REFERENCE 37

G. E. HANSEN, H. C. PAXTON, AND D. P. WOOD, "CRITICAL PLUTONIUM AND ENRICHED-URANIUM-METAL CYLINDERS OF EXTREME SHAPE," NUCL. SCI. ENG. 8: 570-577 (1960).

NUCLEAR SCIENCE and ENGINEERING

THE JOURNAL OF THE AMERICAN NUCLEAR SOCIETY

VOLUME 8, NUMBER 6, December 1960

NUCLEAR SCIENCE AND ENGINEERING: 8, 570–577 (1960)

Critical Plutonium and Enriched-Uranium-Metal Cylinders of Extreme Shape*

G. E. HANSEN, H. C. PAXTON, AND D. P. WOOD

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

Received July 12, 1960

Critical configurations have been established with enriched uranium in the form of squat 15.0-in. diameter cylinders and elongated 3.24-in. diameter cylinders. These cores were reflected by depleted uranium, polyethylene, graphite, and water; also, the squat cylinder was unreflected and reflected by beryllium of various thicknesses. Critical systems of plutonium were squat 6.0-in. diameter cylinders and elongated 2.25-in. diameter cylinders reflected by normal uranium, graphite, water, and in one case, polyethylene. Observed critical heights and diameters were corrected to correspond to standard enriched-uranium and plutonium densities and concentrations. These are tabulated along with effective extrapolation distances.

Early critical measurements on families of small cylinders (*1*, *2*) demonstrated the inadequacy of the simple shape-conversion relationship that is based on fixed extrapolation distance (*3*). The failure of this semitheoretic scheme, which became apparent for cylinders of height/diameter less than $\sim\frac{1}{3}$ and greater than ~ 3, emphasized the need for abundant critical data on systems of extreme shape. As a consequence, critical information on squat and elongated cylinders of enriched uranium and plutonium metal has been obtained at the LASL Critical Assembly Laboratory whenever convenient. This article presents a collection of such results for U^{235} ($\sim 93\%$)-metal cylinders of 15.0 and 3.24-in. diameter and Pu-metal cylinders of 6.0 and 2.25-in. diameter in various reflectors. The 6-in. diameter Pu plates also were clustered to form larger-diameter slabs at reduced density.

The resulting extension of experimental information on the purely "shape" dependence of critical size ("shape factor" data) permits more accurate extrapolation to the two limiting cases of infinite critical cylinder and infinite critical slab. These limiting cases have the two especially useful properties: (1) criticality depends on the distribution of matter along only one spatial coordinate thus facilitating computational checks, and (2) the height of the infinite critical slab and diameter of the infinite critical cylinder represent lower bounds for criticality of cylinders and thus determine a class of cylinders whose nuclear criticality safety is guaranteed.

THE ASSEMBLIES

The elongated enriched-uranium cylinders (93.18 w/o U^{235}) were constructed of six 3.24-in. diameter pieces that ranged from 1 to 6 in. in height, whereas, the squat cylinders (93.4 w/o U^{235}) were built up from twenty-five 15.0 in. o.d. × 0.120-in. thick plates weighing approximately 6.5 kg each. The long delta-phase plutonium cylinders ($\rho \sim 15.61$ g-Pu/cm^3) were formed from ten 2.25-in. diameter pieces, $\frac{1}{2}$ to 3 in. high, each coated with 0.005-in. thick nickel. The squat plutonium cylinders were assembled from thirty-one plutonium plates sealed in thin nickel cans.[1] The average dimensions of the uncoated plutonium were 5.934 in. diameter × 0.123 in. thick and with nickel coating 5.967 in. × 0.135 in. The mass of each plate was ~ 0.9 kg and one plate was segmented for incremental mass adjustment. All plutonium had a composition of $\sim 95\%$ Pu^{239}, $\sim 5\%$ Pu^{240} with traces of Pu^{241} and Pu^{242}.

The reflectors were generally formed of three

---

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] The plutonium plates were kindly loaned by K-Division of LASL after use in a mockup of LAMPRE. The enriched-uranium plates were originally for use in exponential columns.

