REFERENCE 23

C. K. BECK, A. D. CALLIHAN, AND R. L. MURRAY, "CRITICAL MASS STUDIES, PART II," CARBIDE AND CARBON CHEMICALS CORPORATIONS REPORT K-126 (JANUARY 1948).

Approved for issue by: Clifford K. Beck
Date of issue: January 23, 1948

Report Number: K-126

CLINTON ENGINEER WORKS

CARBIDE AND CARBON CHEMICALS CORPORATION
Research and Development Laboratories

CRITICAL MASS STUDIES, PART II

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Report number: K-126
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ABSTRACT

Using uranium enriched to approximately 30% in the 235 isotope and fabricated into one inch cubes having a density of 4.8 gm/cm^3 with the nuclear properties of UF_6, a study has been made of several conditions affecting the mass assembled at criticality. The effect of intermixed hydrogen on the critical mass of U-235 was examined by including small blocks of polyethylene with the active material. Other variables affecting the critical mass about which some information was obtained are: type of reflector surrounding the core, density of U-235, homogeneity of the mixing of the hydrogenous and active materials and the geometrical shape of the assembly.

With no intermixed hydrogen but with the active material surrounded by paraffin, it is possible to assemble more than 100 kg of U-235 into a cube without closely approaching criticality. Increasing the hydrogen content rapidly lowers the critical mass to 7.7 kg and 4.0 kg at hydrogen to U-235 atomic ratios of 32 and 128 respectively.

The removal of the paraffin reflector approximately doubles the critical mass; interposing cadmium or boron between the core and the paraffin also increases the mass by about two in the range of moderation studied.

Provided the cross-sectional area of a parallelepiped assembly is sufficiently small, it is possible to extend its length indefinitely without criticality occurring. At an H/U-235 atomic ratio of 64, an assembly 6" x 7" in cross-section can apparently be so extended.

A few measurements on the density effect indicate the critical mass, at an H/U-235 atomic ratio of 16, to vary as the -1.7 power of the density; at H/U-235 = 1.28, the exponent is about -2.

This paper reports a series of experiments done in Building F-05 of the Carbide and Carbon Chemicals Corporation Research Laboratory during the latter part of 1946. It is the result of work done by:

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CRITICAL MASS STUDIES

PART II

I. INTRODUCTION

Apparatus was set up in Laboratory F-05 of the inoperative Thermal Diffusion Plant at Oak Ridge for continuation of the critical mass experiments begun at Los Alamos. It was the objective of these experiments to investigate the critical masses of U-235, when in a form simulating uranium hexafluoride, with no or only small amounts of hydrogen intermixed, at a density comparable with the maximum of condensed UF_6, surrounded by hydrogeneous reflector, and in geometrical configuration approaching spherical, these conditions being the most hazardous likely to occur when UF_6 is being handled.

Critical Mass Studies, Part I,[1] contained reports of experiments performed under the above conditions with uranium of 95% isotopic concentration of U-235. This report contains the record of a similar set of experiments when the active material contained uranium of approximately 30% isotopic concentration of U-235. Thus, from the two series of experiments, information is obtained relative to the effect of isotopic concentration on critical masses under the experimental conditions studied.

II. MATERIALS USED; PROPERTIES

The critical assemblies of this investigation were composed of two types of small rectangular blocks: "U-cubes" and "H-cubes." Finely ground UF_4 and appropriate quantities of polytetrafluoroethylene (essentially $(CF_2)_n$), mixed and micropulverized together comprised the ingredients of the U-cubes. The uranium was enriched to a U-235 isotopic content of 29.83%. Quantities of this mixture, cold-pressed at 40-50 tons per square inch pressure with suitable dies into one-inch cubes, formed the uranium-bearing building blocks of the critical assemblies. The intention in making these cubes was to secure a suitably shaped block, amenable to handling, possessing chemical constituents and nuclear characteristics simulating those of condensed UF_6. (Polytetrafluoroethylene is abbreviated poly TFE.)

The H-cubes were interspersed in various lattice configurations among the U-cubes to give controlled amounts of moderation to the assembly. Hence, the component material of these cubes was selected for high hydrogen content. Polyethylene (essentially $(CH_2)_n$)* was chosen, since this compound possessed desirable physical properties as well as considerable amounts of hydrogen. Two sizes of H-cubes, "one-inch", (1" x 1" x 1") and "one-half inch", (1" x 1" x ½"), were provided to permit a wide range of H:U ratios in the assemblies. Table I contains a quantitative description of the blocks used in the experiments.

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1 Beck, Callihan, Murray, A-3691, 2-11-47. In subsequent references to this report, it will be designated as Part I.

* Chemical analyses showed the hydrogen to carbon ratio to be equivalent to $CH_{1.92}$.

TABLE I

Materials Used in Critical Assemblies

A. Properties of Uranium Plastic Cubes (U cubes)

Dimensions	1.004" 2.550 cm
Volume	1.012 in^3 16.584 cm^3
Mass	79.53 gm
Density	4.79 gm/cm^3
Total U content	51.56 gm
U-235 content	15.38 gm
Fluorine content	24.69 gm
Carbon content	2.64 gm
Oxygen content	0.08 gm
Impurity (chiefly Al)	0.56 gm

B. Properties of Polyethylene Blocks (H cubes)

	"One-Inch" size	"One-half inch" size
Dimensions	1.005" cube 2.553 cm cube	1.006" x 1.006" x 0.502" 2.555 x 2.555 x 1.275 cm
Volume	1.015 in^3 16.632 cm^3	0.508 in^3 8.324 cm^3
Mass	15.11 gm	7.61 gm
Density	0.91 gm/cm^3	0.91 gm/cm^3
Hydrogen Content	2.09 gm	1.05 gm

Samples of both the polyethylene and the poly TFE materials were examined chemically and spectroscopically for impurities having nuclear characteristics which might affect the values of critical masses. In addition, independent neutron-absorbing tests were made on samples of the materials by Dr. E. O. Wollan of Clinton Laboratories, and Dr. J. R. Dunning of Columbia University. No evidence was found of the presence of undesirable impurities in sufficient quantity to affect the results of the experiments.

III. APPARATUS

Experimental Table

It was the basic purpose in most experiments of this series to determine the critical amount of U-235 under the particular conditions of interest. This purpose could only be realized by actually building an assembly to criticality, or to the near vicinity of criticality, under the desired conditions. The execution of this procedure usually involves some danger to the experimenter, for the possibility is always present, however remotely, that super-critical conditions may inadvertently be achieved, with subsequent release of hazardous radiation.

For this reason, it was decided to effect the actual assembly of all near critical accumulations in these experiments by remote control, with the operator at some distance from the accumulations and protected behind suitable shields from any radiation which might occur. This was accomplished by building the desired accumulation in two parts, separated by a suitable distance, and then, from a remote point, bringing them slowly together. One part was built along an edge of a stationary platform. The other was built along the facing edge of a movable platform of the same height, which could move on ball-bearings along grooved steel tracks toward or away from the stationary platform. In Figure I the stationary platform may be seen on the table at the right and the movable platform is at the left. In practice the movable platform was equipped, on its leading edge, with a 1/16" thick aluminium sheet to add stability to that part of the assembly when in motion. Empirical investigation showed this thickness of aluminium separating the parts of the final assembly to be equivalent to the same thickness of air. An appropriate correction was made to the observed masses for this separation and, except where noted, the results reported here are for zero separation of the assembly parts.

In Figure 2 the movable platform of the experimental table may be seen at the right while the operating position from which assembly is effected is behind the concrete block wall at the left. In Figures 3, 4 and 5 a number of assemblies or partial assemblies may be seen on the stationary and movable platforms.

