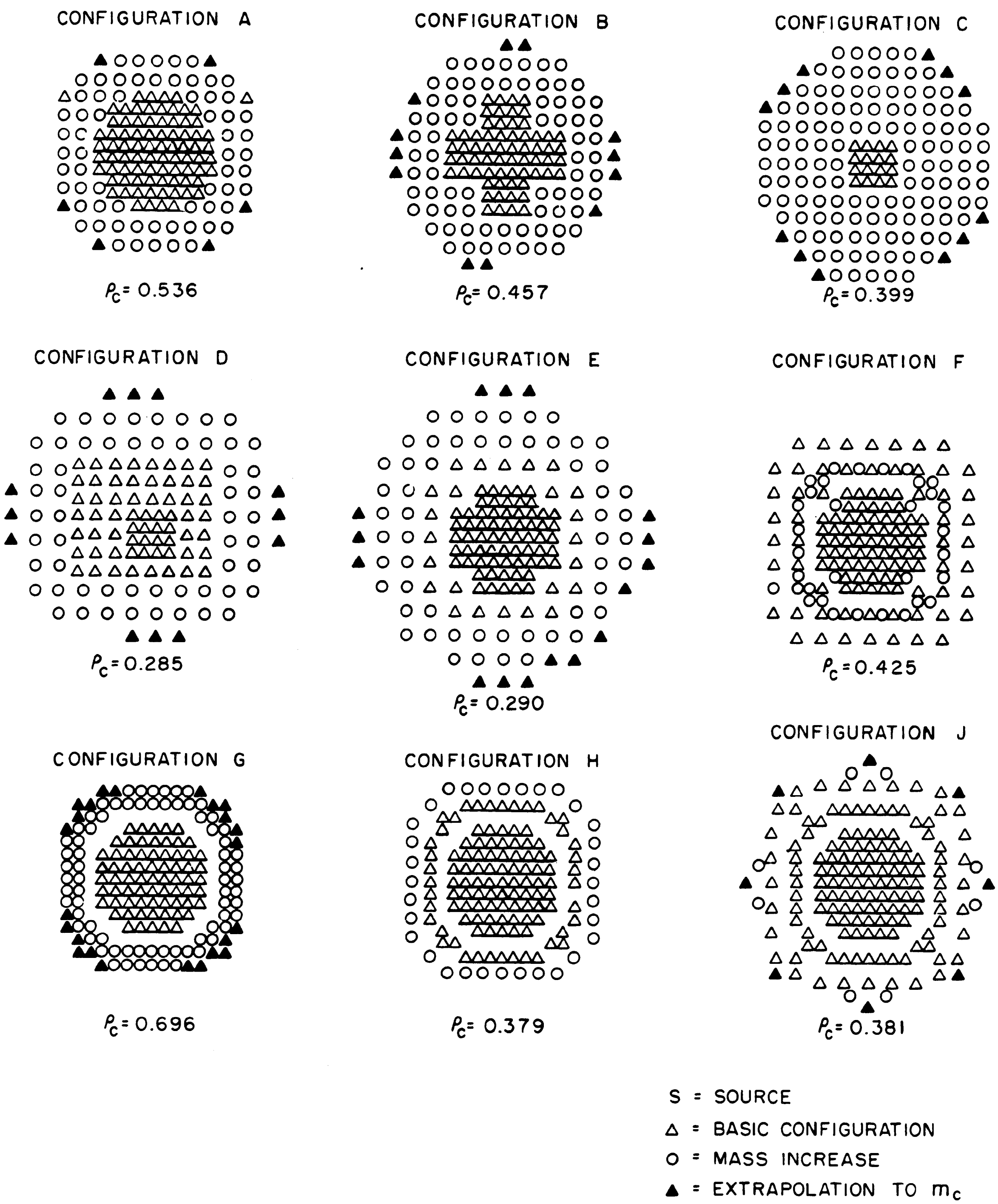

Fig. 12 Cylindrical configurations with varying core densities.