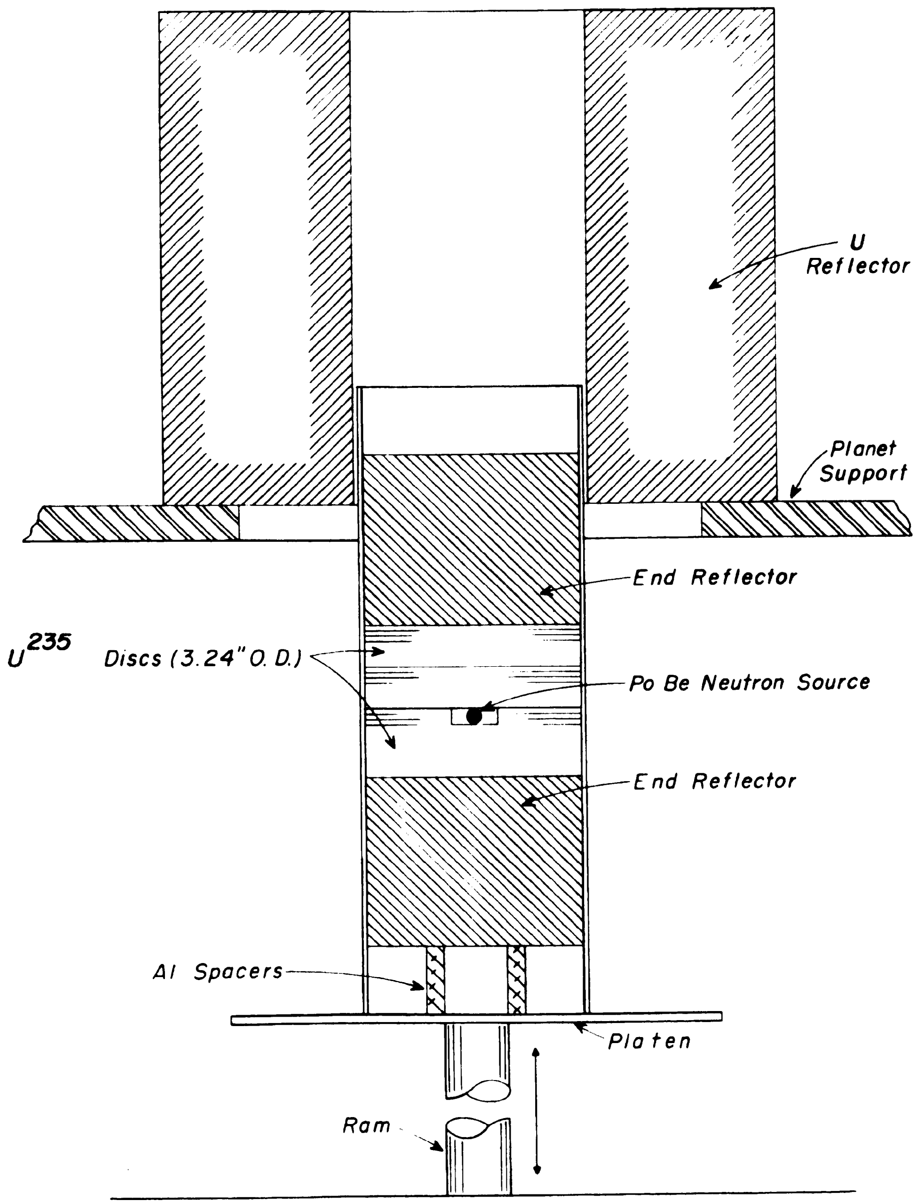

FIG. 1. Setup for 3.24-in. diameter enriched-uranium cylinder in depleted-uranium reflector.

main parts: a peripheral annulus and two end plugs. The peripheral reflector was built up of several rings so that its height could be adjusted to the height of active material in the core. An exception was a beryllium reflector, which consisted of plates and rectangular pieces that introduced unavoidable porosity.

The Planet universal machine and the water immersion tank were utilized for these experiments. The Planet, used for all critical determinations with solid reflectors, consists of a hydraulic lift with a stationary steel platform directly above it. The peripheral portions of the various reflectors (with the exception of beryllium) were supported directly on the platform, so that the active material with the remainder of the reflector could enter from below. The latter portion was placed on the lift either directly or on a minimal mass support. Figures 1 and 2 show the experimental arrangement with a depleted uranium reflector. For an unreflected case, half of the 15-in. diamter enriched-uranium cylinder was supported by a 0.019-in. thick stainless-steel diaphragm which replaced the top platform and the other half was placed on a low-mass support on the lift. The diaphragm was also used to support the upper portion of the beryllium-reflected assemblies (Fig. 3).