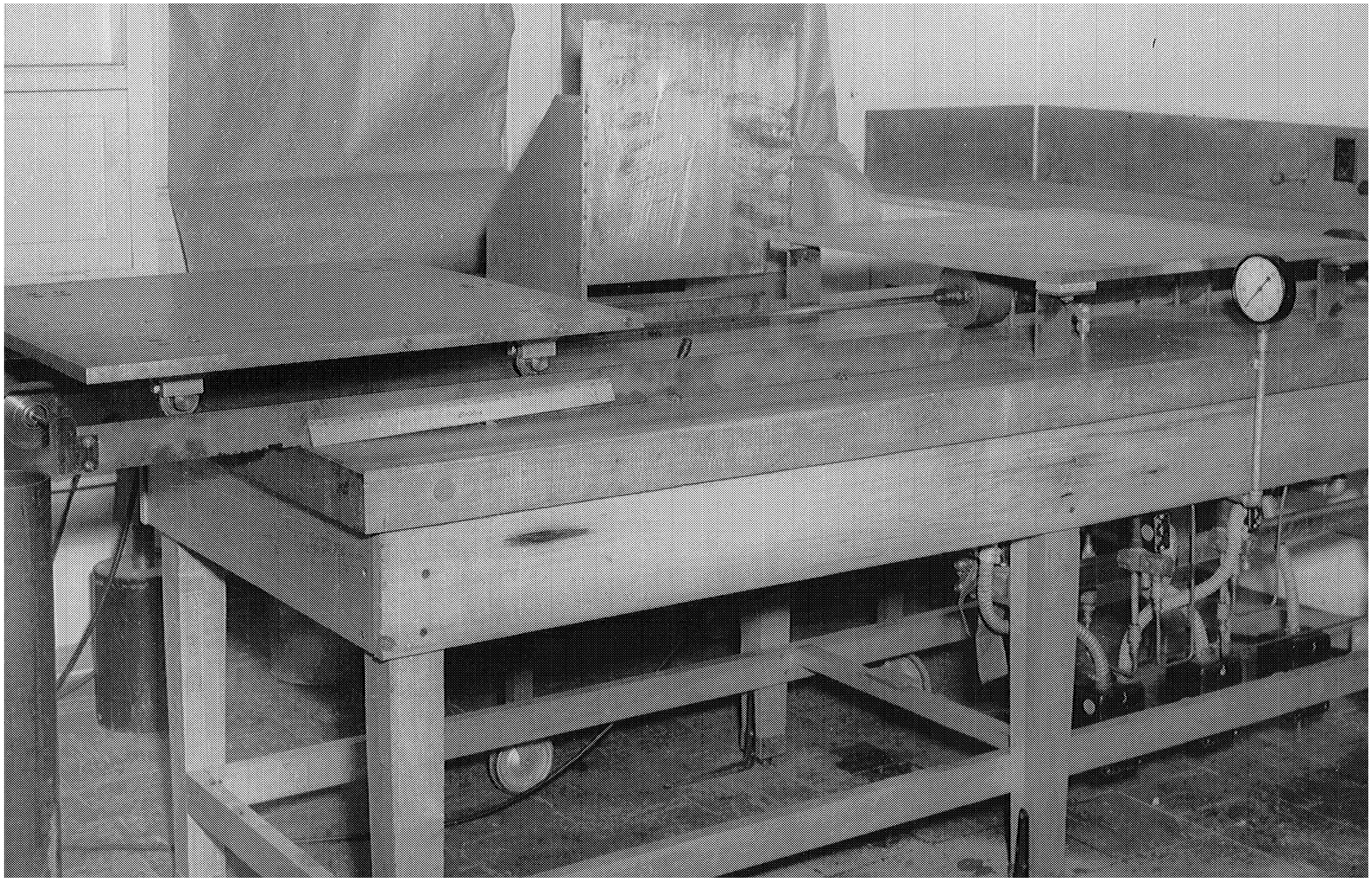

FIG. 1

FIG. 2

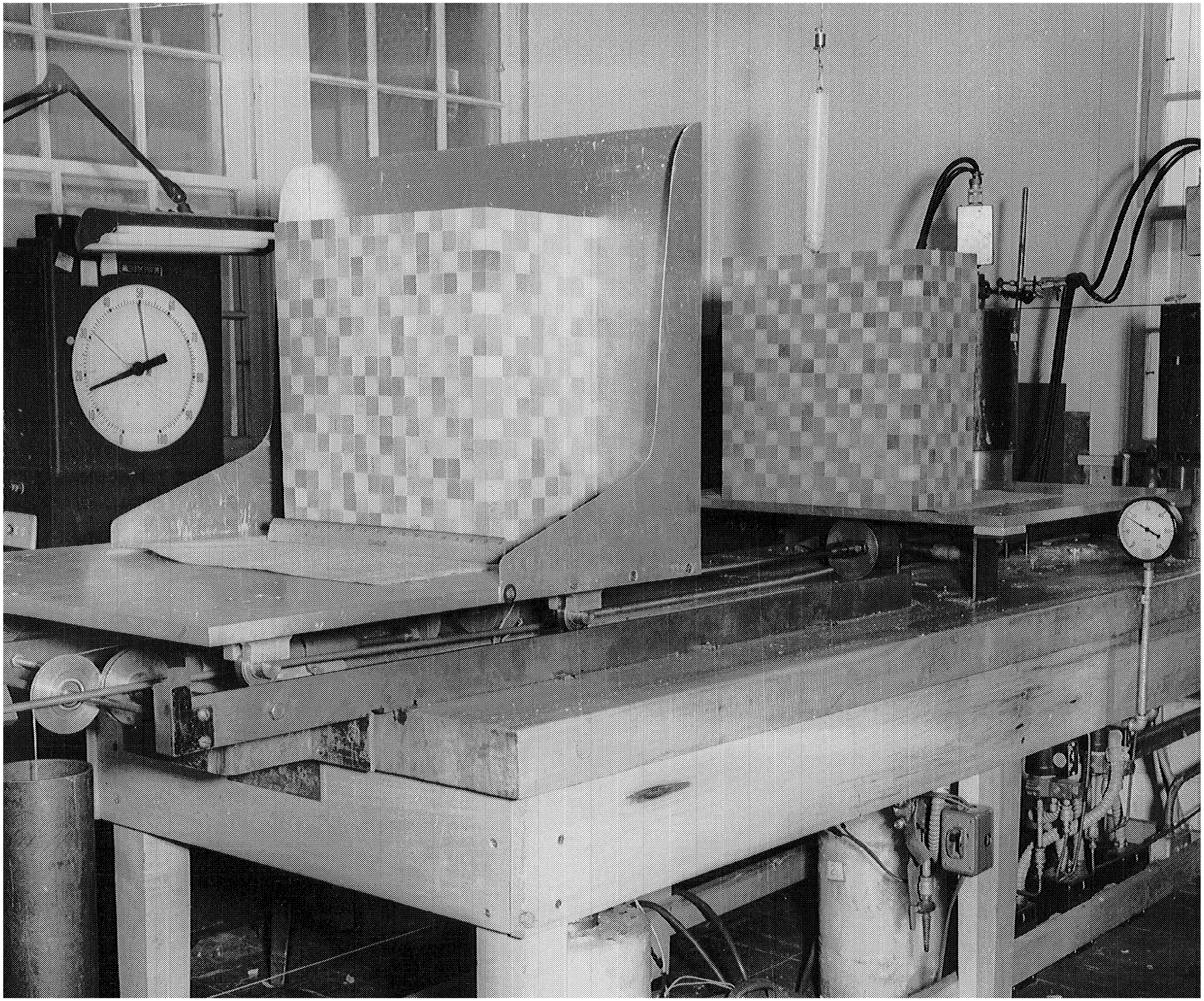

FIG. 3

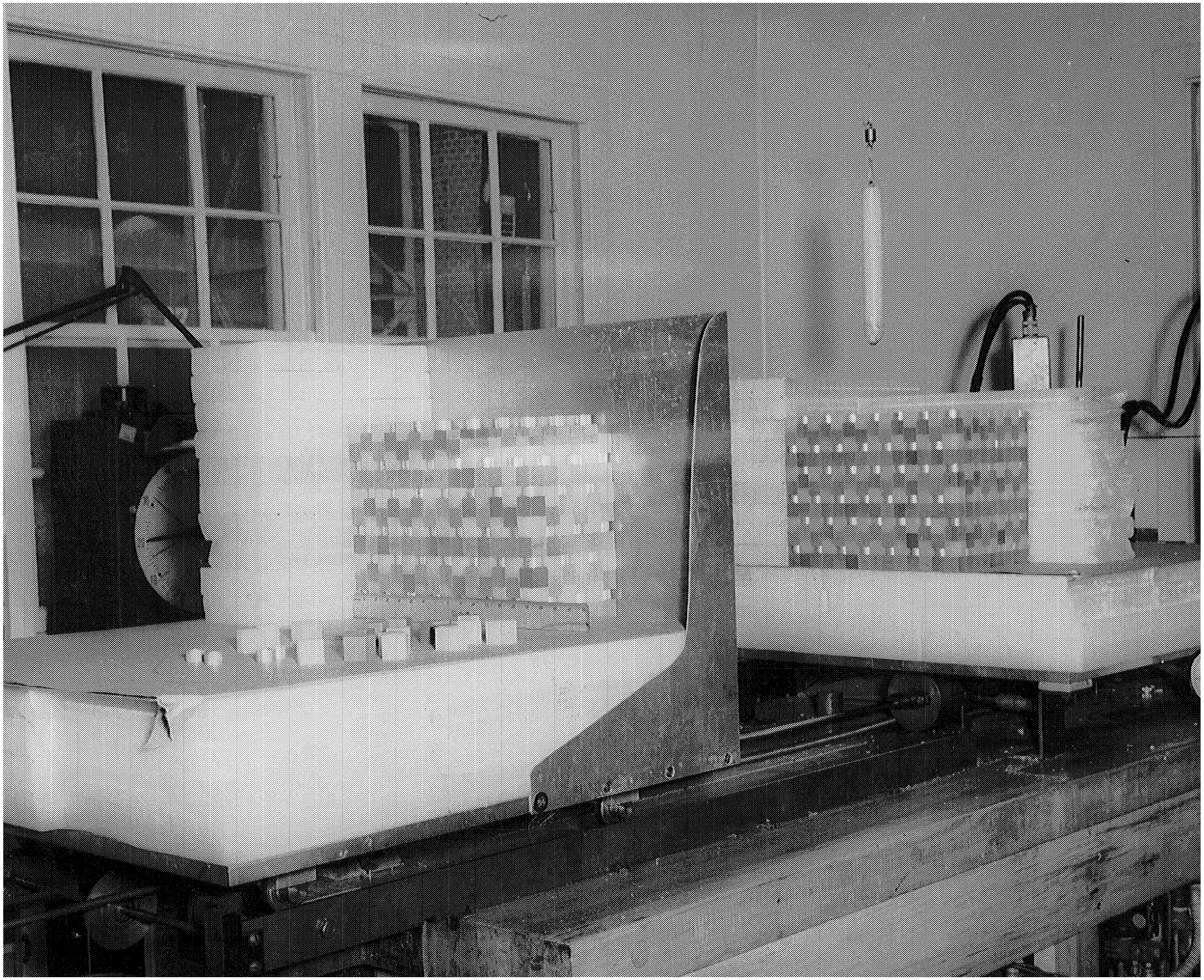

FIG. 4

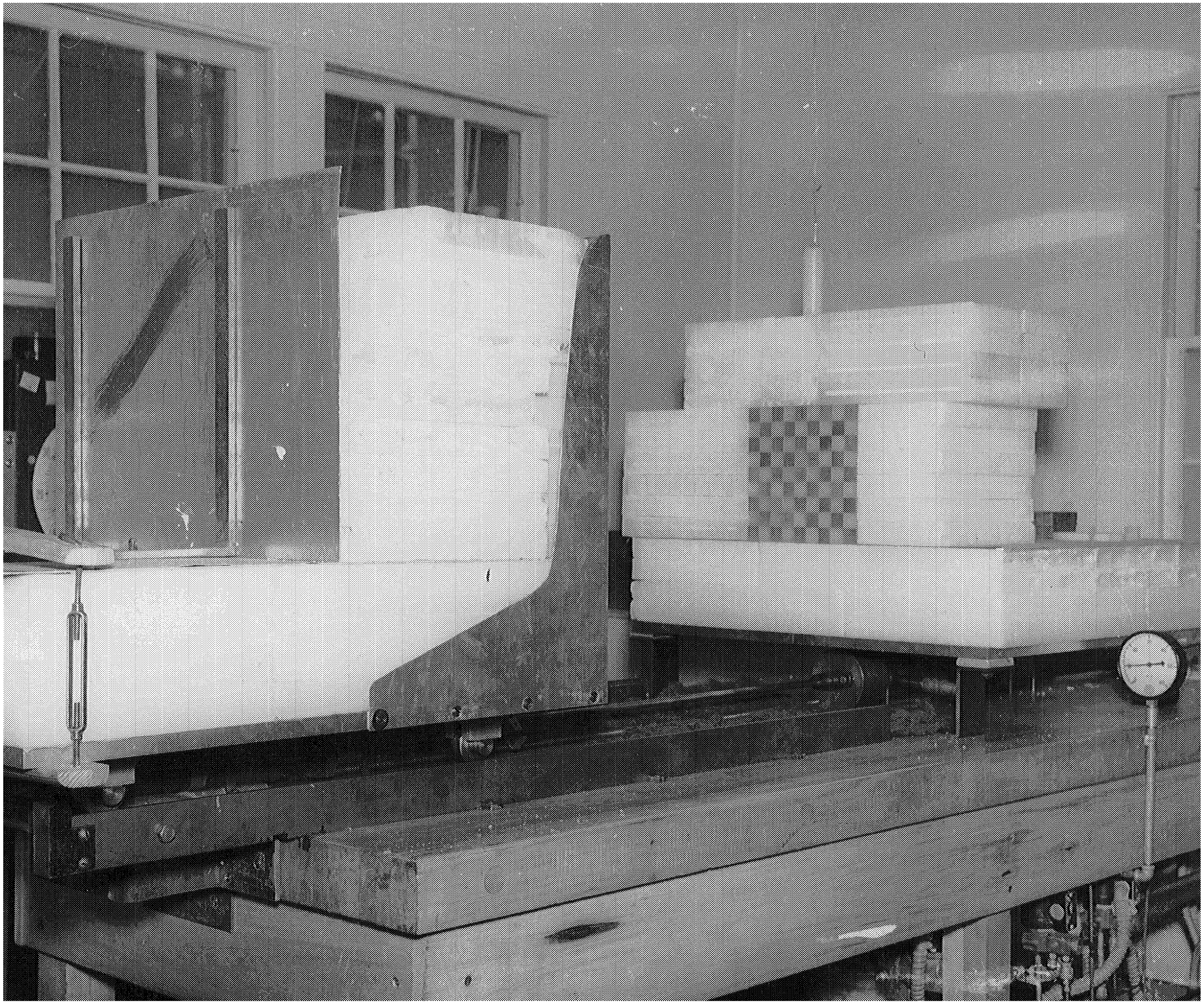

FIG. 5

Control Mechanism

The motion of the platform was controlled by pneumatic pressure on a double acting **piston** in a cylinder mounted under the stationary platform. The piston rod extends from the cylinder through a packing gland and is connected to the movable table through an electro-magnetic coupling. Electric switches at the operating position control the solenoid-operated air-valves which admit pressure to the fore and back face of the piston.

The movable table is free at any time during an assembly operation to move away from the stationary platform to the end of its track, if control or automatic safety devices are actuated. The forward motion of the table during an assembly is limited to the rate at which an operator unscrews a vernier on the end of a rod extending from the movable platform, through the shielding wall, to the operator's position.

Instrumentation

Four completely independent neutron detectors were mounted on or under the experimental table. Two of these were boron trifluoride filled, paraffin encased, ionization chambers connected through amplifying circuits to Brown recording potentiometers located in the operating room which made continuous records of the neutron level near the assembly. One of these detectors was connected to a second recorder, in parallel with the first, located beside the experimental table for the benefit of the experimenter during operations prior to assembly of the two parts. The other two detectors were boron-lined, paraffin encased, proportional counters, connected through amplifying apparatus to scaling circuits and audible counters located in the operating room. An audible impulse counter, in parallel with one of these, was located near the experimental table.