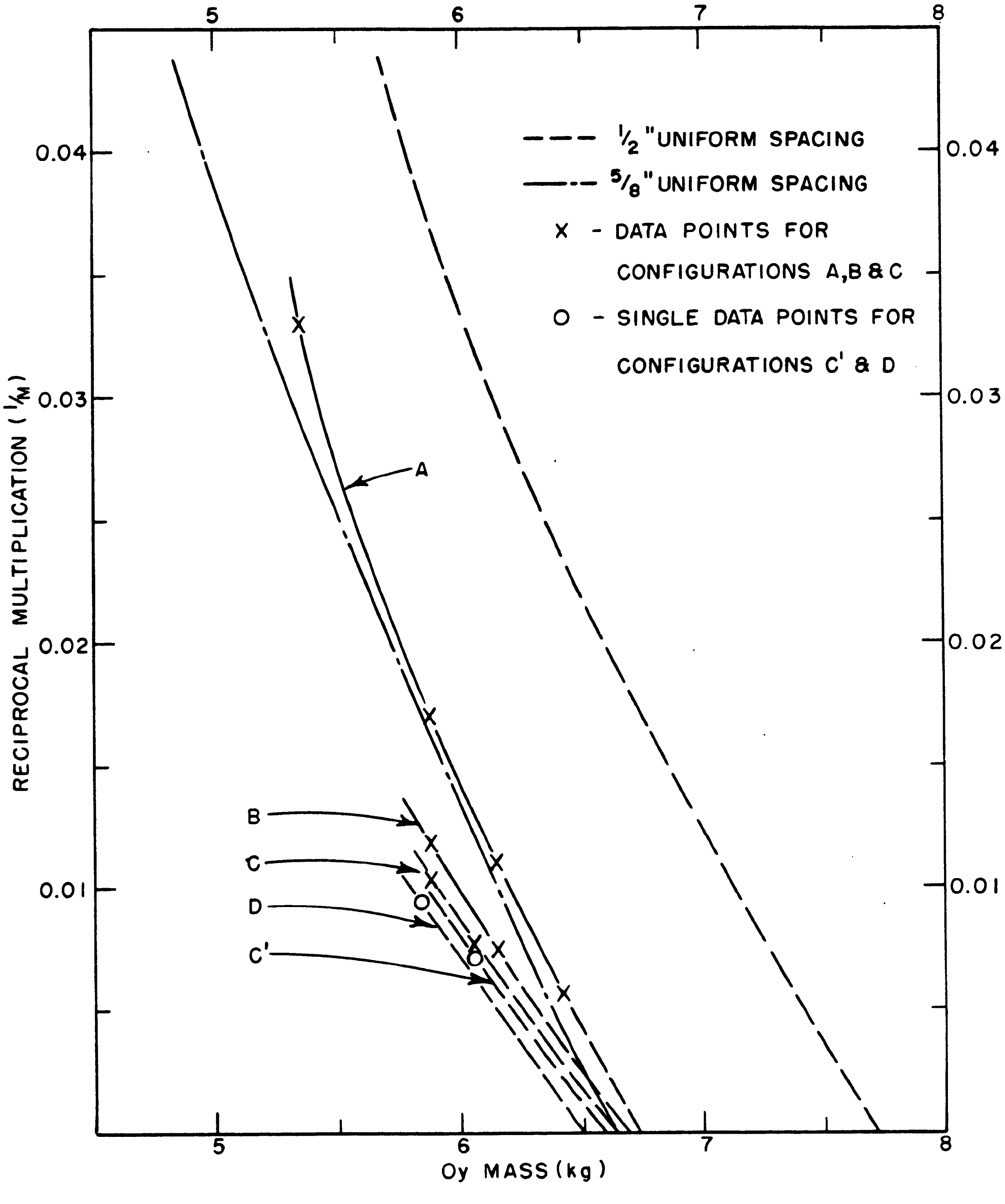

Fig. 13 Determinations of m_c for various 1/8 in. rod configurations having nonuniform Oy density distribution and immersed in water.

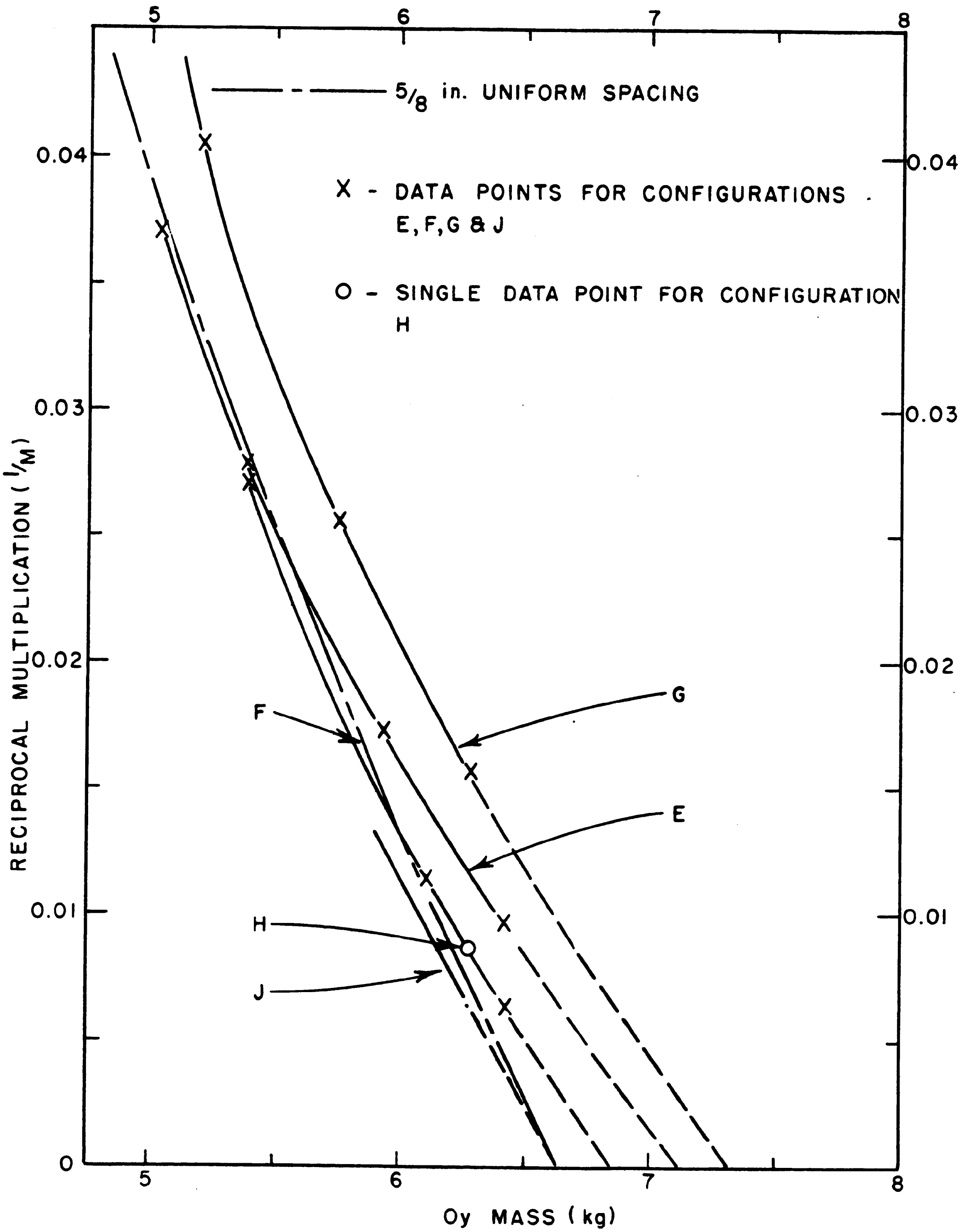

Fig. 14 Determinations of m_c for various 1/8 in. rod configurations having nonuniform Oy density distribution and immersed in water.

