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SPHERICAL AND CYLINDRICAL PLUTONIUM CRITICAL  
MASSES

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ABSTRACT

Experiments to determine critical masses of  $\delta$ -phase plutonium cylinders of three diameters with thin metallic reflectors are reported. Critical reflector thickness measurements were made with two spherical Pu cores; the cylindrical and spherical data are combined to yield shape factors for the spheres for  $U^{238}$  and Be reflection.

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\* Similar information is contained in a previous report, UCRL-4761.

## SPHERICAL AND CYLINDRICAL PLUTONIUM CRITICAL MASSES

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### I. INTRODUCTION

A concept useful in analysis of geometrical effects on critical mass is shape factor. This is defined as the ratio of the spherical critical mass to the critical mass in the configuration of interest, when both systems are reflected with equal thicknesses of the same material.

This report describes a series of spherical and cylindrical critical mass measurements carried out to obtain shape factor data on  $\delta$ -phase plutonium cylinders, with reflectors of 2 and 5 cm of uranium and Be.

The spherical phase consisted of the measurement of critical reflector thicknesses surrounding two plutonium spheres. The masses of these spheres (7.366 and 10.79 kg) were such that the critical reflector thicknesses would be about 2 and 5 cm. Cylindrical critical height determinations were made with cylinders of three diameters: 8.23, 9.87 and 13.85 cm. Shape factors were obtained for these cylinders by interpolation of the spherical results. An experimental relationship between shape factor and the cylindrical height/diameter ratio at critical was thereby obtained.

### II. EXPERIMENTAL PROCEDURE

#### 1. General

The plutonium used was  $\delta$ -phase Pu alloy. Densities of all parts were 15.8. The neutron emission of the  $\text{Pu}^{240}$  in the material served as a distributed source of neutrons for multiplication measurements.

Cylindrical critical height measurements were made by the usual method of plotting reciprocal multiplication ( $1/M$ ) against height as the latter was increased. Extrapolation to  $1/M = 0$  yields the critical height. The plutonium

cores were composed of disks, varying in the thickness from 0.25 to 1.50 inches, depending on diameter. The 8.23- and 9.87-cm diameter parts were coated with nickel, the thickness of which averaged 0.005 inch. The 13.85-cm-diameter disks were enclosed in 0.005-inch copper cans. Diameters quoted are uncoated; the uncoated cylinder height was used in plotting  $1/M$  curves.

The spherical measurements were performed in the same general fashion, the variable being reflector thickness. The 10.79-kg sphere was composed of four segments; the 7.366-kg sphere of two identical hemispheres. The nickel coatings averaged 0.005 inch in thickness. Hemispherical shells, machined to appropriate thicknesses, were used as reflectors.

## 2. Assembly Procedure

The assembly machine consisted of a vertical hydraulic ram, with a horizontal 0.010-inch steel diaphragm centered above it. For the cylindrical measurements, a thin-walled tube supported a horizontal table approximately 1/8-inch thick. The table was perforated with holes to reduce its effective density and thereby the incidental tamping.

In general, part of a plutonium cylinder with its appropriate reflector was placed at the diaphragm center; the remainder was placed on the ram. The two parts were assembled by raising the ram. For the spherical assemblies, the ram was fitted with a thin-walled aluminum tube which held the ram plutonium-reflector hemisphere.

Scram mechanism, allowing the ram to drop, were actuated by counting-rate meters if a preset count rate from the detectors was exceeded during the course of raising the ram to assemble the two halves of a cylinder or sphere.

## 3. Counting Equipment

Two  $\text{BF}_3$  long counters and two moderated LiI (Eu) scintillation detectors were used to measure neutron fluxes from the assemblies. The amount of paraffin surrounding the LiI crystals in such as to give a detector energy response similar to that of the long counters.

## 4. Multiplication Measurements

Small samples of the turnings from the various plutonium parts were available for determinations of their neutron emissions. The samples were

counted in a high-geometry-type counter, calibrated with a small mock fission neutron source. Measurement accuracies were around 3%. The unmultiplied neutron emission from the disks and spherical parts, and thereby from any assembly, could thus be calculated.

The sensitivities of the counters to fission neutrons were determined by measuring count rates due to a calibrated mock fission source placed at the point where the center of the assembly would be. The unmultiplied count rate from an assembly was taken to be the product of its unmultiplied neutron emission and the detector sensitivity.

Multiplication as used here is defined as the ratio of the multiplied count rate from an assembly to the unmultiplied count rate determined as described above. The base rates for a reflected system were corrected by a factor accounting for reflector effects on counting rates. This factor was measured by taking the ratio of count rates due to a mock fission source at the diaphragm center with a reflector shell surrounding it, to those obtained with the shell removed. The multiplications measured by the four detectors generally agreed within 10%.

### 5. Density Corrections

The presence of the nickel or copper coatings and the 0.010-in. -thick steel diaphragm effectively reduced the Pu core density and thereby the reactivity of the assembly. Attempts were made to determine the increase in measured  $1/M$  due to these dilutants, in order to find the  $1/M$  which would be measured if the assembly were composed of a solid Pu cylinder or sphere

These corrections were made by doubling (for convenience) the known coating thickness at every interface in the assembly separately, and measuring the resulting  $1/M$ . The change in  $1/M$  observed, from that of the unmodified assembly, was then due to the presence of the coating in question. This procedure was carried out at each interface, the  $1/M$  corrections for each being obtained. The sum of these was subtracted from the  $1/M$  obtained from the original unmodified assembly, the result being taken as the value which would be measured from a solid Pu assembly. These corrected points were extrapolated to find the critical height or reflector thickness.

The linear relationship between  $1/M$  and coating thickness was verified in many cases. Figure 1 shows the changes in  $1/M$  observed when the effective coating thicknesses in a cylindrical assembly were increased.