A one curie polonium-beryllium neutron source was attached to an endless cable passing near the four detectors in order that the operator from a remote position could make periodic checks of the neutron response of the detectors. A second neutron source of about 4 curies was maintained in close proximity to one part of the accumulation during building or assembly. A second cable connected to this source permitted the operator to remove it as desired during "source-jerk" tests for approach to criticality. The use of the response of the reactivity of an assembly to the removal of the neutron source as a measure of the nearness to criticality is described in Part I.

Automatic Safety Features

Several automatic safety devices were provided to disassembly the material in case of high radiation, or of power or air failure.

A counter-weight was attached by a cable over a pulley to the movable platform so that automatic separation of the two platforms would occur if the magnetic coupling connecting the movable platform to the control pistor should become disengaged.

Switches and relays were so adjusted that failure of electrical power would (a) automatically operate the proper air valves to separate the platforms and (b) de-energize the magnetic coupling, thus allowing the counter-weight to separate the platforms. These operations were also effected when the air supply pressure dropped below a predetermined value well above the normal operating pressure in the cylinder.

Two of the neutron detectors were connected independently to this safety mechanism so that automatic separation of the platforms could be effected by either detector in case the neutron density exceeded a predetermined value.

All automatic safety devices were tested before each experiment.

IV. OUTLINE OF A TYPICAL EXPERIMENT

A standard procedure was adopted for determination of criticality under any chosen set of conditions.

Prior Planning

All investigators present agreed in advance on the plan of the experiment, its objectives, and the responsibility of each person. Five persons usually participated in an experiment.

Instrument Check

Before each experiment all automatic safety devices were tested, and the response of each neutron detector was checked.

Accumulation

The two platforms on the experimental table were separated their maximum distance, and on each of the two adjacent edges of the respective platforms a predetermined quantity of U and H cubes was assembled in a chosen configuration. A neutron source was kept nearby to insure a constant supply of "input" neutrons into the assemblies. Figures 3, 4 and 5 show typical examples of assembly construction.

Assembly

When the two parts were completed and the thin aluminum shields suitably placed to prevent spillage or shifting of the load on the movable platform in case of rapid disassembly, the investigators retired behind the protecting wall. The parts of the assembly were slowly brought together while careful watch was maintained over the neutron level. If criticality was reached before they were completely together, or if complete assembly was obtained without reaching criticality, the platforms were separated and the amount of active material in the assembly decreased or increased as necessary. The parts were again brought slowly together.

This was repeated until satisfactory information on the criticality of the particular configuration under investigation was obtained. When the value was established, the platforms were fully separated and the experiment was then considered complete.

It should be pointed out that the term "criticality" when used in describing these experiments refers to the condition of assembly when, in the absence of an external neutron source, the neutron level remains constant after the lapse of sufficient time for the delayed neutrons to effect reproduction. That is, assemblies were made "delayed critical".

V. EXPERIMENTAL RESULTS

Following the procedure outlined above, determinations of critical mass were made for various conditions of accumulation. The one-inch "U-cubes" of mock-up UF_6, at 30% U-235 isotopic concentration, were alternated with appropriate numbers of the polyethylene "H-cubes," in regular lattice configurations of cubical or rectangular shape, and the size of the assembly was adjusted to criticality under various conditions of interest: untamped, paraffin tamped, cadmium or boron shielded, etc. In addition to the information on critical masses established, a number of incidental facts of interest were obtained.

The pertinent experimental arrangements and the results obtained are described in the following sections.

A. EFFECT ON CRITICAL MASS OF CONCENTRATION OF INTERMIXED HYDROGEN

1. Paraffin enclosed:

Completely paraffin encased assemblies were built to criticality with H-cubes intermixed with U-cubes in various ratios from $\frac{1}{2}$H:2U to 7H:1U. In addition, extrapolated values of critical mass were obtained from neutron multiplication measurements on two assemblies containing lower amounts of intermixed hydrogen for which sufficient material to reach criticality was not available. In one of these no H cubes were used and in the other the ratio was 1H to 7U. The data are presented in Table 2 and Figure 6, Curve A.

2. With no reflector (air only):

Three critical assemblies were built, with H-cube to U-cube ratios of 1:2, 1:1 and 4:1, respectively, and in these cases no paraffin was placed around the assemblies. The pertinent data are presented in Table 3 and Figure 6, Curve B.

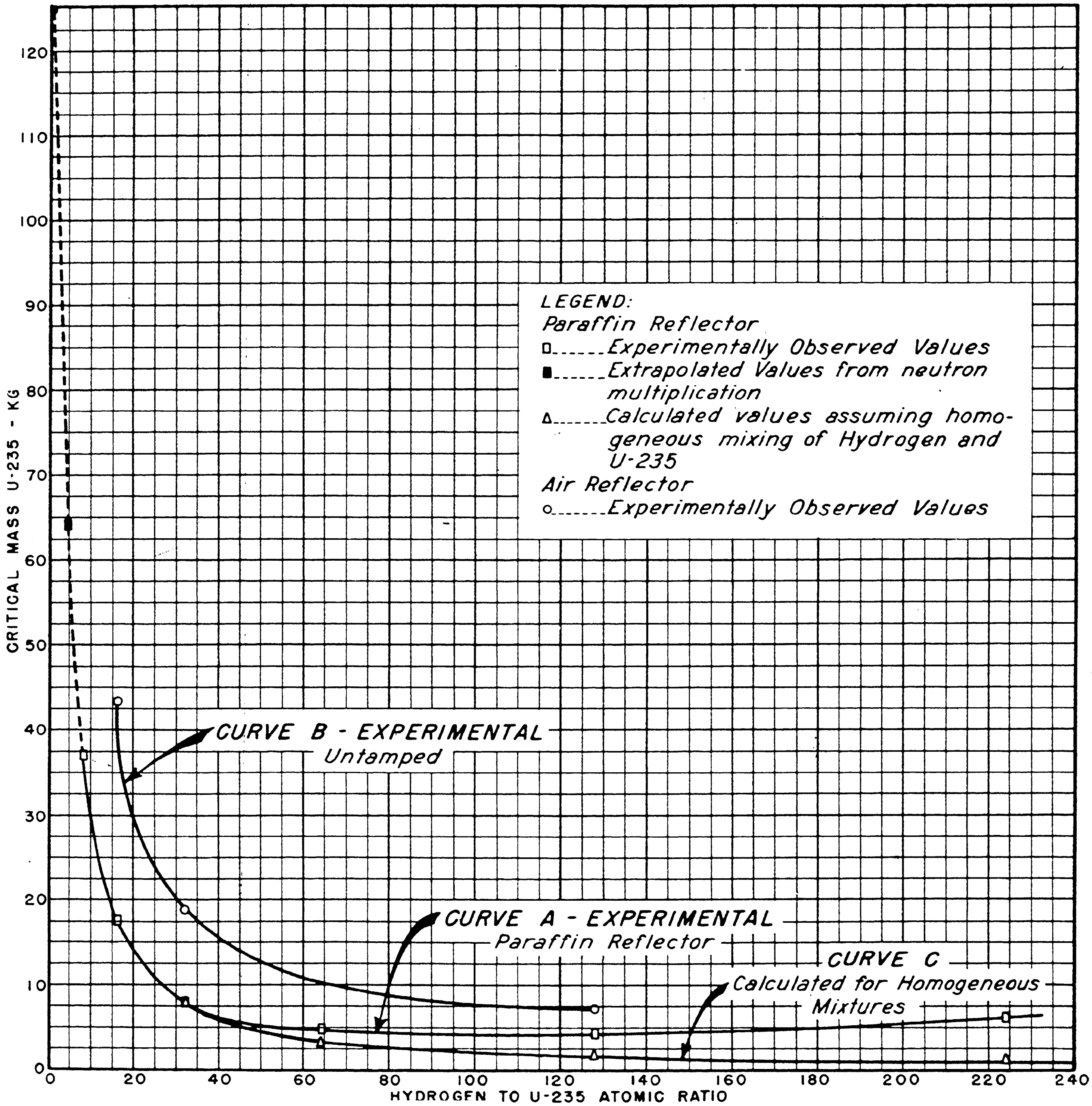

FIGURE 6

VARIATION OF CRITICAL MASS U-235 WITH CONCENTRATION OF INTERMIXED HYDROGEN

TABLE 2

EFFECT OF INTERMIXED HYDROGEN ON CRITICAL MASS (PARAFFIN REFLECTOR)