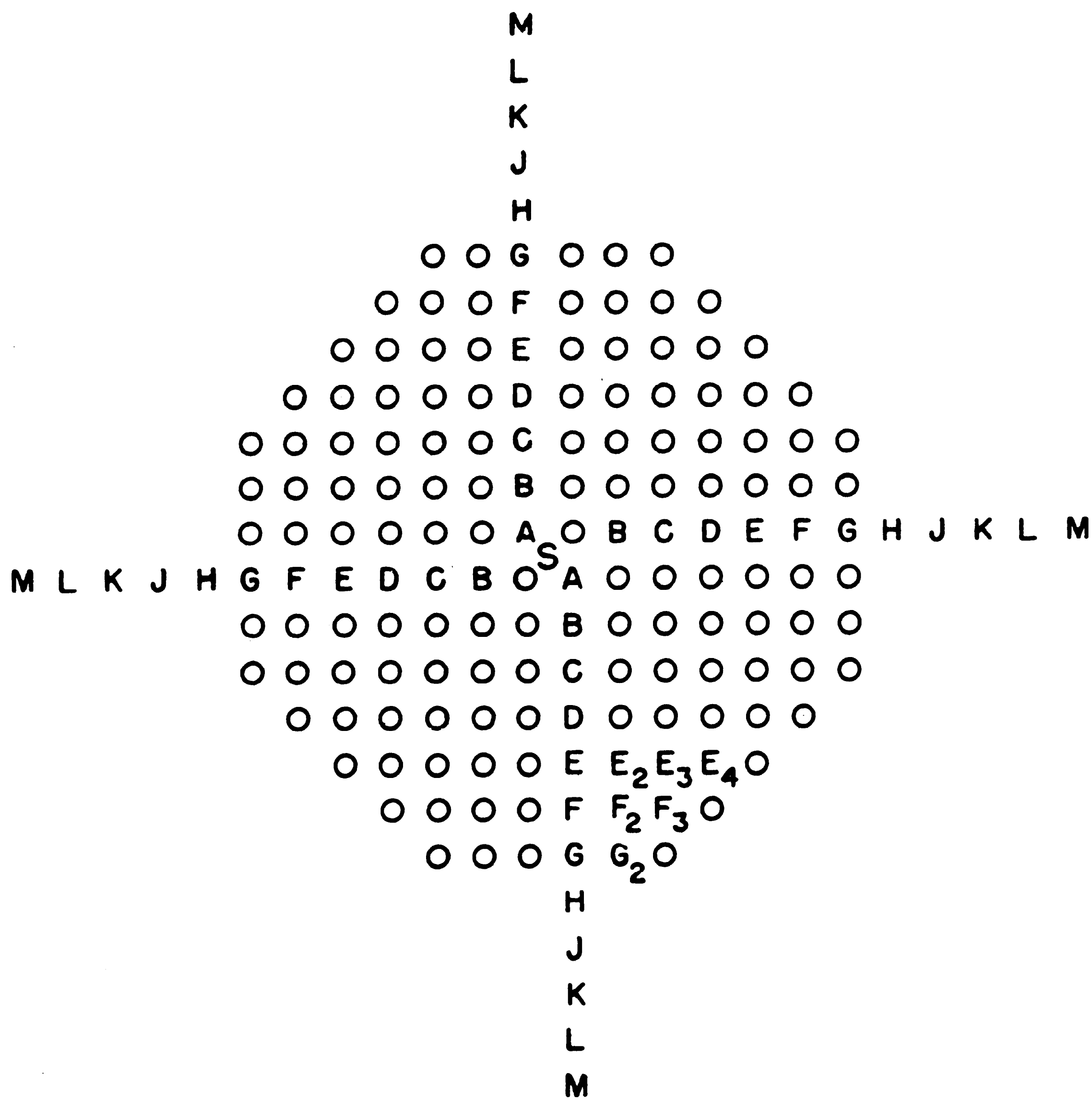

Fig. 15 Cross-sectional view of rod positioning for placement of various materials.