The immersion Tank was used for measurements with effectively infinite-water reflector. The core of interest was placed in the empty tank and the

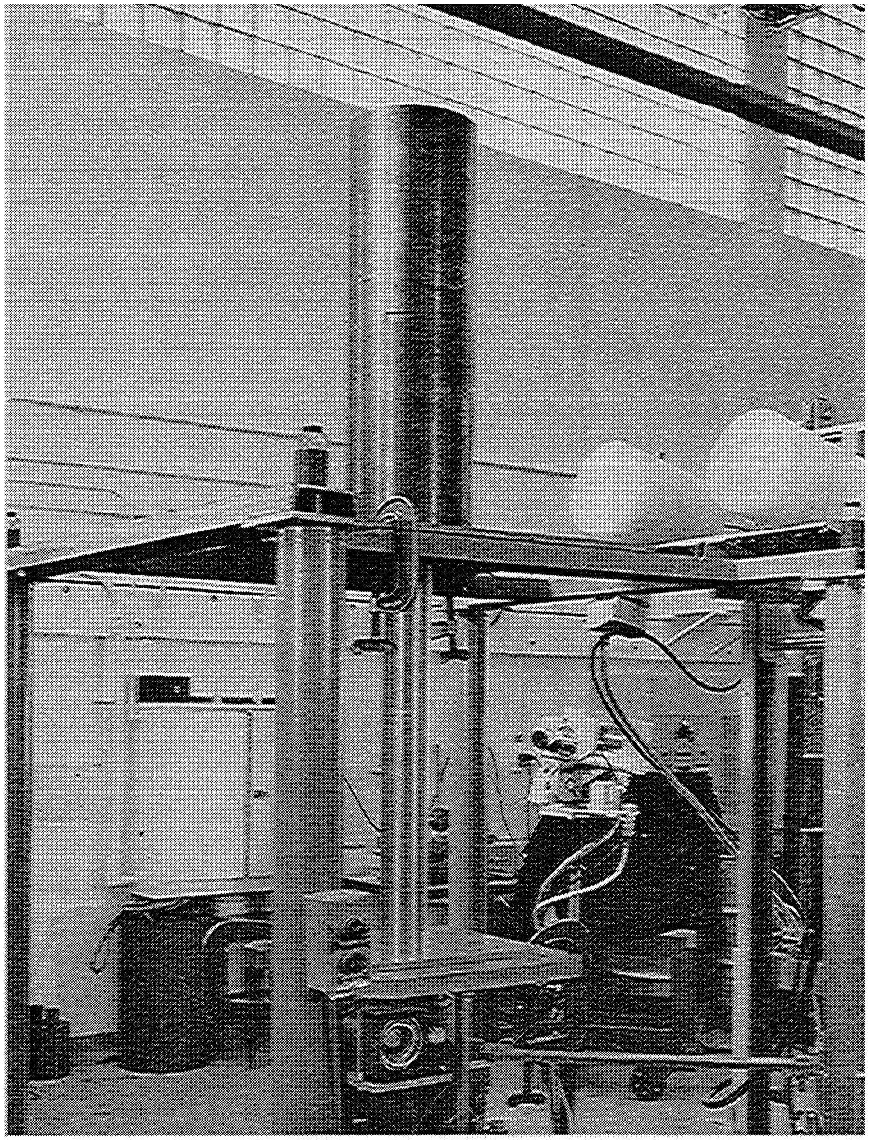

FIG. 2. Reflector of 2.75-in. thick depleted uranium on Planet machine. The guide tube surrounds the 3.24-in. diameter enriched-uranium core.

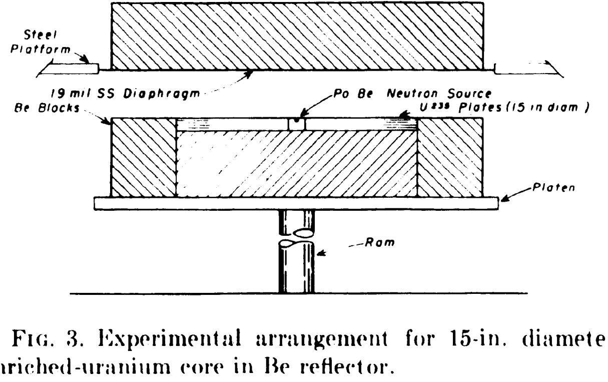

FIG. 3. Experimental arrangement for 15-in. diameter enriched-uranium core in Be reflector.

water level raised remotely (Fig. 4). Slabs built up of the plutonium plates were protected from the water by the $\frac{1}{2}$-in. thick Lucite containers shown in Fig. 5. BF_3 neutron counters in water-tight polyethylene sleeves, were positioned approximately 6 in. distant from the core surface. All assemblies with the 3.24-in. U^{235} and 2.25-in. Pu cylinders had a 0.030-in. thick steel guide sleeve between the core and reflector.