The averaged measured densities of the various reflector materials used are tabulated below.

The lithium hemispheres were formed by pouring molten lithium into copper cans; the density quoted is lower than theoretical, probably because of air bubbles in the castings.

Table I. Reflector Densities

Material	Density, g/cm <sup>3</sup>
Uranium	18.8
Beryllium (2% BeO)	1.86
Carbon (Pile Grade C-18)	1.632
Titanium	4.46
Lithium	0.5

### III. RESULTS

#### 1. Spherical

Table II lists the reflector configurations used and the results obtained. The values in column 3 are the average of results obtained by extrapolation of the corrected inverse multiplication curves of the four counters. The spread in corrected counter values is given in column 4. Columns 5 and 6 are the average uncorrected critical thicknesses and counter spreads. Figure 2 shows the uranium, Be and C results.

Multiplications reached with the thickest reflectors were about 100. Correspondingly the accuracies of the results are around 2% if effects of the coatings are considered. The measurements on the 10.79-kg sphere reflected with Li and Ti, as well as the 7.366-kg sphere reflected with carbon, were somewhat inconclusive since critical thicknesses were appreciably greater than the thickest reflectors practical for use. The  $1/M$  curves are shown in Fig. 3. Figure 4 is typical of the curves obtained with other reflecting arrangements. In some cases, for the thickest reflector used in a measurement, the correction applied was larger than the measured  $1/M$ ; the critical thickness was then taken to be the value at which the uncorrected curve passed through the  $1/M$  correction value. The average multiplications of the unreflected 10.79- and 7.366-kg spheres were 5.3 and 3.4, respectively.

The general procedure followed to make a spherical measurement was to obtain a preliminary  $1/M$  curve by use of relatively thin reflectors. The thicknesses of these were based on preliminary calculations of the critical thickness. On the basis of the preliminary curve, two relatively thick reflector shells were machined down to an appropriate diameter. The assembly of the sphere with these reflectors was carried out by decreasing separation distance between the two halves, plotting  $1/M$  as the separation was decreased. Extrapolation to zero separation yields the fully assembled  $1/M$ . Examples of curves so obtained are shown in Fig. 5. For comparison, the  $1/M/mil$  values obtained when steel shims were inserted in the mid-plane assembly interface are given; the two generally differ by 20-40%.

Table II. Critical Spherical Reflectors

1	2	3	4	5	6
Sphere mass, kg	Reflector material	Av corrected reflector thickness at critical, cm	Counter spread	uncorrected reflector k. at critical, cm	Counter spread
10.79	Uranium	1.93	1.94	2.08	2.12
			1.91		2.04
"	Beryllium	1.77	1.78	1.96	1.97
			1.75		1.94
"	Carbon	3.83	3.78	4.45	4.55
			3.84		4.25
"	Lithium	$\infty$			
"	Titanium	8		10	
7.366	Uranium	6.74	6.71	7.16	7.25
			6.77		7.10
"	Beryllium	5.25	5.36	5.50	5.63
			5.20		5.35

## 2. Cylindrical Results

Tables III, IV, and V give reflector configurations and results of the cylindrical critical-height measurements. The values of column 3 were obtained by extrapolation of the corrected  $1/M$  vs uncoated Pu height curves;



they are accurate within 1%, except as noted. Columns 4 and 5 are based on column 3. The uncoated assembly critical height is given in column 6, obtained by extrapolation of the uncorrected  $1/M$  vs uncoated  $Pu$  height curve. Total thicknesses of coatings and diaphragm are in column 7.

Examples of the  $1/M$  curves obtained are shown in Fig. 6. The variation of critical mass with the ratio of the critical height to the diameter is shown in Fig. 7.

### 3. Shape Factor Results

In order to obtain shape factor data, spherical critical-mass values were obtained for 2- and 5-cm reflector thicknesses by interpolation of the experimental spherical curve (Fig. 2). An uncertainty of approximately  $\pm 1\%$  in the critical-mass value is introduced by the interpolation of the uranium and Be curves. Since only one carbon point is available, the uncertainty is around  $\pm 2\%$ . Uncertainties in the experimental shape factors are, therefore,  $\pm 3\%$  for uranium and Be,  $\pm 4\%$  for carbon reflectors.

The spherical critical masses used in shape factor determinations are:

Uranium,	5 cm:	8.06 kg
	2 "	10.68 "
Beryllium,	5 "	7.48 "
	2 "	10.35 "
Carbon	5 "	10.05 "
	2 "	12.75 "

Shape factor results on this basis are shown in column 8 of Tables III, IV and V, and are shown graphically in Fig. 8.

Table III. 8.23-cm-diam  $\delta$  -Phase Pu Critical Cylinders

1	2	3	4	5	6	7	8
Reflec- tor mat- erial	Reflec- tor thick- ness, cm	Correct- ed uncoat- ed Pu crit- ical ht., cm	Correct- ed Pu <sup>239</sup> critical mass, kg	h/d at crit- ical	Uncorrect- ed uncoat- ed critical ht., cm	Total thick- ness of Ni + steel in assm'y, cm	Shape factor
Be	5	9.14	7.62	1.11	9.37	0.092	0.982
Be	2	15.0	12.5	1.82	15.63	.166	.828
Uranium	5	10.0	8.33	1.22	10.28	.122	.968
Uranium	2	15.5	12.9	1.88	16.06	0.141	0.828