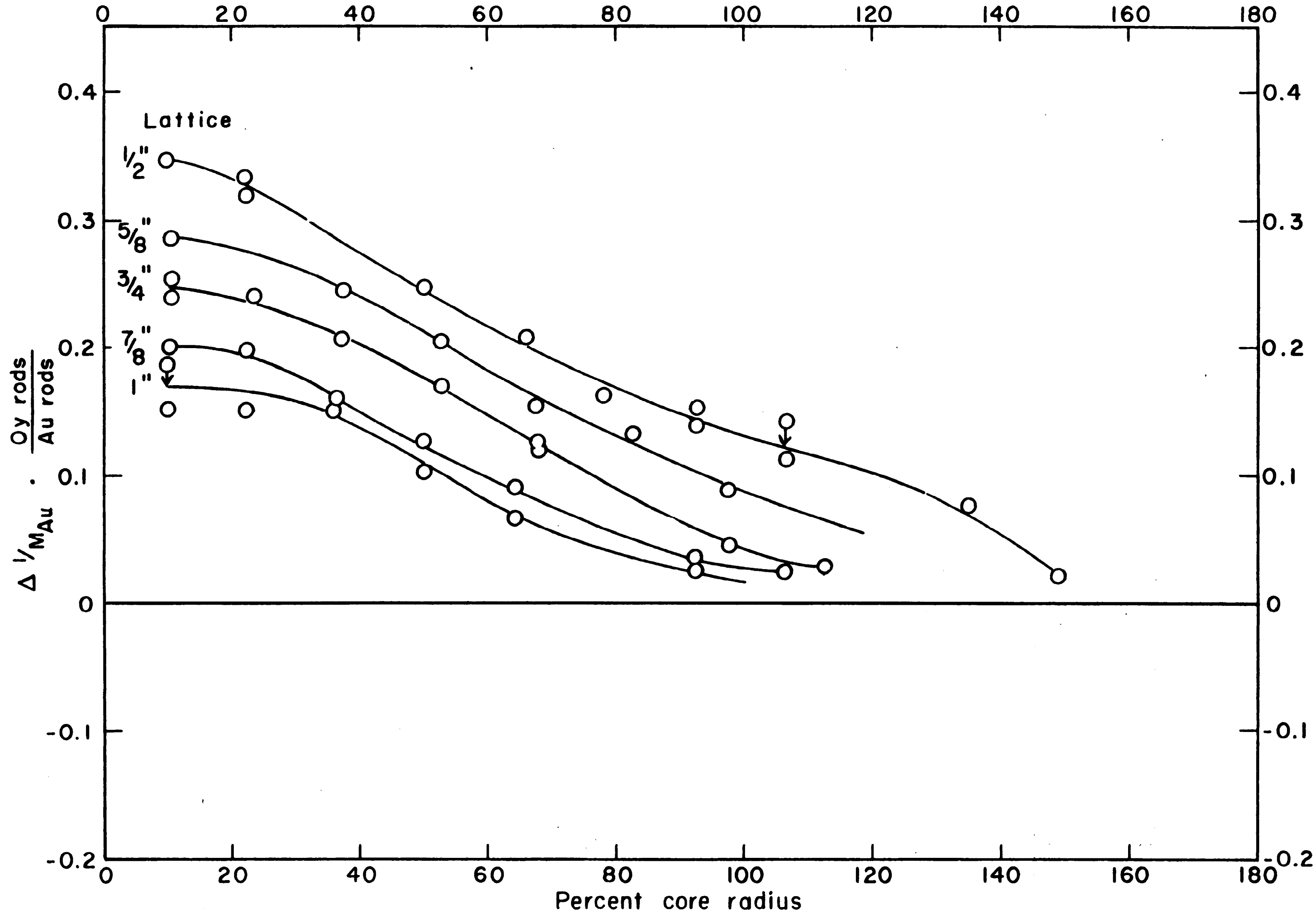

Fig. 16 $(1/M_{Au} - 1/M_{H_2O})$ per Au rod x number of Oy rods in the configuration vs percent core radius.

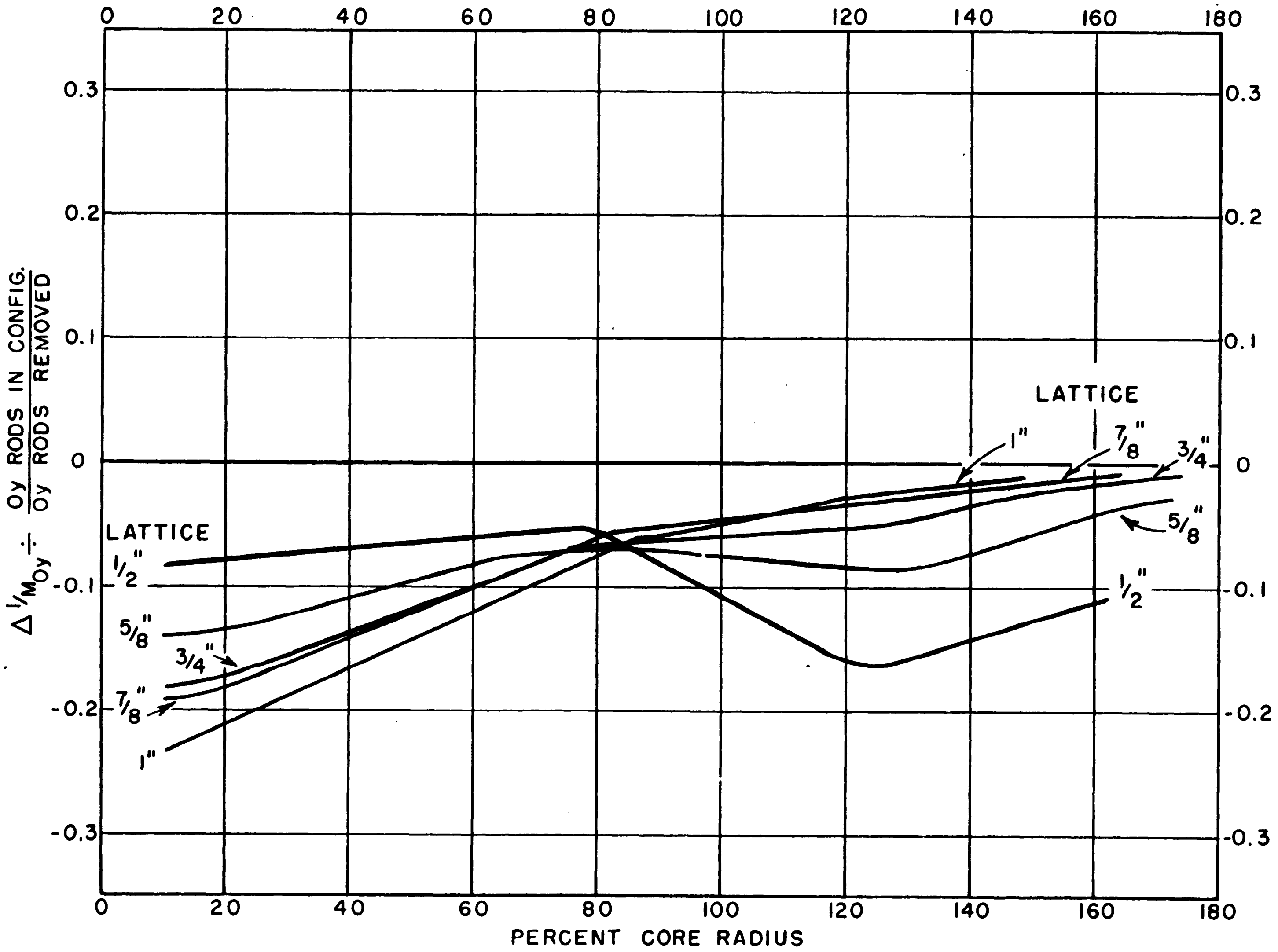

Fig. 17 $(1/M_{Oy} - 1/M_{H_2O})$ per Oy rod removed x number of Oy rods in the configurations vs percent core radius.