Experimental Conditions:
Cubical assembly
Completely paraffin enclosed
Lattice Configurations refer
to Figures 19, 20, 21 and 22

H-Cubes: U-cubes	0:1	1:7	1:4	1:2	1:1	2:1	4:1	7:1
Lattice Configuration	A	H	S	C	B	E	G	I
Size of Assembly (Cube Units)**	17x16x16*	16x15x15*	15x14x14+	12x12x12-	10x10x10-	10x10x9	11x11x11-	16x14x14+
Overall Density gm/cm^3	4.8	4.3	4.0	3.5	2.8	2.2	1.7	1.4
H-Cubes	0	454	598	570	498	596	1032	2800
U-Cubes	4351***	3179*	2392	1140	498	298	258	400
Total Cubes	4351***	3633	2990	1710	996	894	1290	3200
Atoms H/Atoms U-235	0	4.6	8	16	32	64	128	224
Critical Mass U, kg	-	215****	123.3	58.8	25.7	15.4	13.3	20.6
Critical Mass U-235, kg	125***	64****	36.8	17.5	7.7	4.6	4.0	6.2

* Not critical

** These assembly dimensions are given in the number of cubes (H and U) along an edge. The actual dimensions, in inches, are slightly larger.

*** This assembly was made of 1105 cubes each containing 48.76 gm U-235 (95.3% assay) and 3246 cubes each containing 15.38 gm U-235 (29.8% assay, which are described in this report) making a total of 223.9 kg of U and 103.8 kg U-235. This assembly was not critical and a very conservative estimate of the critical mass from multiplication measurements is 125 kg. The value is more probably near 160 kg.

**** Extrapolated value from neutron multiplication in 49 kg U-235.

Four striking facts appear from the data in Tables 2 and 3, and Figure 6:

a. The critical mass of U-235 with no intermixed hydrogen, either with or without reflector, appears to be well over 125 kg. Further discussion of this will be given in Section V, F.

b. The value of the critical mass decreases sharply with the addition of small amounts of hydrogeneous material.

c. The values of critical masses of assemblies with no paraffin reflector are considerably higher than those of comparable assemblies paraffin encased, and the difference is much greater in assemblies of low moderation. In those with little moderation, there is a much larger proportion of fast neutrons than in well-moderated assemblies. Neutrons in relatively larger numbers from deep within the pile reach the surface and escape if no reflector is present. In a well moderated assembly, on the other hand, neutrons from locations relatively near the surface are the only ones able to escape whether a reflector is present or not; hence, the influence of a reflector is much less pronounced than in the former case.

d. A minimum appears in experimental curve A where the hydrogen-U-235 atomic ratio is about 125. This effect, which is due to increasing inhomogeneity in the H and U mixing in the assemblies with increasing hydrogen content, will be discussed further in the next section. Curve C of Figure 6 shows the calculated critical masses for this range of moderation.

B. EFFECT ON CRITICAL MASS OF INHOMOGENITY OF H:U MIXING IN MODERATED ASSEMBLIES

The stacking of unit quantities of active material and moderator in the core of an assembly according to some regular pattern other than uniform and homogeneous mixing, may, according to the particular circumstances, result in either larger or smaller critical masses than those for homogeneous mixing. In the case of small rectangular H and U cubes used in the experiments described in this report, the inhomogeneity effect when small relative numbers of H-cubes were used appears to be negligible. But for assemblies containing a 1:1 or larger ratio of H to U cubes, the effect is appreciable.

Greuling, of Clinton Laboratories, following methods indicated briefly in Part I, has made calculations which are of considerable importance in the study of inhomogeneity effects. Using known nuclear constants for the materials involved and for the inhomogeneous lattice configurations employed in the experiments, the values

TABLE 3

EFFECT OF INTERMIXED HYDROGEN ON CRITICAL MASS (AIR REFLECTOR)

Experimental conditions:
Cubical assembly
Air reflector
Lattice configurations
refer to Figures 19 and 20

H-cubes: U-cubes	1:2	1:1	4:1
Lattice Configuration	C	B	G
Size of Assembly (Cube units)	16x16x16+	14x13x13+	13x14x13-
Overall Density gm/cm^3	3.5	2.8	1.7
H-cubes	1405	1232	1856
U-cubes	2810	1232	464
Total cubes	4215	2464	2320
Atoms H/Atoms U-235	16	32	128
Critical Mass U, kg	144.9	63.5	23.9
Critical Mass U-235, kg	43.2	18.9	7.1

of the critical masses to be expected for assemblies having H:U cube ratios of 1:1 and greater, were calculated. In Table 4, a comparison is made of these calculated values with those experimentally obtained. The agreement is quite good for H:U cube ratios above 1:1. At lower H concentrations than this, the effect of fast neutrons in causing fission, which have been neglected in the calculations, became appreciable, and the assumptions on which the calculations are based become invalid.

Calculations were also made of critical masses which would have been obtained had the active material and the moderator been homogeneously mixed in the core of the assembly. These values, presented also in Table 4, and in curve C of Figure 6, are not subject to verification in the present experiments, but the good agreement, noted above, between theory and experiment in instances of appreciable inhomogeneity, leads to confidence in them.

Further, the values are not inconsistent with the data obtained with the "Water Boiler" which gives a critical mass of about 1.2 kg U-235 at H/U-235 atomic ratio ≅ 500.[2]

An attempt was made to experimentally estimate the effect of the inhomogeneous placing of active and moderating materials on the critical mass of an assembly having a given moderation. This was achieved by exaggerating the inhomogeneity at an H to U-235 atomic ratio of 32 (1:1 H to U cube ratio). Four assemblies were built to criticality. In the first, H and U cubes were alternated along each coordinate; in the second, single cubes were alternated in two directions and groups of two H cubes were alternated with pairs of U cubes in the third. The third assembly had single cubes alternately placed along one axis with pairs alternating along two. The fourth was built with two H-cubes alternately placed with two U cubes in each of the three dimensions. In each instance the assembly was approximately cubical and was paraffin enclosed. The data are given in Table 5. If arbitrary "inhomogeneity factors" of 1, 2, 4 and 8 are assigned, respectively, to these four cases and the plot of it against critical mass, shown in Figure 7, extrapolated to "zero inhomogeneity" one obtains about 7.5 kg as the mass of U-235 which would be critical at this moderation if they were intimately mixed.

This agrees, within the limits of the experimental accuracy and the assumptions of the theory, with the calculated value, 7.9 kg, given in Table 4.

Also in Table 5 are given the data obtained from two additional and similar experiments with H:U-235 atomic ratios of 16 and 8. In cases of such low moderation no theoretical analyses have been made and the data are too meagre to lead to definite conclusions. In the experiment with the H:U-235 ratio of 16 stacking arrangements of ½/1, 1/2 and 2/4 were studied. Of these the intermediate "inhomogeneity", 1/2, gave a critical mass less than the others by an amount greater than the precision of the measurement. This implies an arrangement of the moderator and active material which is more favorable for criticality.