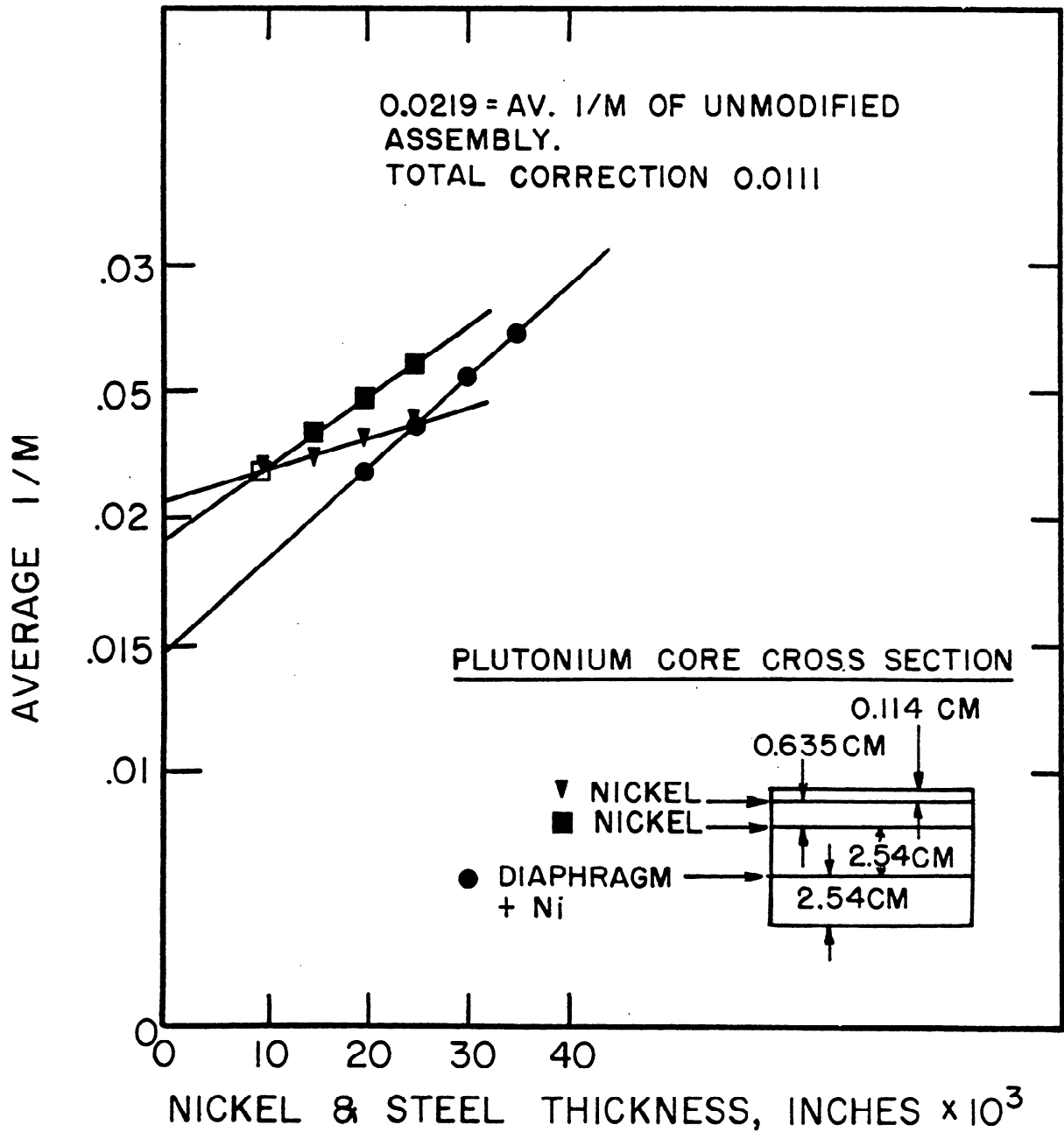
Table IV. 9.87-cm-diameter Cylinder Results

Uranium	5	6.91	8.24	0.700	7.02	0.102	0.978
Uranium	2	9.14	10.90	0.926	9.38	.127	.980
Be	5	6.35	7.58	0.643	6.43	.076	.987
Be	2	8.91	10.63	0.903	9.09	.102	.974
C	5	8.46	10.09	0.857	8.64	.102	.996
C	2	10.63	12.68	1.077	10.90	.152	.816
Steel	10	7.92*	9.45	0.802	7.98*	.102	
Poly- ethylene	10	7.01*	8.36	0.710	7.06*	.076	
None	-	17.3*	20.6	1.75	18.2*	0.300	

\* Accuracies:  $\pm 4\%$ .