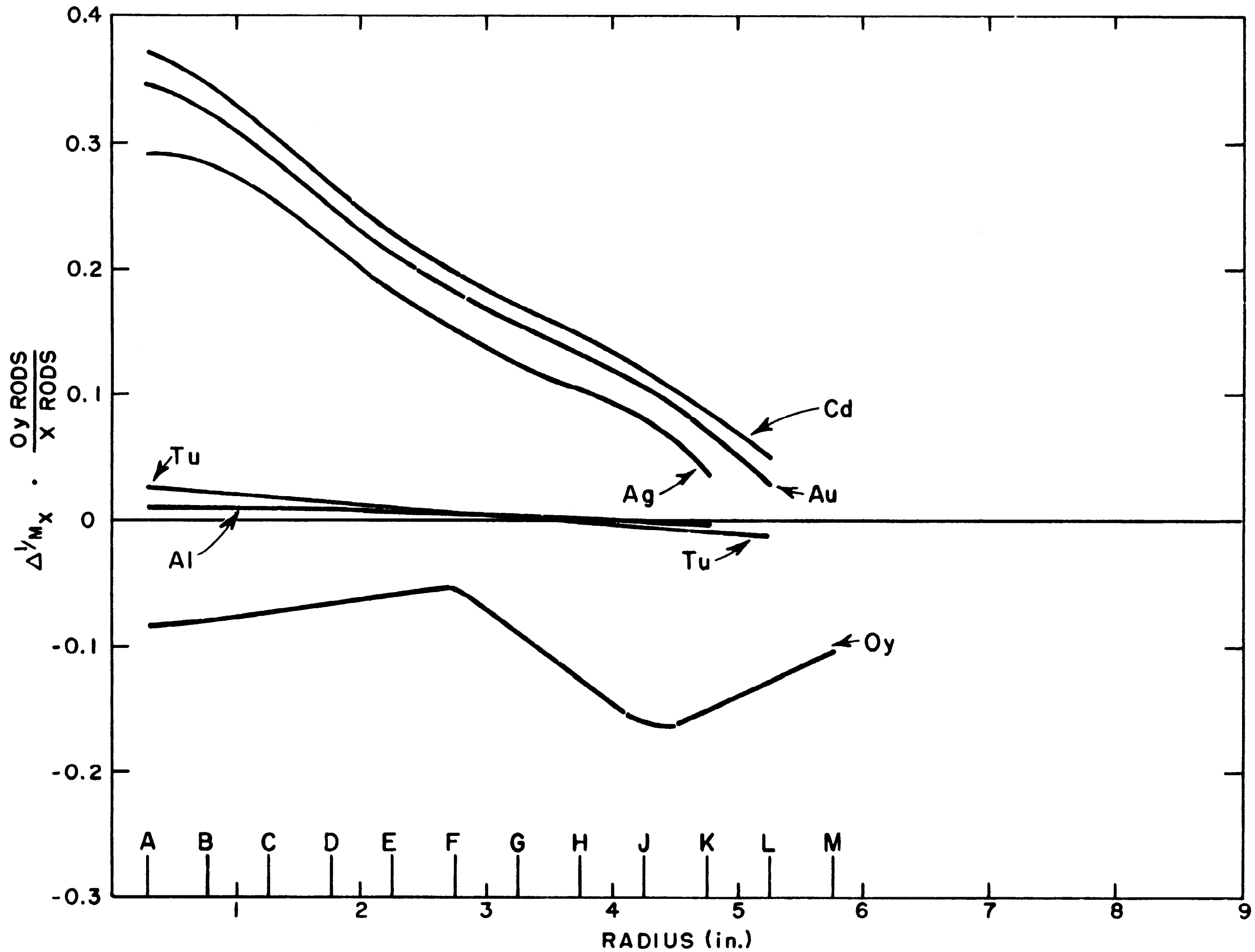

Fig. 18 $(1/M_x - 1/M_{H_2O})$ per replaced rod x number of Oy rods in the configuration vs radius as determined for the 1/2 in. lattice spacing.

5. Data Summary - Discussion

5.1 Minimum Critical Mass Relations

Table V summarizes the critical data at optimum lattice spacing for uniform configurations. For convenience of interpolation, Fig. 19 gives minimum critical mass as a function of the effective dimension of Oy unit. Figure 20 shows how minimum critical mass and optimum H:U depend upon the size of Oy unit. Included with the data from this experiment are determinations made at Oak Ridge[6,7] for flooded lathe turnings and for UO_2F_2 solutions. The Oak Ridge measurements on lathe turnings did not pin-point a minimum critical mass (at optimum H:U), but did provide an upper limit to this mass.

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(6) McLendon and Morfitt, op. cit.

(7) Beck, op. cit.

TABLE V.

SUMMARY OF CRITICAL DATA FOR FLOODED ARRAYS AT OPTIMUM H:U

Description	Oy Cubic (solid)	Oy Cubics (1 x 1 x 1 in.)	Oy Cubics (1/2x1/2x 1/2 in.)	Oy Rods (1/8 dia. x 12 in. long)	Lathe Chips ~0.005 in. thick	Solution ($UO_2F_2 + H_2O$)
Minimum m_c (kg Oy)	24.0	22.3	14.0	6.5	2.97*	0.947**
H:Oy atomic ratio (core vol.)	--	5.4	15	53	113	310
ρ_{Oy} (core vol.)	18.72	3.84	1.59	0.48	0.213	0.0787
Vol. (core) at critical (cm^3)	1282	5803	8790.4	13,541	12,864	11,347
Diameter (unit) (cm) (sph. mass equiv.)	13.48	3.1512	1.5756	0.3175	0.0127	3.416×10^{-8}
Gms/unit (Oy)	24,000	306.8	38.35	44.561	--	3.904×10^{-24}

*(1) Report Y-A2-71; lowest determined m_c by immersion of lathe chips in water.

**(2) Report K-343; lowest determined m_c for solutions.

5.2 Nonuniform Arrays of Oy Rods

Data on effectiveness of Oy rods in uniform lattices, Fig. 17, guided the selection of some of the nonuniform configurations of Fig. 12. Near optimum spacing, the Oy effectiveness changes with radius within the core, so it appears that some nonuniform rearrangement of rods should give a critical mass below the minimum that was measured for uniform lattices. As mentioned with reference to Table IV, no such reduced minimum was observed, presumably because there was insufficient flexibility of rod spacing.