TABLE 4

COMPARISON OF EXPERIMENTAL AND

CALCULATED VALUES OF CRITICAL MASS

H:U Cube Ratio	H:U-235 Atomic Ratio	Critical Mass U-235 Experimental Latticed	Calculated Latticed	Calculated Homogenous
1:1	32	7.7 kg	8.62 kg	7.91 kg
2:1	64	4.6	4.31	3.44
4:1	128	4.0	3.57	1.80
7:1	224	6.8	5.76	1.22

[2] Greuling, E., "Theory of Water Tamped Water Boiler", LA-399

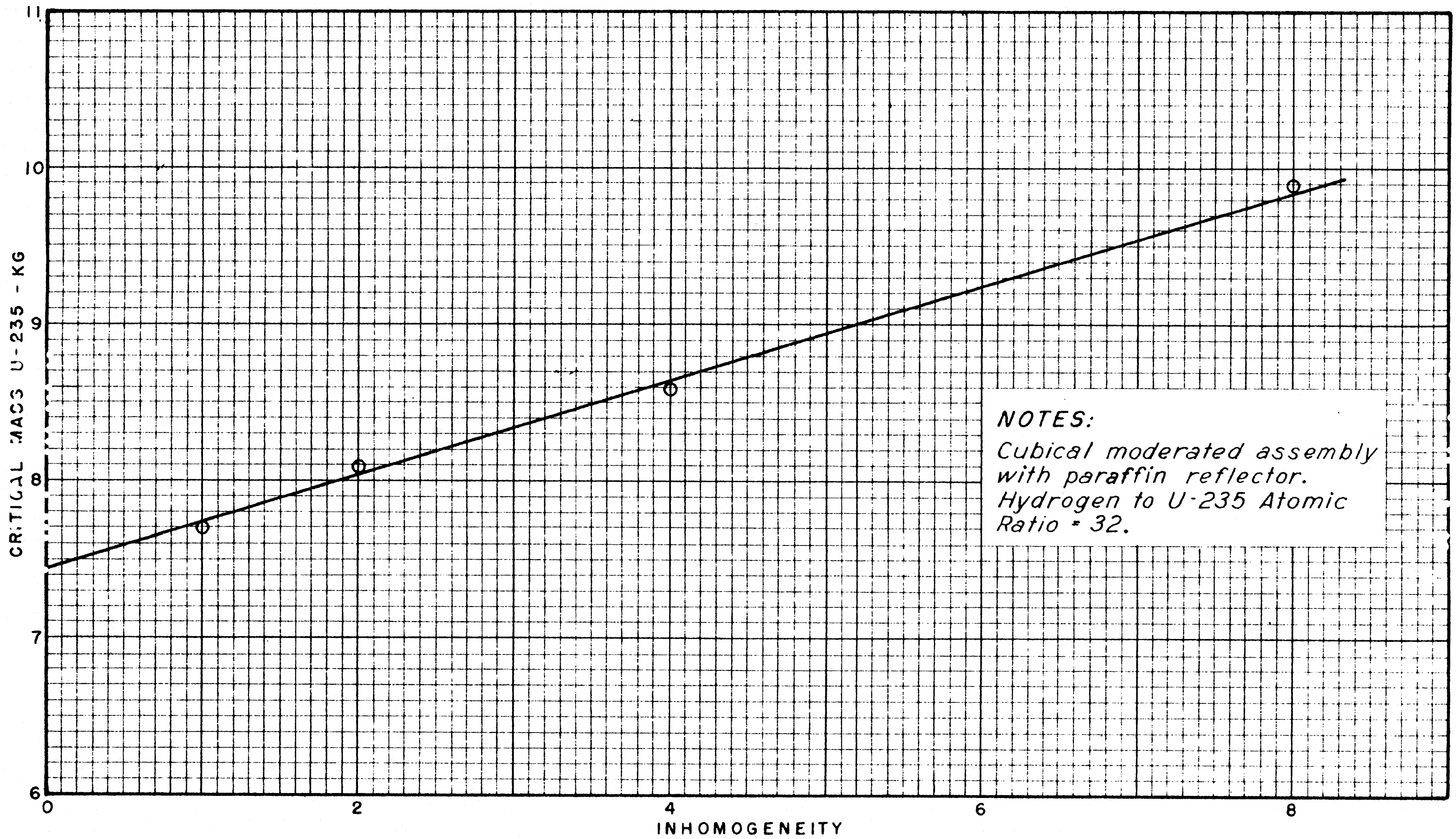

FIGURE 7

EFFECT OF INHOMOGENEITY ON CRITICAL MASS

TABLE 5

EFFECT OF INHOMOGENEITY OF H:U MIXING

Cubical assembly
Completely paraffin enclosed
Lattice configuration refers
to Figures 19, 20, 21, & 22.

Overall H:U cube ratio	1:1				1:2			1:4	
H to U-235 Atomic Ratio	32				16			8	
Overall Density gm/cm³	2.8				3.5			4.0	
Test Number	1/1	2/2	4/4	8/8	½/1	1/2	2/4	½/2	1/4
Lattice Configuration	B	L	K	J	F	C	R	D	S
Size of Assembly (cube units)	10x10x10-	10x10x10+	11x10x11-	11x11x12-	13x12x11+	12x12x12-	13x12x12+	15x15x14+	15x14x14+
H-Cubes	498	527	558	646	585	570	645	612	598
U-Cubes	498	527	558	646	1170	1140	1290	2450	2392
Total Cubes	996	1054	1116	1292	1755	1710	1935	3062	2990
Critical Mass U, kg	25.7	27.2	28.8	33.3	60.5	58.8	66.5	126.3	123.3
Critical Mass U-235, kg	7.7	8.1	8.6	9.9	18.1	17.5	19.8	37.7	36.8

C. EFFECT OF REFLECTOR ON CRITICAL MASS

A series of experiments was designed and executed with the object of determining the effect of surrounding various fissionable assemblies with materials having different neutron absorbing and reflecting properties. Boron, cadmium, paraffin and air were investigated for shielding and reflecting effects. Fissionable assemblies of H and U cubes in ratios of 1:2, 1:1 and 4:1 were studied.

The details of the experimental arrays and the results obtained are presented in Table 6 and in Figure 8. It is seen from the three curves in Figure 8 that the critical masses of assemblies surrounded by cadmium, in the region of moderation studied, are roughly double those of the same assemblies surrounded by paraffin. Assemblies having only air as a reflector have higher critical masses than those surrounded by cadmium or boron. At the moderation studied, boron possesses better shielding properties, resulting in higher critical mass, than cadmium.

D. EFFECT OF GEOMETRY ON CRITICAL MASS

With other conditions fixed, the smallest critical mass is obtained when a given assembly is in spherical shape, since the ratio of surface, through which neutrons can escape, to volume is smaller in this case than in any other. For practical reasons, it would be desirable to know the relations between critical masses in spherical shapes to those in cylindrical shapes. Experimentally, efforts were directed to the determination of differences in critical masses between cubes and long rectangular parallelepipeds.

Assemblies of H:U cubes in ratios of 1:2 and 2:1 completely enclosed in paraffin were studied. In each series of experiments, the (minimum) critical mass was determined with the material in cubical configuration. Then critical masses were determined with the material in rectangular parallelepipeds having successively smaller cross-sectional areas, necessitating, of course, greater lengths. A cross-section was soon reached for which criticality apparently could not be achieved, regardless of extension in length.

The experimental details and data are presented in Tables 7 and 8 and in Figures 9 and 10. In the first series of experiments, where the ratio of H cubes to U cubes was 1:2, criticality was reached when a parallelepiped of 9 inches by 9 inches cross-section was extended to 40 inches. With a cross-section of 8 inches by 9 inches, it appears that criticality could not have been achieved at indefinite extension.