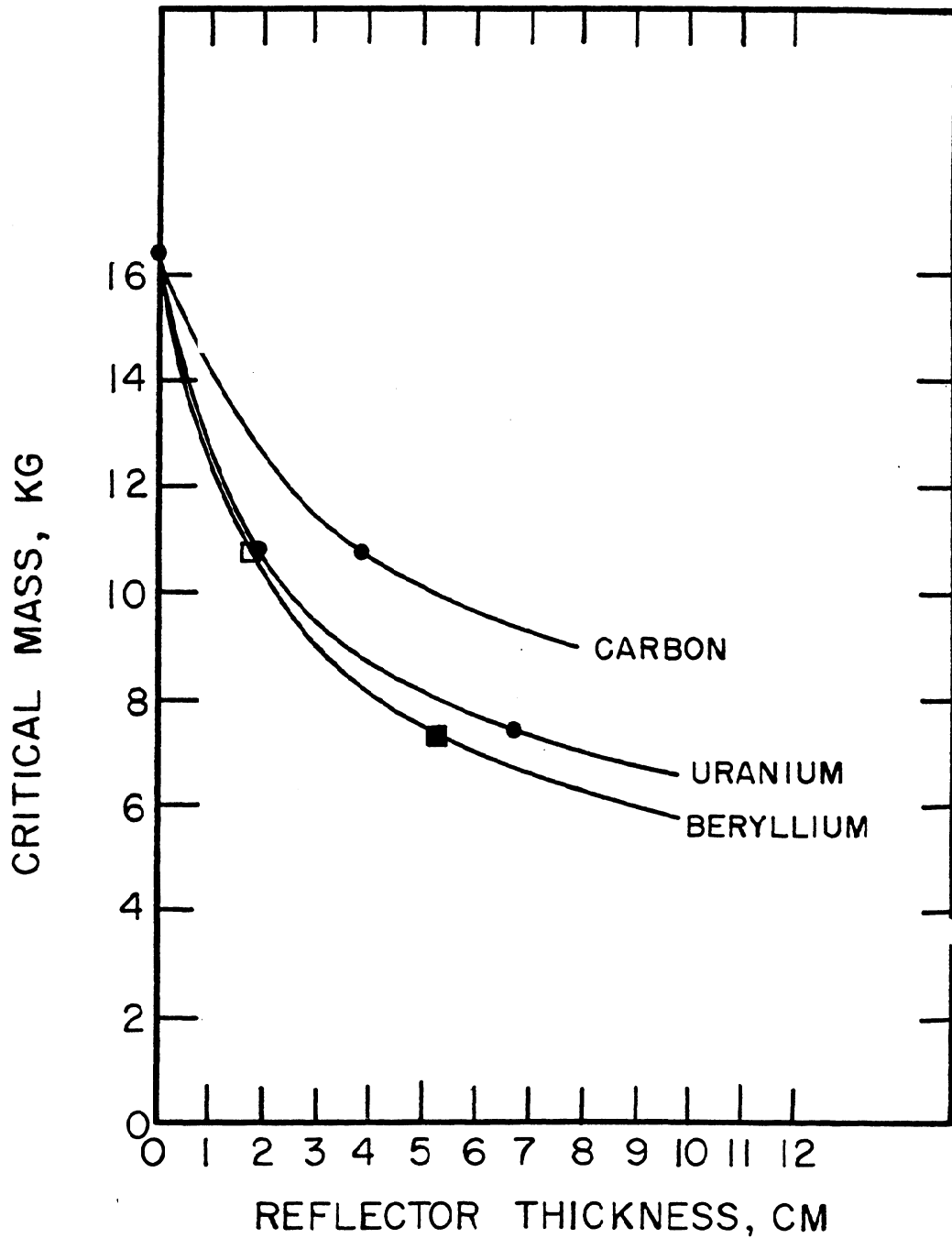
Table V. 13.85-cm-diameter  $\delta$  -Phase Pu Cylinders

None	-	8.00	18.72	0.577	8.30	0.230	-
Uranium	5	4.38	10.25	.316	4.45	.102	0.786
Uranium	2	5.545	12.98	.400	5.685	.127	0.823
Be	5	3.94	9.22	.284	4.04	.102	0.811
Be	2	5.428	12.70	0.392	5.57	0.127	0.815



MUL-3669

Fig. 1. Change of  $1/M$  with coating thickness for 9.87-cm-diam Pu cylinder (5-cm uranium reflector).



MUL-3670

Fig. 2. Variation of spherical critical mass with reflector thickness.