5.3 Analysis of Replacement Data

Replacement data, $\Delta 1/M(x,r,L)$, for nonfissionable elements may be expressed as

$$\Delta 1/M(x,r,L) = f(r,L)\ \sigma_{abs}\ (x,r,L)\ N(x)$$

where $\sigma_{abs}\ (x,r,L)$ is closely the spectrum-averaged cross section of material x at position r in the lattice L. N is the number of atoms in the sample, and $f(r,L)$ is a function which is independent of x. Thus, one may construct the effective cross-section ratios

$$\frac{\sigma_{abs}(x,r,L)}{\sigma_{abs}(y,r,L)} = \frac{\Delta 1/M(x,r,L)}{\Delta 1/M(y,r,L)}\ \frac{N(y)}{N(x)}$$

Analysis of the multiplication data tables (Appendix) reveals that for a set of materials Au, Cd, and Ag, these effective cross-section ratios are, within experimental error, independent of r and L. Table VI gives the mean values of $\sigma(x,L)/\sigma(y,L)$ obtained by averaging over r for each of the five different lattice densities examined.

For fissionable materials, $\Delta 1/M(x,r,L)$ has the more complicated functional form

$$\Delta 1/M(x,r,L) = f(r,L)\left[\sigma_f(x,r,L) + \sigma_c(x,r,L) - g(r,L)\,\nu(x)\sigma_f(x,r,L)\right]N(x)$$

where g(r,L) is the relative effectiveness of fission spectrum to absorption spectrum neutrons, and ν is the number of neutrons produced per fission by material x ($\sigma_c + \sigma_f$ being the capture and fission cross section of the material). The function g(r,L), for given L, is known to assume a minimum value somewhat less than unity at r = 0 and to become arbitrarily large as $r \to \infty$. If one proceeds as above for the nonfissionable materials, one has for example

$$\frac{\sigma_{abs}(Oy,0,L)}{\sigma_{abs}(Au,0,L)} \equiv \frac{\left[\sigma_f+\sigma_c-g(0,L)\nu\sigma_f\right](Oy,0,L)}{\sigma_{abs}(Au,0,L)} = \frac{\Delta 1/M(Oy,0,L)\ N(Au)}{\Delta 1/M(Au,0,L)\ N(Oy)}$$

Values of this ratio for the various lattice densities are given in Table VI and indicate a strong dependence of g(0,L) on L.

TABLE VI.

EFFECTIVE CROSS-SECTION RATIOS DETERMINED WITH REPLACEMENT DATA

ρ_{Oy}(core)	0.870	0.551	0.379	0.276	0.209	Av.
$\frac{\sigma_{abs}(Ag)}{\sigma_{abs}(Au)}$	0.87 ± 0.03	0.84 ± 0.05	0.87 ± 0.03	0.85 ± 0.03	0.85 ± 0.05	0.86
$\frac{\sigma_{abs}(Cd)}{\sigma_{abs}(Au)}$	1.58 ± 0.04	1.55 ± 0.03	1.58 ± 0.02	1.61 ± 0.05	1.60 ± 0.05	1.58
$\frac{\sigma_{abs}(Oy)^{*}}{\sigma_{abs}(Au)}$	-0.328	-0.65	-0.99	-1.30	-1.86	--

*Central points only.

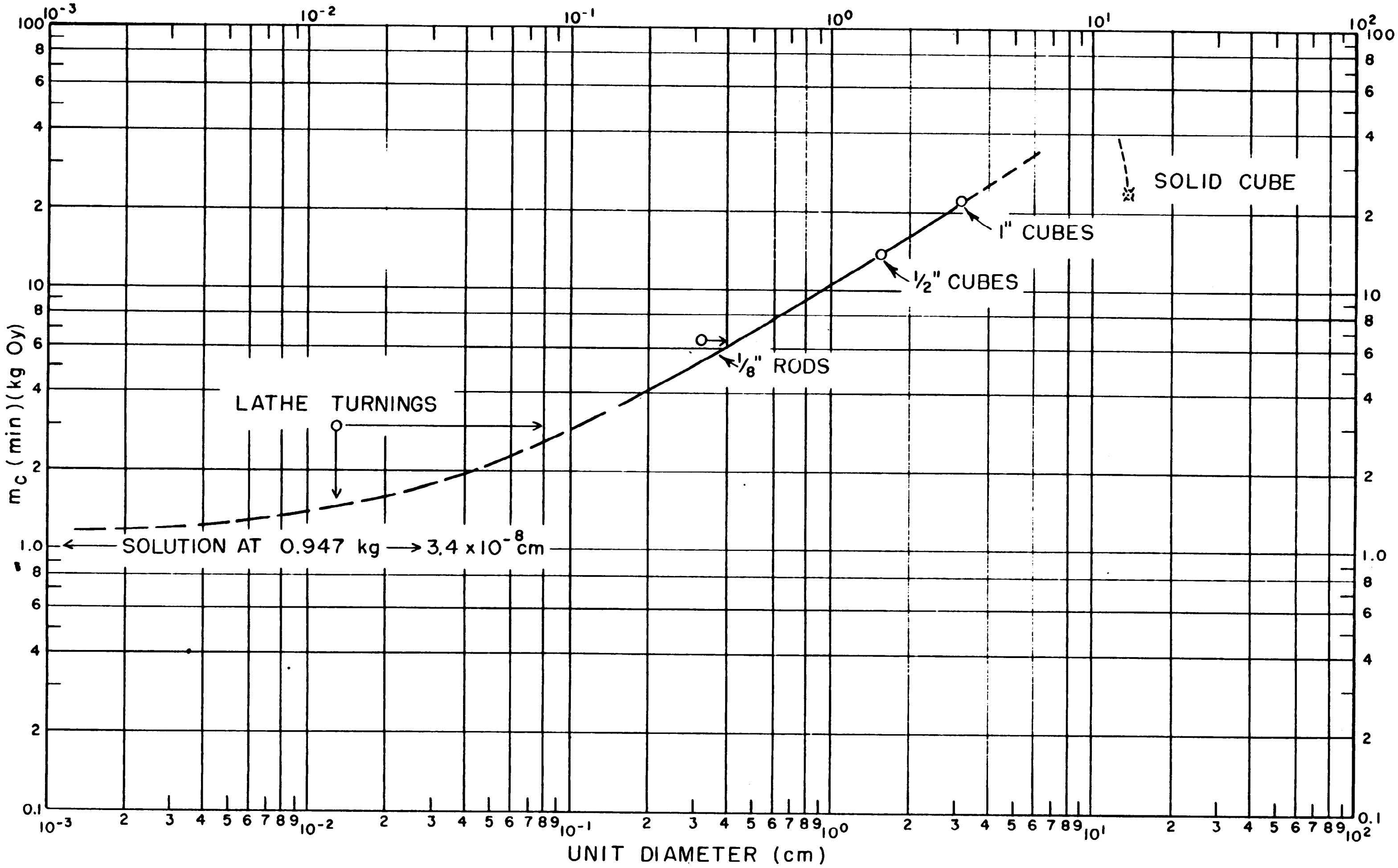

Fig. 19 Relation between m_c(min) and effective spherical diameter of Oy unit.