PROCEDURE

Though the specific mechanics of determining an extrapolated critical height for the different

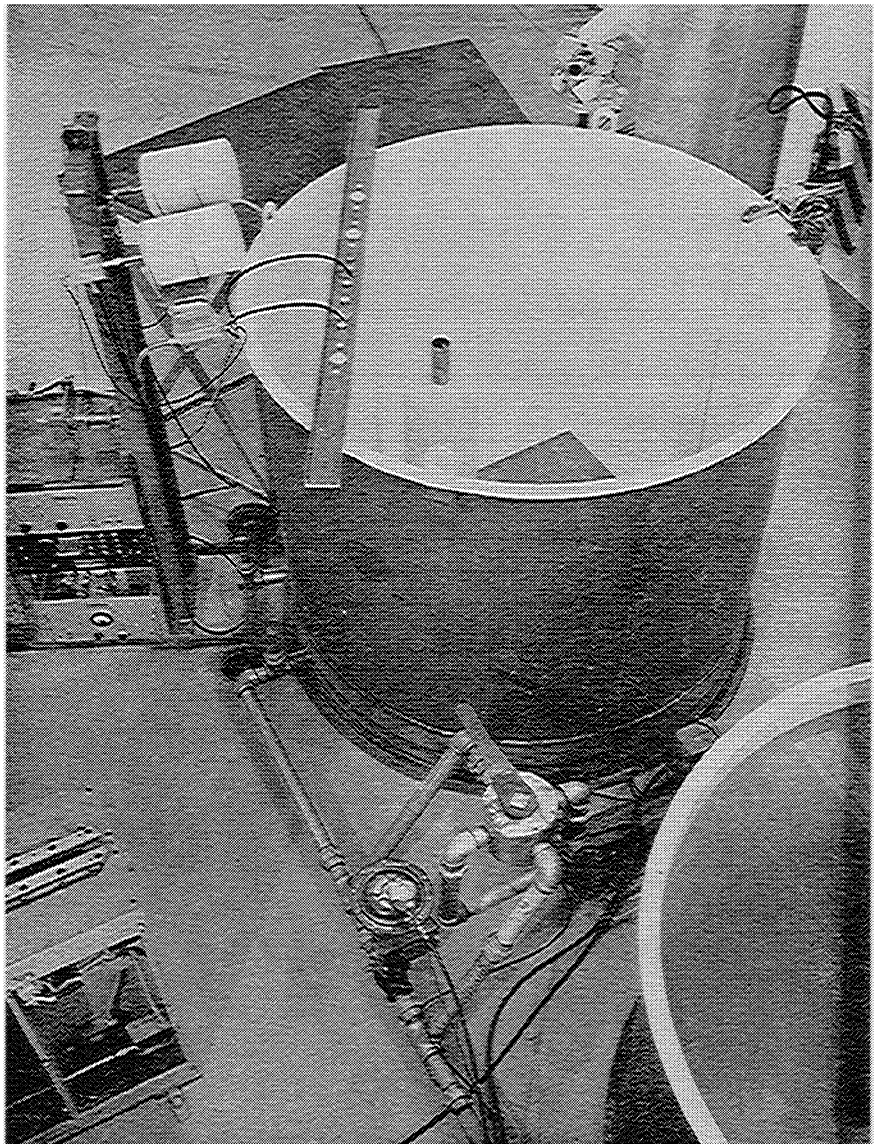

FIG. 4. Immersion tank for water-reflected systems.