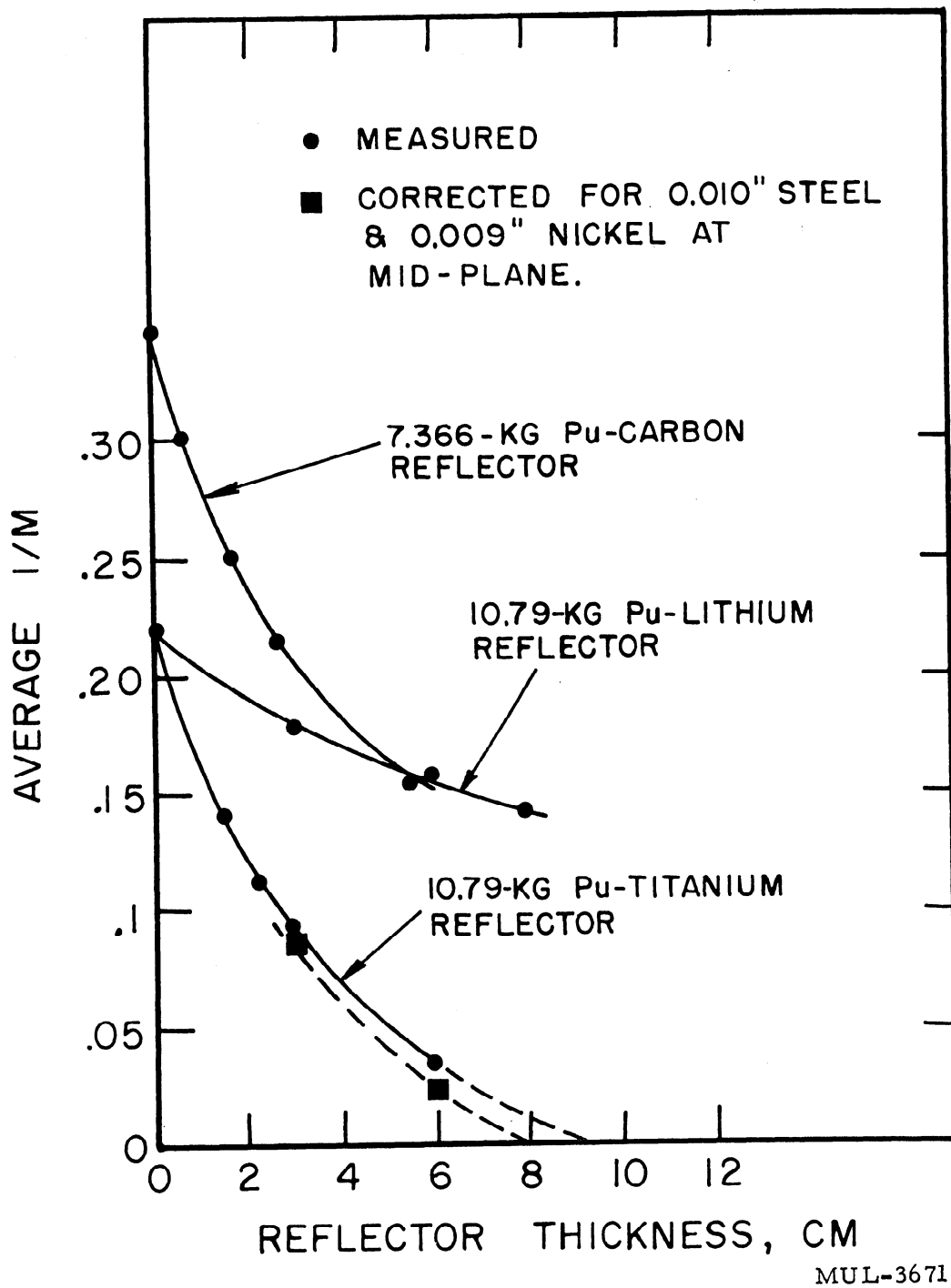
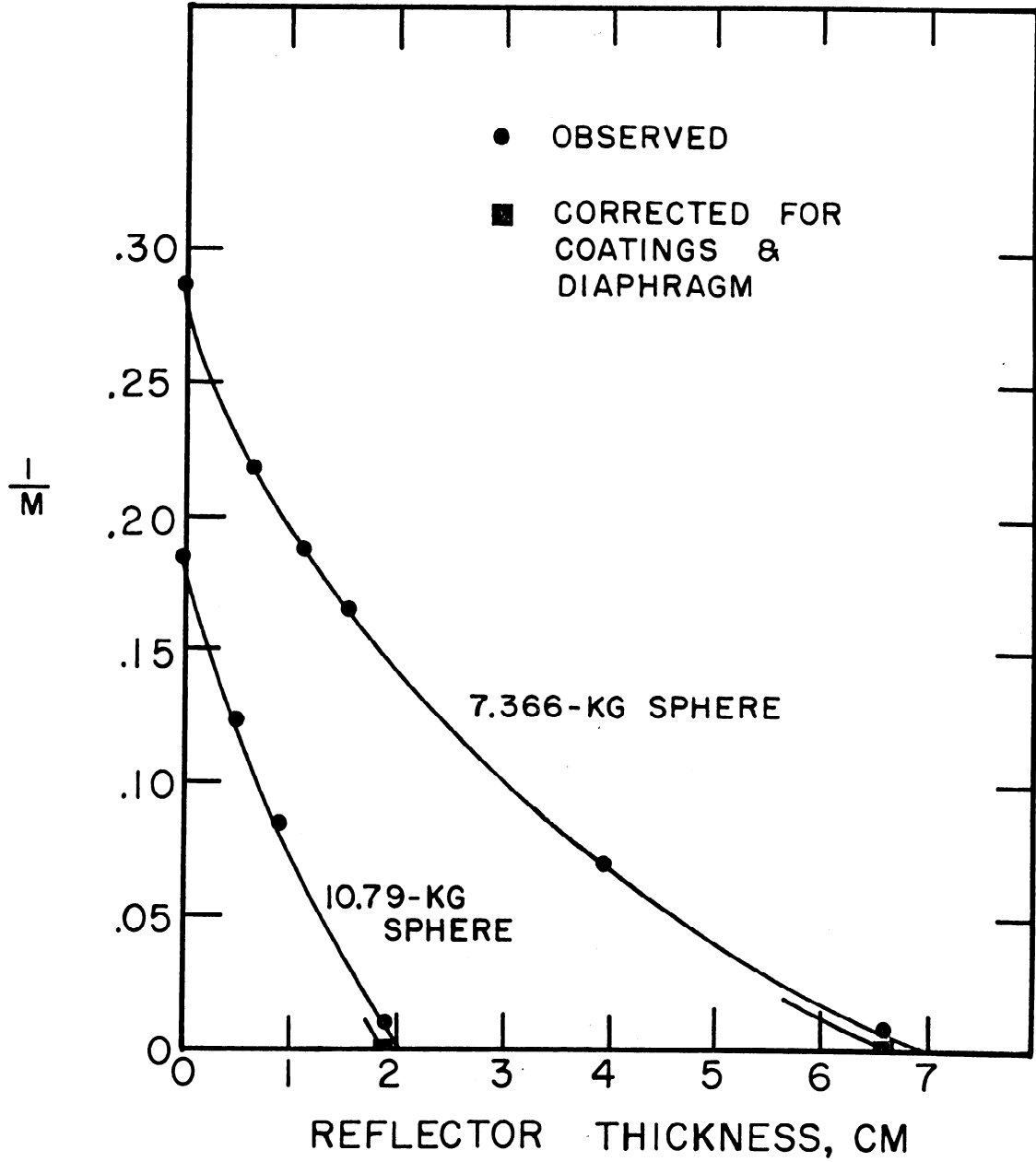