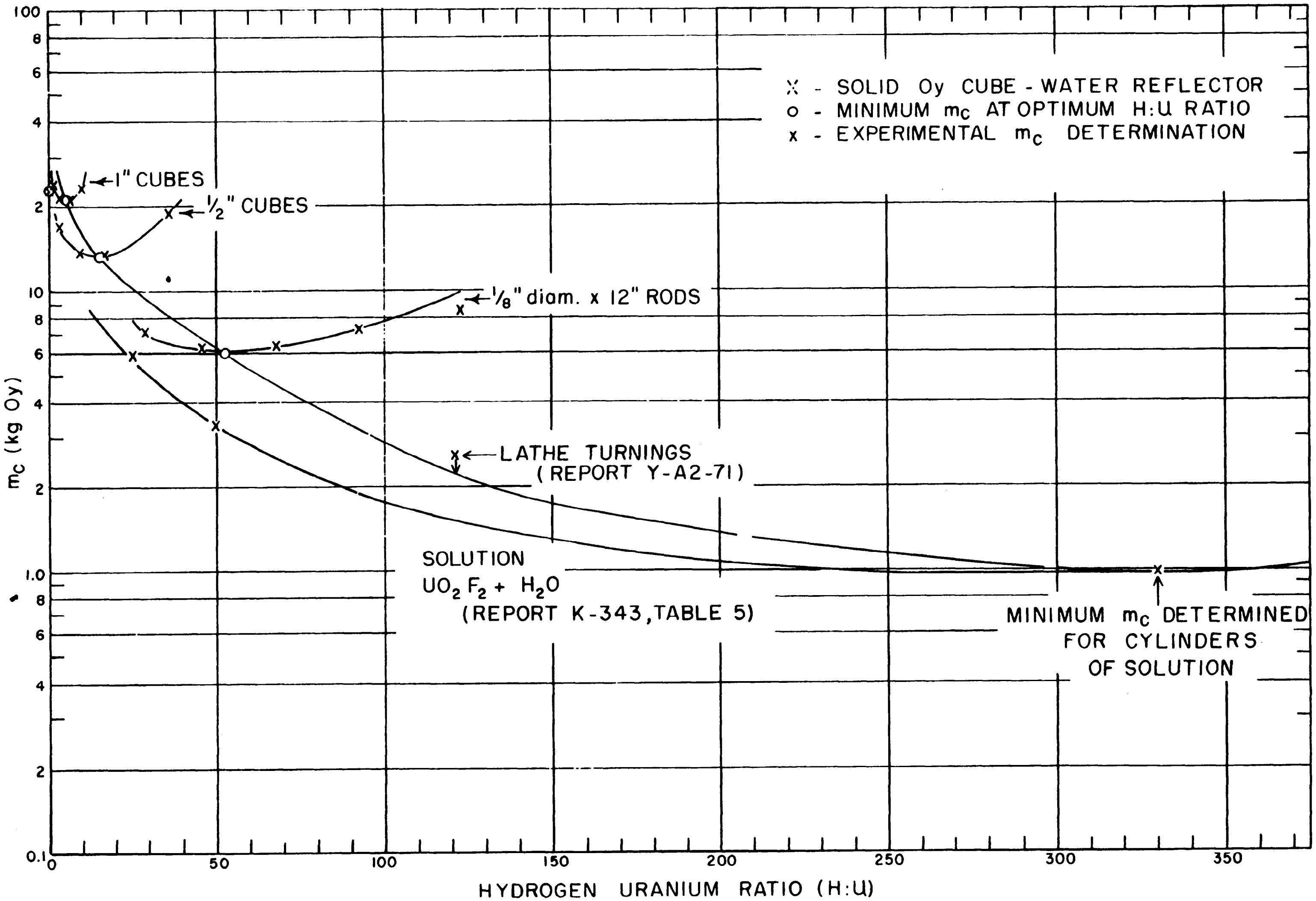

Fig. 20 Relations of m_c(Oy) vs H:U ratios for various Oy unit sizes.

APPENDIX

1/M VALUES FOR REPLACEMENT MEASUREMENTS

TABLE A-1.

$\Delta 1/M \times 10^3$ FOR CORES HAVING $\rho_{Oy} = 0.870$

$10^3 \times (1/M_x - 1/M_{H_2O})$ for 1/2 in. Spaced Configuration of 3.524 in. Radius

Position	Radius (in.)	% Core Radius	Oy	Au	Ag	Cd	Tu	Al
				1 Rod Replacement				
A	0.354	10.03	-0.705	2.235	1.885	2.395	0.225	0.095
B	0.791	22.43	-0.71	2.150	1.850	2.330	0.140	0.120
C								
D	1.768	50.16	-0.57	1.600	1.430	1.720	0.120	0.120
E	2.264	64.24	-0.359	1.345	1.155	1.435	0.009	-0.15
F	2.761	78.36	-0.254					
G	3.260	92.49	-0.614	0.890	0.800	1.070	0	-0.030
H	3.758	106.65	-0.795	0.914	0.634	0.960	0.024	
				2 Rod Replacement				
A	0.354	10.03	-1.09					
B				4.175				
C								
D								
E								
F				2.12				
G				2.005				
H				1.484				
J								
K				0.984				
L				0.284				
				4 Rod Replacement				
B	0.791	22.43	-2.015		7.025	8.775	0.615	0.305
C	1.275	36.17	-1.850					
D	1.768	50.16	-1.730					
E	2.264	64.24	-1.590					
F	2.761	78.36	-1.387		3.830	5.00	0.260	0.080
G	3.260	92.49	-2.323		3.475	4.745	0.450	0.015
H	3.758	106.65	-3.320		2.644	3.694	-0.096	-0.086
J	4.258	120.82	-4.180					
K	4.757	134.98	-3.970		1.704	2.414	-0.266	-0.096
L	5.256	149.15				1.174	-0.326	
M	5.756	163.33	-2.810					

TABLE A-2.