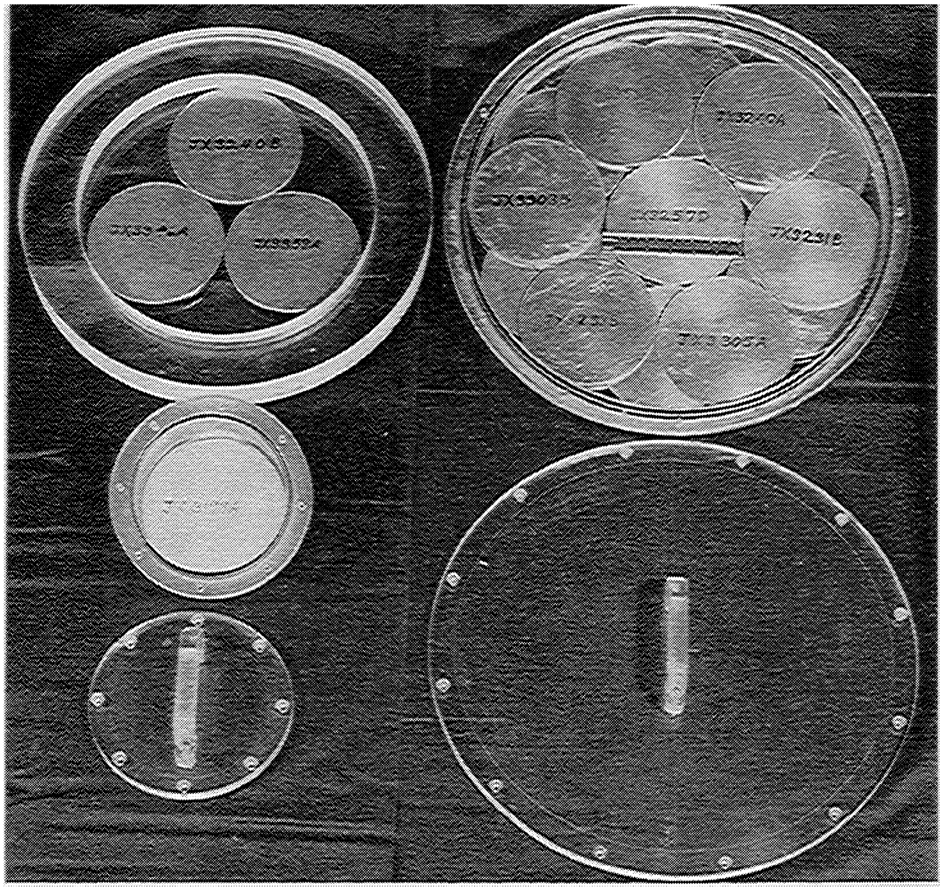

FIG. 5. Plutonium disks in their Lucite containers prior to water immersion.

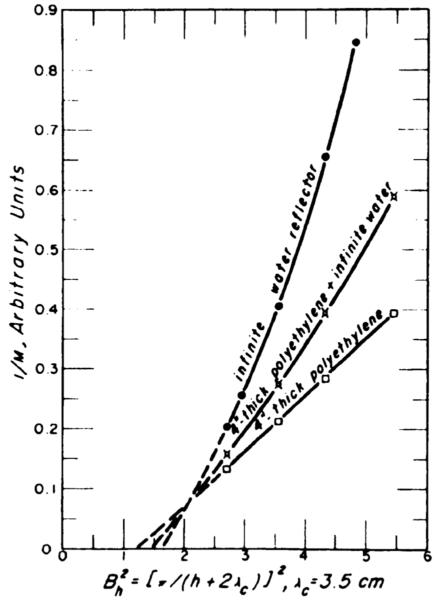

FIG. 6. Extrapolations to critical longitudinal buckling for 2.25-in. diameter plutonium in water and polyethylene.

fissionable cylinders varied somewhat from one setup to another, the general procedures for the enriched-uranium systems were as follows:

(1) Four BF_3 neutron counters in polyethylene geometries monitored the neutron leakage from the assembly.

(2) With the reflector materials in place, a Po-Be neutron source was centered within the reflector to eatablish an unmultiplied count.

(3) A known-safe amount of the fissionable material was placed on the lift generally between the reflector plugs, and the neutron source positioned within it. The lift was then raised remotely and a multiplied count recorded. The ratio of these two counts determined the multiplication (M) of the system.

(4) As additional material was added, a series of multiplication values was determined. The resulting plot of $1/M$ versus height of fissionable material permitted an extrapolation to critical.

Measurements on the plutonium cylinders were similar except that no neutron source other than that from spontaneous fission was required. Plutonium mass divided by neutron counting rate was taken as the index of reciprocal multiplication.