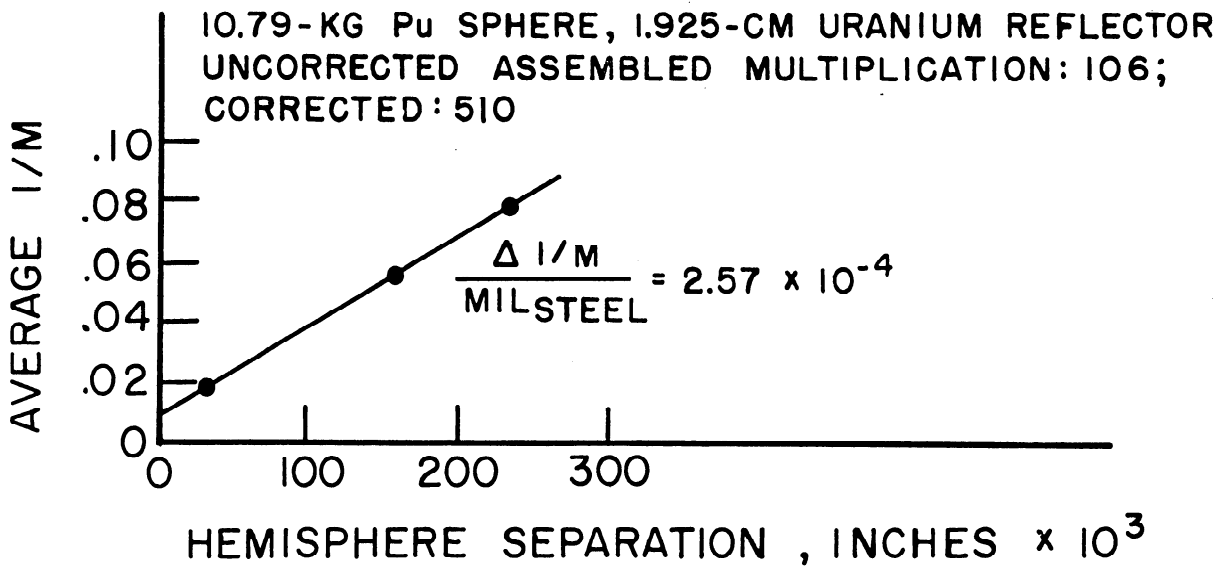
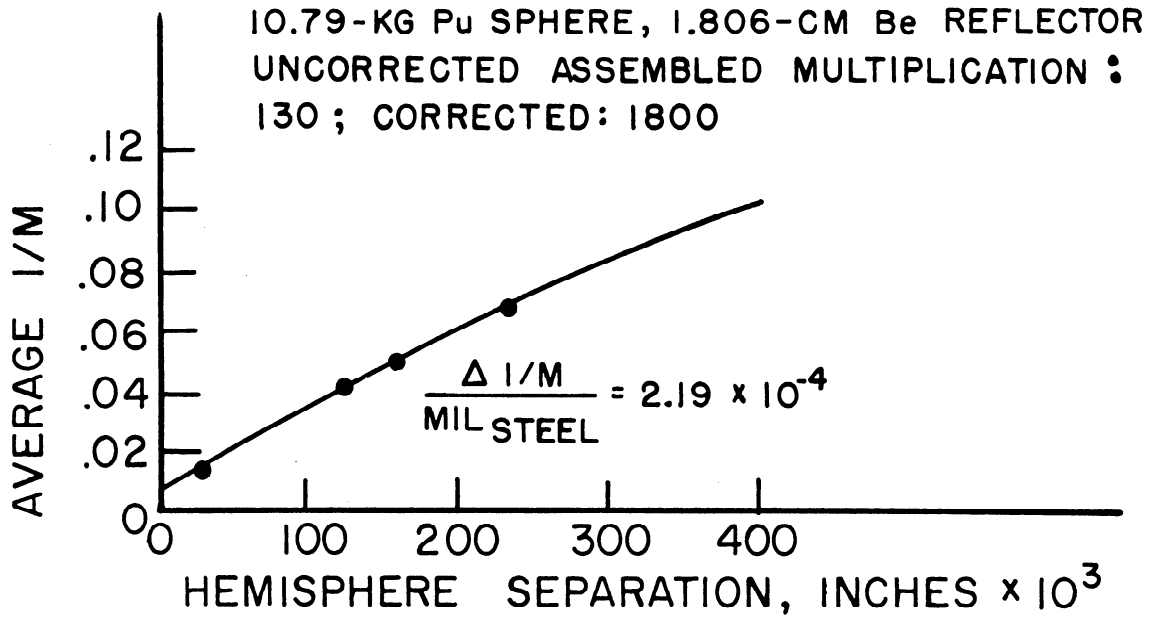