$\Delta 1/M \times 10^3$ FOR CORES HAVING ρ_{Oy} = 0.551

$10^3 \times (1/M_x - 1/M_{H_2O})$ for 5/8 in. Spaced Configuration of 4.173 in. Radius

Position	Radius (in.)	% Core Radius	Oy	Au	Ag	Cd
			2 Rod Replacement			
A	0.442	10.59	-2.024	4.235	3.565	4.435
B						
C				3.620		
D				3.025		
E				2.230		
F				1.970		
G				1.277		
			4 Rod Replacement			
B	0.988	23.68	-3.877			
C	1.593	38.18	-3.282		6.15	8.20
D	2.210	52.95	-2.704		5.135	6.895
E	2.830	67.81	-2.237		4.00	5.340
F	3.452	82.71	-1.917		2.920	3.940
G	4.074	97.64	-2.183		2.217	3.057
H	4.698	112.58	-2.340			
J	5.322	127.53	-2.530			
K	5.946	142.48	-2.000			
L	6.570	157.44	-1.350			
M	7.194	172.40	-0.850			
E_3	3.217	77.09	-3.21			
F_2	3.563	85.38	-2.86			
G_2	4.169	99.90	-2.668			

TABLE A-3.

$\Delta 1/M \times 10^3$ FOR CONFIGURATION HAVING ρ_{Oy} = 0.379

$10^3 \times (1/M_x - 1/M_{H_2O})$ for 3/4 in. Spaced Configuration at 5.008 in. Radius

Position	Radius (in.)	% Core Radius	Oy	Au	Ag	Cd	Tu	Al
			1 Rod Replacement					
A	0.530	10.59	-1.287	1.824	1.524	2.014		
B	1.186	23.68	-1.181	1.980	1.71	1.990		
C	1.912	38.18	-1.091	1.490	1.23	1.62		
D	2.652	52.95	-0.887	1.214	1.084	1.414		
E	3.396	67.81	-0.698	0.897	0.771	1.043		
F	4.142	82.71	-0.471					
G	4.889	97.64	-0.494	0.357	0.357	0.472		
H	5.638	112.58	-0.489	0.250	0.199	0.290		
			2 Rod Replacement					
A	0.530	10.59	-2.714	3.625	3.115	4.175	0.075	0.035
B				3.63				
C				3.135				
D								
E				1.80				
F								
G				0.66				
H				0.373				
			4 Rod Replacement					
B	1.186	23.68	-5.109		6.04	8.280	0.070	0.060
C	1.912	38.18	-4.213		5.335	7.165		
D	2.652	52.95	-3.377					
E	3.396	67.81	-2.591		3.070	4.29		
F	4.142	82.71	-2.007					
G	4.889	97.64	-1.901		1.250	1.74	-0.160	-0.15
H	5.638	112.58	-1.652		0.724	1.034	-0.146	-0.056
J	6.387	127.53	-1.453					
K	7.135	142.48	-0.953					
L	7.885	157.44	-0.573					
M	8.634	172.40	-0.353					
E_3	3.861	77.10	-2.747					
E_4	4.276	85.38	-1.220*					
F_2	4.276	85.38	-2.467					
G_2	5.003	99.90	-2.007					

*Two rods only.

TABLE A-4

$\Delta 1/M \times 10^3$ FOR CONFIGURATIONS HAVING $\rho_{Oy} = 0.276$

$10^3 \times (1/M_x - 1/M_{H_2O})$ for 7/8 in. Spaced Configuration of 6.167 in. Radius

Position	Radius (in.)	% Core Radius	Oy	Au	Ag	Cd
			1 Rod Replacement			
A			-1.299	1.344	1.105	1.495
B			-1.206	1.307	1.078	1.482
C			-1.016	1.066	0.906	1.239
D			-0.844	0.829	0.719	0.961
E			-0.644	0.608	0.491	0.740
F			-0.448			
G			-0.388	0.244	0.137	0.264
H			-0.248	0.154	0.162	0.261
			2 Rod Replacement			
A	0.618	10.03	-2.540			
			4 Rod Replacement			
B	1.383	22.43	-4.700			
C	2.231	36.17	-4.030			
D	3.094	50.16	-3.200			
E	3.962	64.24				
F	4.832	78.36	-1.710			
G	5.704	92.49	-1.310			
H	6.577	106.65	-1.130			
J	7.451	120.82	-0.860			
K	8.324	134.98	-0.590			
L	9.198	149.15				
M	10.073	163.33	-0.200			
F_3	5.286	85.71	-1.31			

TABLE A-5.

$\Delta 1/M \times 10^3$ FOR CONFIGURATIONS HAVING $\rho_{Oy} = 0.209$

$10^3 \times (1/M_x - 1/M_{H_2O})$ for 1.0 in. Spaced Configurations of 7.045 in. Radius

Position	Radius (in.)	% Core Radius	Oy	Au	Ag	Cd	Tu
			1 Rod Replacement				
A			-1.590	1.20	0.88		-0.23
D			-0.72	0.74	0.66		0.09
G			-0.20	0.17	0.07		-0.14
			2 Rod Replacement				
A	0.707	10.03	-3.088	2.00	1.70	2.36	
B				2.005			
C				2.010			
E				0.875			
G				0.367			
			4 Rod Replacement				
B	1.581	22.43	-5.415		3.375	4.805	
C	2.550	36.17	-4.616		3.22	4.33	
D	3.536	50.16	-3.30				
E	4.528	64.24	-2.501		1.565	2.115	
F	5.523	78.36	-1.656				
G	6.519	92.49	-1.365		0.667	0.837	
H	7.517	106.65	-1.105				
J	8.515	120.82	-0.755				
K	9.513	134.98					
L	10.517	149.15	-0.315				
M	11.511	163.33					