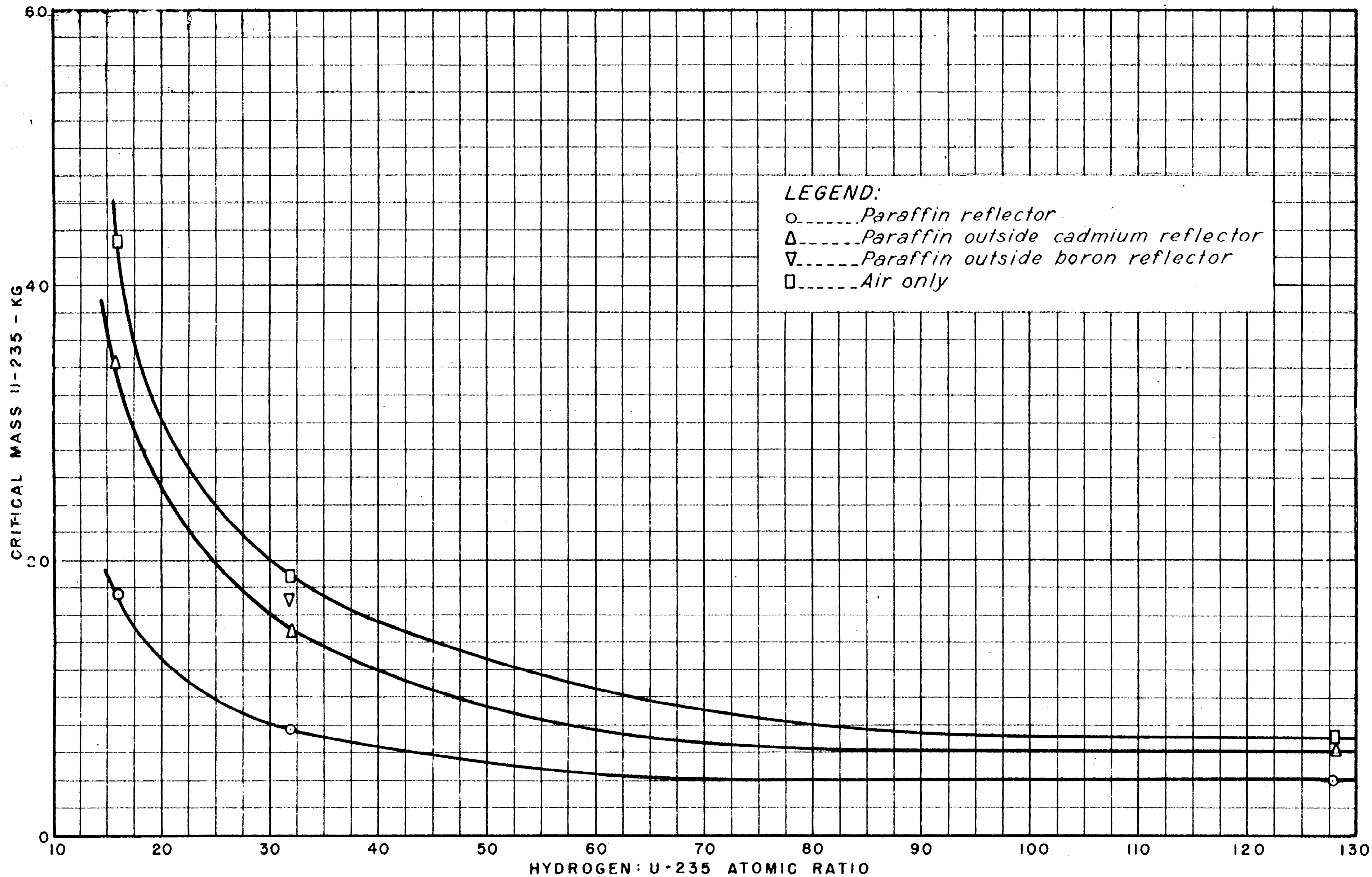

FIGURE 8

EFFECT OF REFLECTOR ON CRITICAL MASS

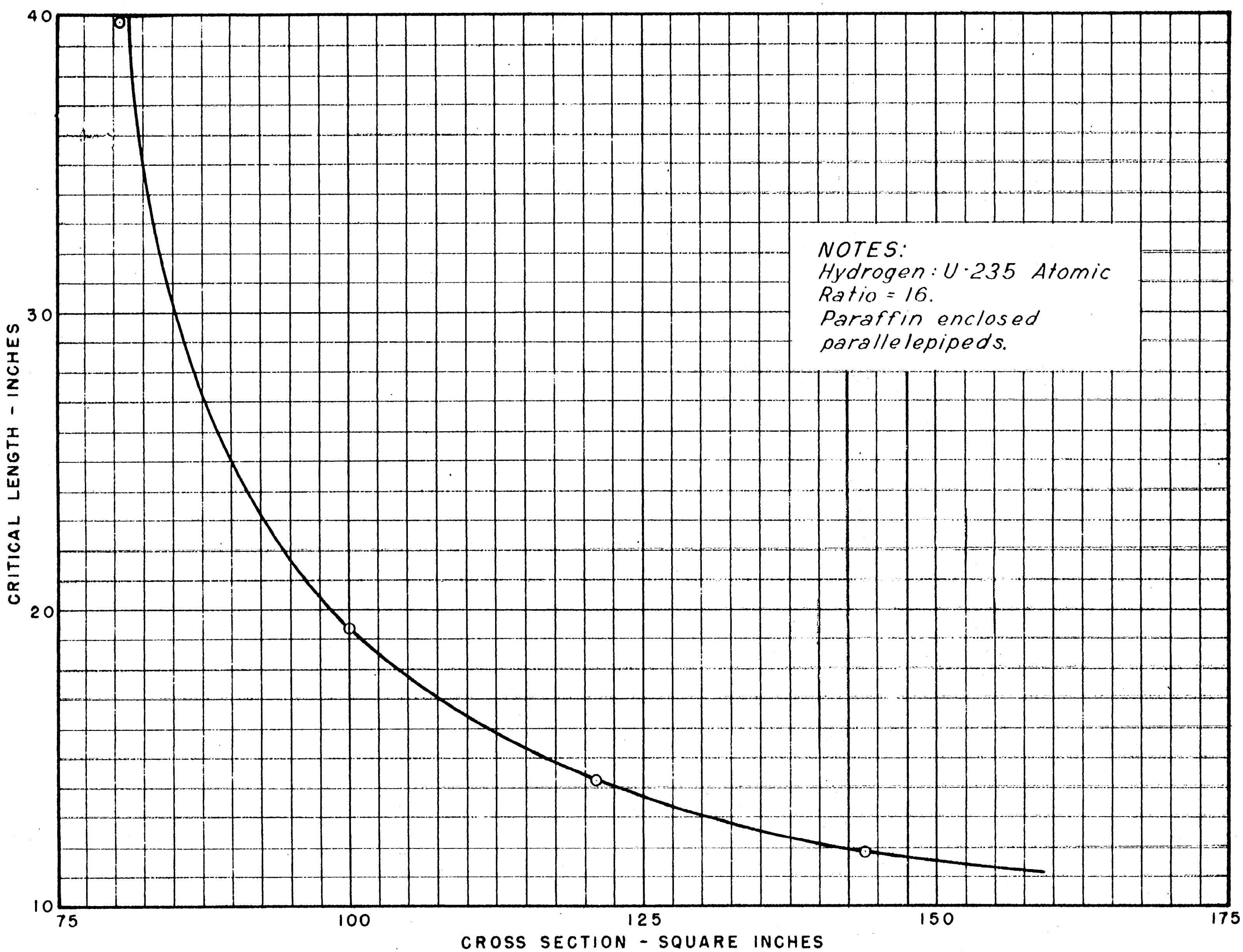

FIGURE 9

EFFECT OF GEOMETRY ON CRITICAL DIMENSIONS

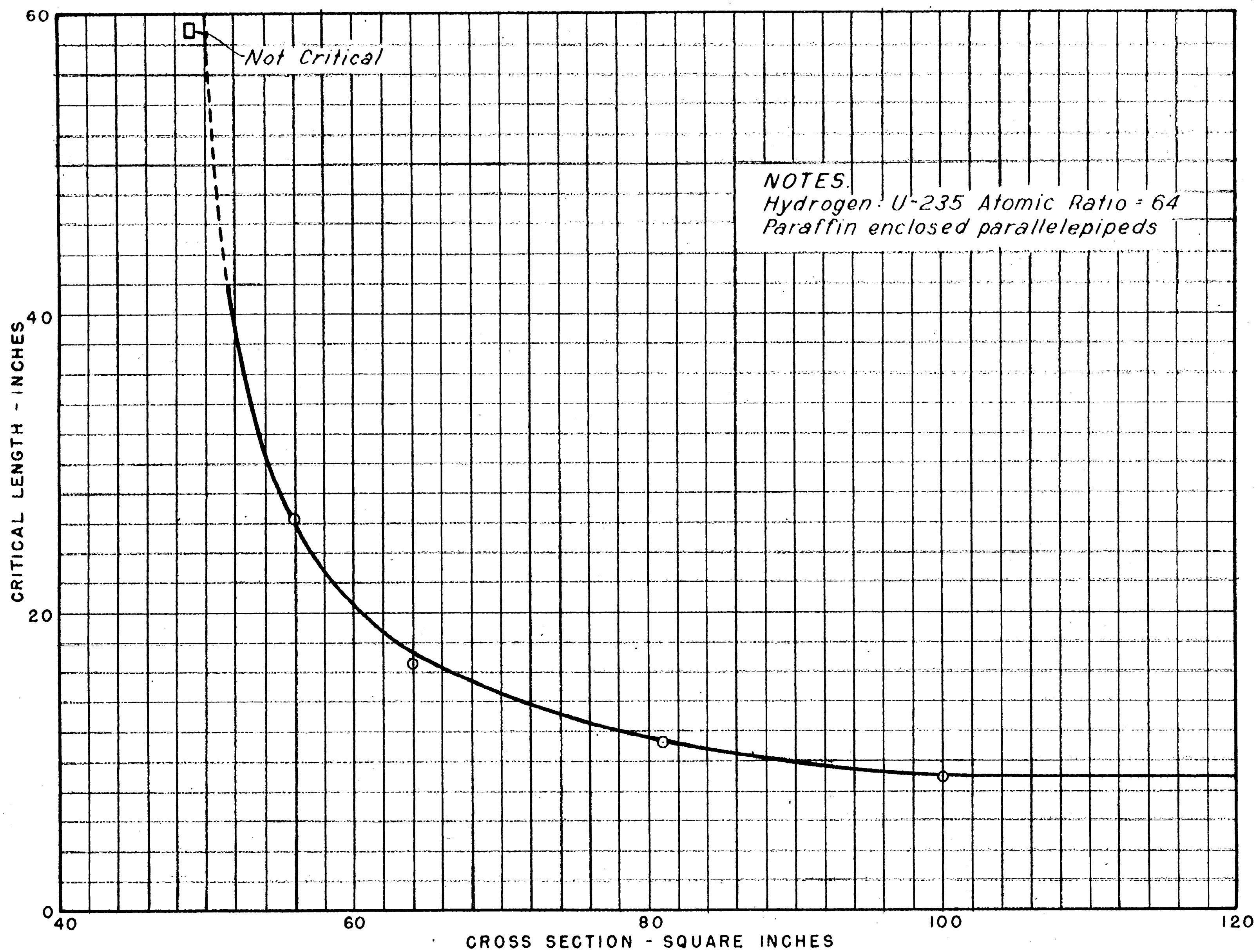

FIGURE 10

EFFECT OF GEOMETRY ON CRITICAL DIMENSIONS

TABLE 6

EFFECT OF REFLECTOR ON CRITICAL MASS

Cubical assembly
Lattice configuration refers
to Figures 19 and 20

H-cubes: U-cubes	1:2			1:1				4:1		
Lattice Configuration	C			B				G		
H to U-235 Atomic Ratio	16			32				128		
Shielding	Paraffin	Paraffin outside cadmium 0.38 gm Cd/cm^2	Air (Untamped)	Paraffin	Paraffin outside cadmium 0.38 gm Cd/cm^2	Paraffin outside boron 0.46 gm B/cm^2	Air (Untamped)	Paraffin	Paraffin outside cadmium 0.38 gm Cd/cm^2	Air (Untamped)
Size of Assembly (cube units)	12x12x12-	15x15x15-	16x16x16+	10x10x10-	12x13x12+	13x13x13+	14x13x13+	11x11x11-	13x13x12-	13x14x13
H Cubes	570	1117	1405	498	965	1118	1232	1032	1600	1856
U Cubes	1140	2235	2810	498	965	1118	1232	258	400	464
Total Cubes	1710	3352	4215	996	1930	2236	2464	1290	2000	2320
Critical Mass U, kg	58.8	115.2	144.9	25.7	49.8	57.6	63.5	13.3	20.6	23.9
Critical Mass U-235, kg	17.5	34.4	43.2	7.7	14.8	17.2	18.9	4.0	6.2	7.1

TABLE 7

EFFECT OF GEOMETRY ON CRITICAL MASS

Experimental conditions:
1:2 ratio of H-cubes to U-cubes
Lattice configuration: C, Figure 19
Atoms H/Atoms U-235 = 16
Overall density = 3.5 gm/cm^3
Paraffin reflector

Size of Assembly (cube units)	12x12x12-	11x11x14+	10x10x19+	9x9x40-
H Cubes	570	578	648	1075
U Cubes	1140	1155	1295	2150
Total Cubes	1710	1733	1943	3225
Critical Mass U, kg	58.8	59.6	66.8	110.9
Critical Mass U-235, kg	17.5	17.8	19.9	33.1

TABLE 8

EFFECT OF GEOMETRY ON CRITICAL MASS

Experimental conditions:
2:1 ratio of H-cubes to U-cubes
Lattice configuration E, Figure 20
Atoms H/Atoms U-235 = 64
Overall density = 2.2 gm/cm^3
Paraffin reflector

Size of Assembly (cube units)	10x10x9	9x9x11+	8x8x17-	8x7x26+	7x7x59*
H-cubes	596	604	712	976	1924
U-cubes	298	302	356	488	962
Total Cubes	894	906	1068	1464	2886
Critical Mass U, kg	15.4	15.6	18.4	25.2	(49.6)*
Critical Mass U-235, kg	4.6	4.6	5.5	7.5	(14.8)*

*Not critical

For the case of H:U cube ratio of 2:1, an assembly 7 inches by 7 inches in cross-section was extended to a length of 59 inches without its being critical, and indications were that indefinite extension would not have produced criticality.

E. EFFECT OF DENSITY ON CRITICAL MASS