Fig. 3. Average inverse multiplication vs reflector thickness.



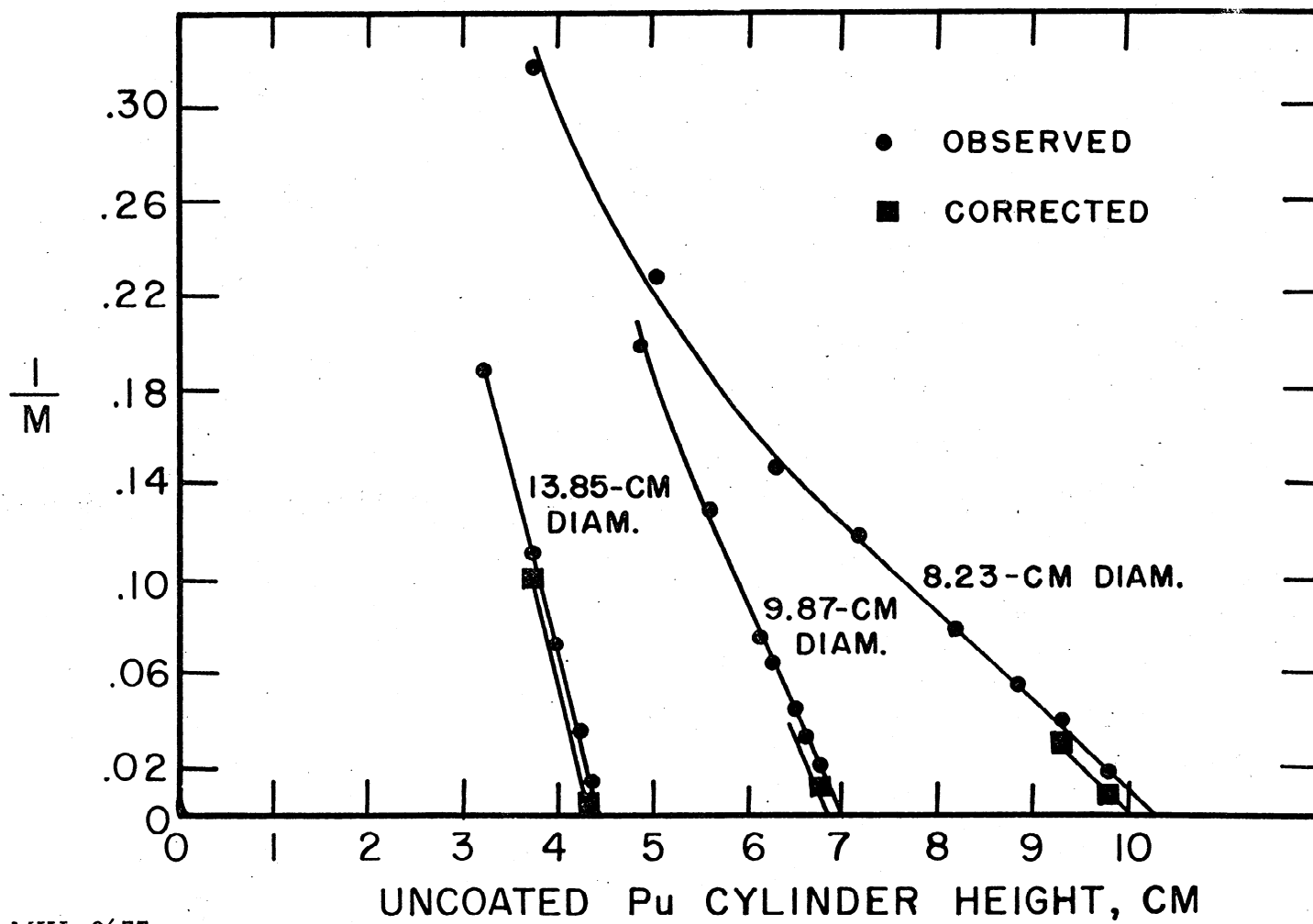
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Fig. 4.  $1/M$  curves for Pu spheres with uranium reflectors.



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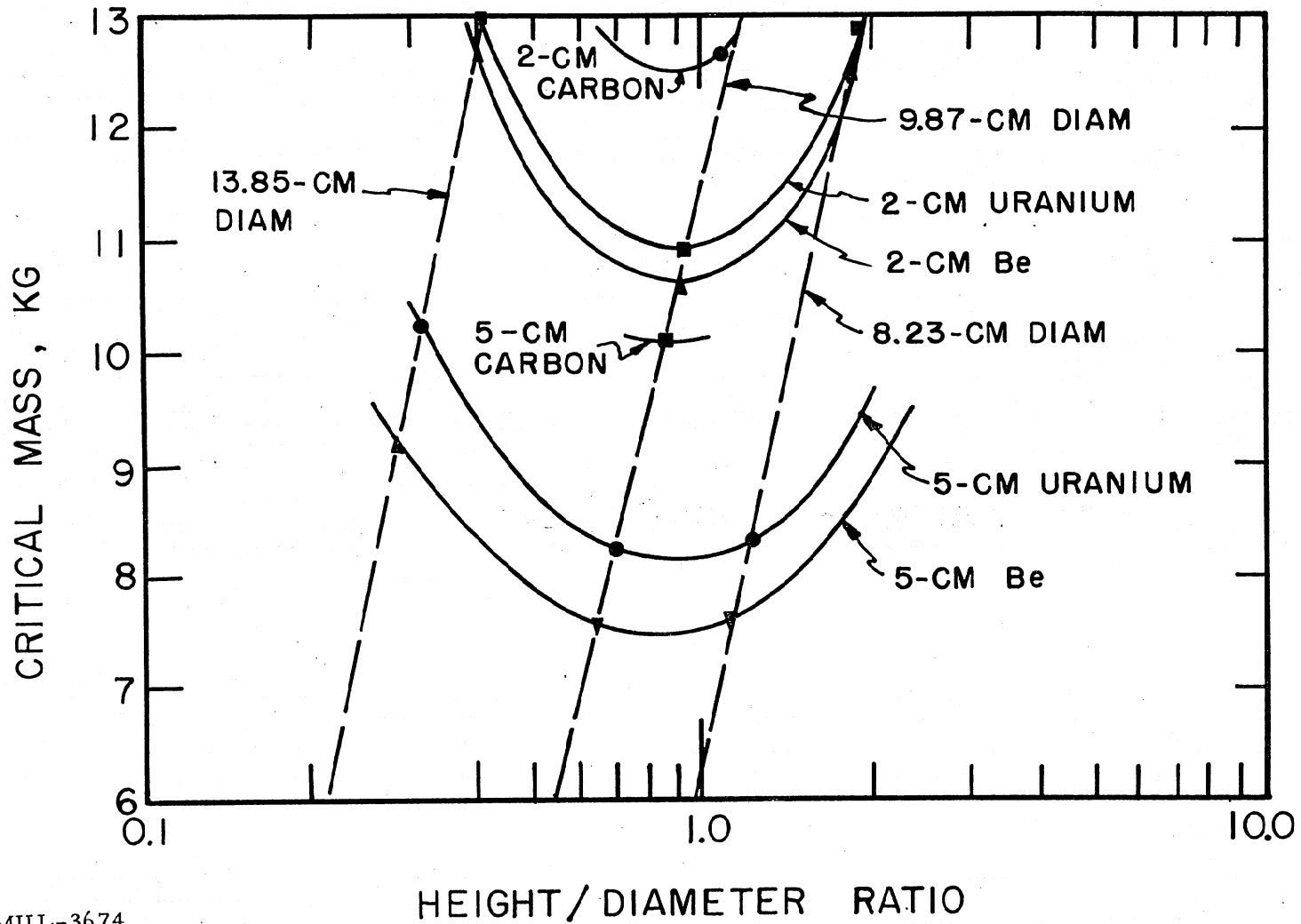
Fig. 5. Variation of 1/M as reflected spheres are assembled.