The critical height of the 15.0-in. diameter U^{235} cylinder in water was measured with and without 0.030-in. thick steel plates on the ends to provide a

TABLE I

Critical Specifications for 3.24-in. Diameter U^{235} (93.2%) and 15.0-in. Diameter U^{235} (93.4%) Cylinders in Various Reflectors

Core				Reflector			Estimated core height and diameter and effective extrapolation distance for 18.80 g/cm³ U^{235} (93.5%)				
Height, *h* (in.)	Diameter, *d* (in.)	Density (g/cm³)	Mass (kg)	Material	Thickness (in.)	Density (g/cm³)	Height, *h* (cm)	Diameter, *d* (cm)	*h*/*d*	Extrapolation distance (cm)[a]	
										λ_c for cylinder	λ_s for sphere
27.8 ± 0.4	3.24	18.70	70.3	Depleted uranium	2.75	18.9	70.3 ± 1.0	8.19	8.6	4.30	4.28
22.0 ± 0.4	3.24	18.70	55.5	Graphite	4.85	1.60	55.5 ± 1.0	8.19	6.8	4.34	4.10
16.1 ± 0.3	3.24	18.70	40.7	Graphite	5.75	1.60	40.7 ± 0.7	8.19	5.0	4.42	4.24
14.3 ± 0.3	3.24	18.70	36.1	Graphite	6.25	1.60	36.1 ± 0.7	8.19	4.4	4.47	4.30
26.0 ± 0.4	3.24	18.70	65.8	Polyethylene	4.00	0.92	65.8 ± 1.0	8.19	8.0	4.31	4.10
39.5 ± 2.0	3.24	18.70	100	Water	∞	1.00	100 ± 5	8.19	12.2	4.26	4.10
3.25 ± 0.03	15.00	17.70	165.7	None			7.73 ± 0.07	35.9	0.215	2.10	2.150
1.96 ± 0.03	15.00	17.70	100.5	Beryllium	1.00	1.80	4.70 ± 0.07	36.0	0.131	3.54	3.53
1.35 ± 0.02	15.00	17.70	69.5	Beryllium	2.00	1.80	3.26 ± 0.05	36.1	0.090	4.22	4.27
1.02 ± 0.02	15.00	17.70	52.5	Beryllium	3.00	1.80	2.47 ± 0.05	36.2	0.068	4.60	4.74
0.79 ± 0.01	15.00	17.70	40.5	Beryllium	4.00	1.80	1.91 ± 0.03	36.2	0.053	4.87	5.06
0.635 ± 0.01	15.00	17.70	32.5	Beryllium	5.00	1.80	1.53 ± 0.03	36.3	0.042	5.05	5.28
1.37 ± 0.02	15.00	17.70	70.0	Normal uranium	3.00	18.9	3.29 ± 0.05	36.0	0.091	4.21	4.35
1.09 ± 0.02	15.00	17.70	55.8	Graphite	7.00	1.60	2.62 ± 0.05	36.1	0.073	4.53	4.36
1.43 ± 0.02	15.00	17.70	73.2	Polyethylene	2.00	0.92	3.43 ± 0.05	36.0	0.095	4.14	3.89
1.23 ± 0.01	15.00	17.70	63.2	Water	∞	1.00	2.97 ± 0.03	36.1	0.082	4.36	4.10

[a] Values depend upon choice of bare-sphere extrapolation distance = 2.15 cm.

TABLE II

Critical Specifications for 2.25 and 6.0-in. Diameter Plutonium Cylinders in Various Reflectors

Core				Reflector			Estimated core height and diameter and effective extrapolation distance for 15.65 g/cm³ plutonium (96% Pu^{239})				
Height, *h* (in.)	Diameter, *d* (in.)	Density (g/cm³)	Mass (kg)	Material	Thickness (in.)	Density (g/cm³)	Height, *h* (cm)	Diameter, *d* (cm)	*h*/*d*	Extrapolation distance (cm)[a]	
										λ_c for cylinder	λ_s for sphere
19.77 ± 0.1	2.25	15.44	20.0	Depleted uranium	3	18.7	49.6	5.65	8.8	3.63	3.58
16.11 ± 0.1	2.25	15.44	16.3	Graphite	7	1.60	40.4	5.65	7.2	3.65	3.59
33.0 ± 2.0	2.21	15.44	32.2	Polyethylene	4	0.92	82.7	5.55	14.9	3.63	3.42
27.8 ± 1.5	2.21	15.44	27.1	Water	∞	1.00	69.7	5.55	12.6	3.64	3.42
29.4 ± 2.0	2.21	15.44	28.6	Polyethylene + water	4 + ∞		73.7	5.55	13.3	3.64	3.42
1.54 ± 0.01	6.0	14.3	10.14	Normal uranium	3	18.7	3.68	14.3	0.26	3.40	3.58
2.33 ± 0.01	6.0	14.3	15.44	Graphite	1	1.60	5.55	14.3	0.39	2.69	2.76
1.63 ± 0.01	6.0	14.3	10.8	Graphite	7	1.60	3.89	14.3	0.27	3.31	3.59
1.67 ± 0.03	6.0	14.3	11.1	Water	∞	1.00	3.98	14.3	0.28	3.28	3.42
1.05 ± 0.03	11.0[b]	13.1[b]	21.4	Water	∞	1.00	2.37	24.8	0.095	3.37	3.42
0.79 ± 0.03	16.0[b]	13.1[b]	34.1	Water	∞	1.00	1.78	36.1	0.049	3.45	3.42