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Fig. 6. Inverse multiplication curves for Pu cylinders with 5-cm uranium reflection.





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Fig. 7. Cylinder critical masses.

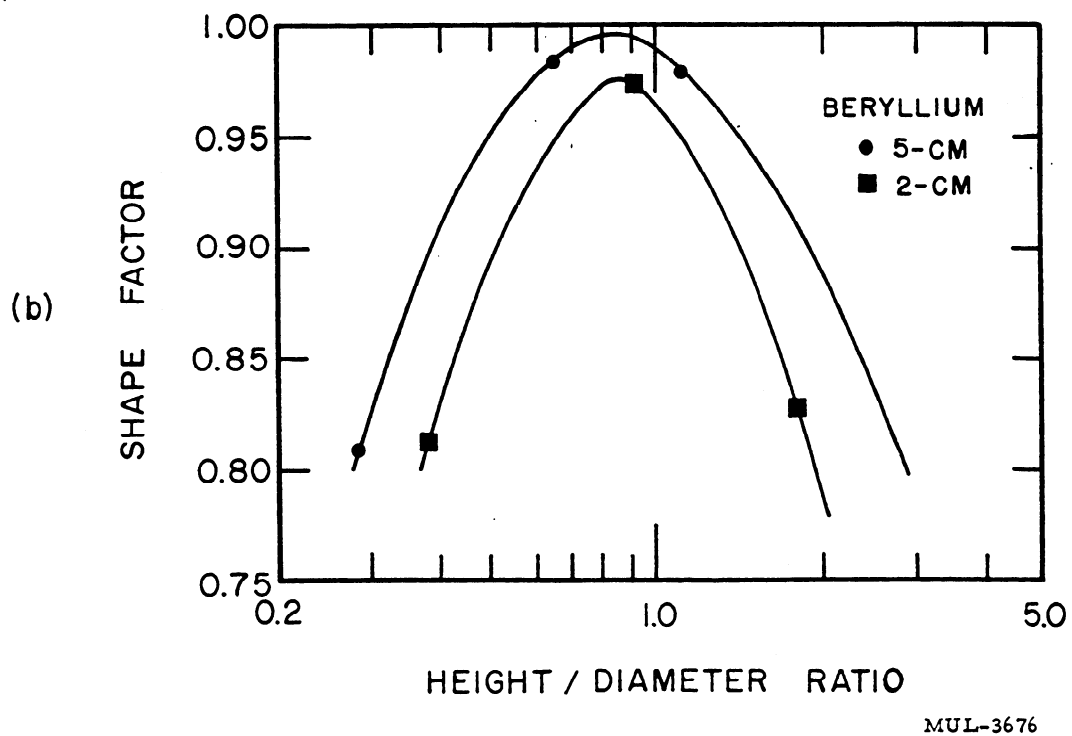
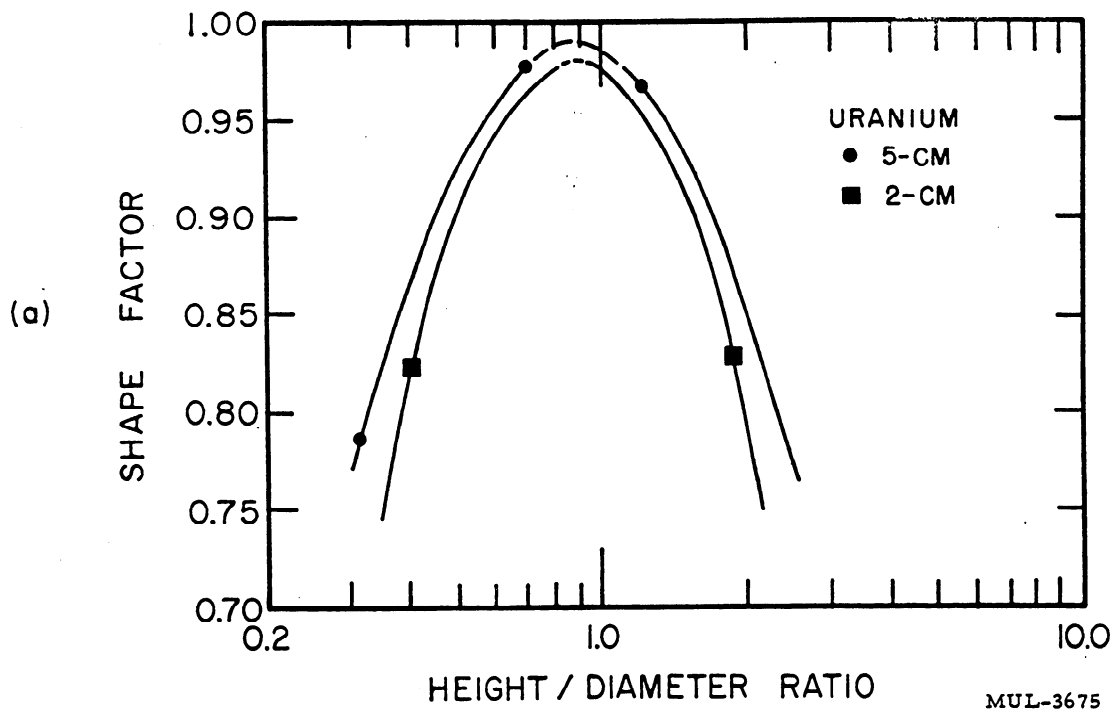


Fig. 8. Shape factors for cylinders with (a) uranium reflection and (b) Be reflection.