[a] Values depend upon choice of bare-sphere extrapolation distance = 2.06 cm.

[b] Estimated equivalent diameter and density for the pseudo-cylinder.

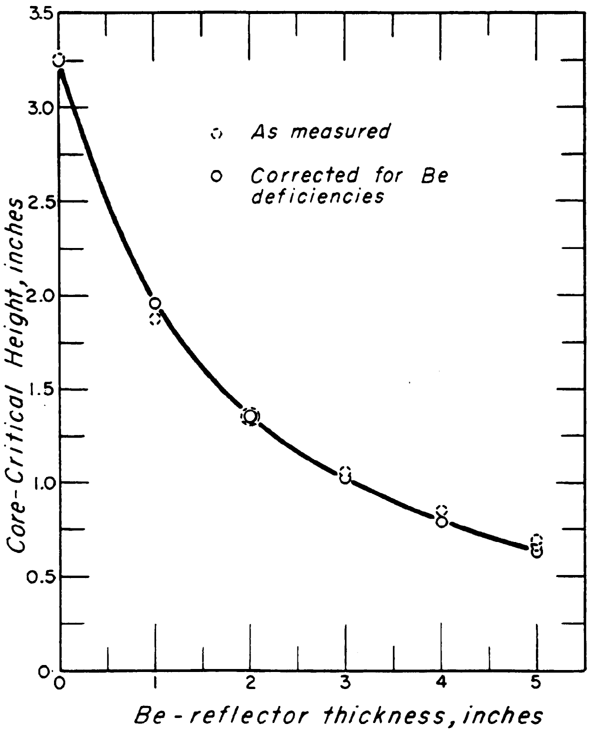

FIG. 7. Critical height of 15-in. diameter U^{235} (93.4%) cylinder (ρ = 17.7 g/cm^3) vs Be reflector thickness.

basis for estimating the effect of the steel guide sleeve of similar thickness that surrounded the elongated cylinders. The containers shown holding the three-plate and seven-plate layered arrays illustrated in Fig. 5 were employed in a special set of multiplication measurements designed to check the consistency of the extrapolation procedure for estimating the critical thickness of the water-reflected infinite plutonium slab. Multiplications were measured for both staggered and eclipsed-layer structures of these submerged pseudo-cylinders and void coefficients of reactivity were determined for the assemblies of highest multiplication.

RESULTS

Figure 6 shows special extrapolation curves, $1/M$ vs longitudinal buckling, that were used to establish critical conditions for the elongated plutonium cylinders in water and polyethylene. In other cases, extrapolations against height were satisfactory.

Specifications of the critical configurations for the U^{235} cylinders are listed in Table I and for the plutonium cylinders in Table II. The critical conditions correspond to configurations with snugly-fitting reflectors without guide sleeves. The final portions of these tables give critical diameters and heights corrected to standard densities and concentrations of enriched uranium and plutonium.[2] For the squat U^{235} cylinders imperfectly reflected by Be, the magnitude of corrections to full-density reflector and core appears in Fig. 7. Results also are corrected for incidental reflection by the lift.

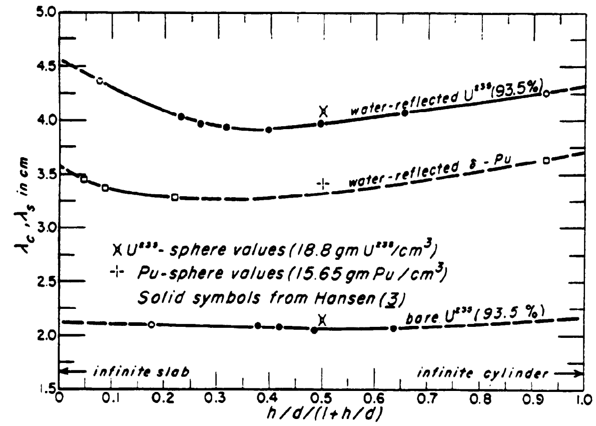

FIG. 8. Effective extrapolation distances for cylinders of U^{235} (93.5%) and delta-phase plutonium metal. Ordinates depend upon choices of bare-sphere extrapolation distances as 2.15 cm for U^{235} (93.5) and 2.06 cm for Pu.

The effective extrapolation distance, λ_c, listed in Tables I and II is that which satisfies the relation

$$B^2 \text{ (sphere)} = \left(\frac{2.405}{d/2 + \lambda_c}\right)^2 + \left(\frac{\pi}{h + 2\lambda_c}\right)^2$$

for a cylinder of diameter d and height h, when bare-sphere extrapolation distances are chosen as 2.15 cm for U^{235} (93.5%) and 2.06 cm for plutonium. The extrapolation distance, λ_s, in the final column

---

[2] To convert the critical cylinder of core height, h, core diameter, d, fissile element density, ρ, and impurity density, ρ_i, to a similarly reflected critical cylinder with dimensions d_0, $h_0 = d_0(h/d)$, and fissile element density, ρ_0, use was made of the dilution exponent m_i for which $h_0/h \equiv d_0/d = (\rho_0/\rho)^{-m_i}$. The exponent m_i is simply related to the dilution exponent n_i introduced in the accompanying article "Reactivity Contributions of Various Materials in Topsy, Godiva, and Jezebel," by Engle *et al.* viz., $3m_i = n_i - 1$. Specifically, m_i was evaluated from the diffusion-theory perturbation formula for a rectangular solid

$$m_i = \frac{\left\{1 + \frac{1}{2}\left(\frac{\sigma_{a,i}}{\sigma_a} + \frac{\sigma_{tr,i}}{\sigma_{tr}}\right)\frac{\rho_i A}{(\rho - \rho_0)A_i} + \frac{1}{2}\left(\frac{\sigma_{a,i}}{\sigma_a} - \frac{\sigma_{tr,i}}{\sigma_{tr}}\right)\frac{\rho_i A}{(\rho - \rho_0)A_i}\frac{2B\lambda}{\pi}\left(\frac{B_x^3 + B_y^3 + B_z^3}{B^3}\right)\right\}}{1 + \frac{4B^3(\lambda^3 - \lambda_0^3)}{3\pi}\left(\frac{B_x^3 + B_y^3 + B_z^3}{B^3} - \frac{B_x^5 + B_y^5 + B_z^5}{B^5}\right)}$$

where the one-group cross sections $\sigma_{a,i}$, $\sigma_{tr,i}$ for impurity i, and σ_a, σ_{tr} for the fissile element were obtained from the tables listed by Engle *et al.* A, A_i designate atomic weights of impurity i, and fissile element; B and λ_0 designate the buckling and bare extrapolation distance and are obtained from the bare-sphere measurements on Godiva (U^{235}) and Jezebel (Pu^{239}); the buckling components such as $B_z = \pi(h + 2\lambda)$ determine the effective extrapolation distance, λ, from $B_x^2 + B_y^2 + B_z^2 = B^2$.

of the tables, is for a sphere with reflector of the same composition, density, and thickness as that of the corresponding cylinder (*2*). Of particular significance is the difference between λ_c and λ_s for this measures departure from the conditions for a simple, universal-shape transformation.

In the case of water-reflected U^{235} cylinders, there now exist critical data for a reasonably complete range of height/diameter values. Such data are presented in Fig. 8 as effective extrapolation distance vs $h/d(1 + h/d)$, with suggested extrapolations to the cases of infinite slab and infinite cylinder. Also shown are the less-complete data for water-reflected plutonium and for bare U^{235}.

REFERENCES

1. A. D. Callihan, J. W. Morfitt, and J. T. Thomas, *Proc. 1st Intern. Conf. Peaceful Uses Atomic Energy, Geneva* **5**, 145–155 (1955).
2. G. A. Graves and H. C. Paxton, *Nucleonics* **15**, 90–92 (1957).
3. G. E. Hansen, *Proc. 2nd Intern. Conf. Peaceful Uses Atomic Energy, Geneva* **12**, 84–88 (1958).