A brief investigation was made of the effect of density on critical mass by stacking the H and U cubes in a cubical assembly with aluminum spacers between the cubes. These spacers were sections of aluminum tubing 7/8" OD, 5/8" ID which were $\frac{1}{2}$" or 1" long. The actual ratio of the H and U cubes in the assembly was not changed, but the interposition of aluminum spacers reduced the overall stacking density of the assembly. Comparisons were made of the critical masses of cubical assemblies containing H-cube:U-cube:Al spacer ratios of 4:1:0, 4:1:$\frac{1}{2}$ and 4:1:1, and also of assemblies containing H-cube:U-cube:Al spacer ratios of 1:2:0, 1:2:$\frac{1}{2}$, and 1:2:1. The experimental details and results are given in Tables 9 and 10.

From arguments presented in Part I, it would be expected that in the ideal case, critical mass should vary approximately inversely with the density squared. In actual assemblies it would be expected that greatest deviations from this "inverse square rule" would occur in assemblies relatively largely affected by the surrounding reflector, i.e., small, unmoderated assemblies. Here, one would expect the numerical value of the negative exponent of density to be somewhat less than two. Large, well moderated assemblies would be expected to follow more nearly the inverse square rule.

The results obtained in these experiments are in moderate agreement with these expectations. In the assembly with low moderation, H:U-235 = 16, the mass varied as the -1.7 power of the density. In one instance of higher moderation, with an H:U-235 ratio of 128 the exponent is -2.1; in the other experiment it is -2.9. There is no apparent reason for this difference and the experiment was not repeated.

F. CRITICAL MASS BY EXTRAPOLATION OF NEUTRON MULTIPLICATION MEASUREMENTS.

If enough material is not available to build a critical assembly, or if one does not desire to risk the hazards involved in making an assembly critical, an estimate of the value of critical mass under a given set of conditions may be obtained by measuring the neutron multiplication by two or more sub-critical assemblies which approach criticality by different amounts. With fixed geometry of counters, assembly, reflector, etc., and a fixed location and strength of source in successive assemblies, the multiplication of "input" neutrons in each assembly should be proportional to the nearness of approach of that assembly to criticality. At criticality, of course, the multiplication of each "input" neutron is infinite.

If a graph is plotted of the reciprocal of neutron multiplication as a function of the U-235 mass in each respective assembly, a more or less linear curve is obtained, depending on the geometry, characteristics of the counters, etc. An extrapolation of this curve to zero reciprocal multiplication gives, as the intercept, an approximate value of the critical mass.

TABLE 9

EFFECT OF DENSITY ON CRITICAL MASS

Experimental conditions:
4:1 ratio of H-Cubes to U-cubes
Cubical assembly
Completely paraffin enclosed
H to U-235 atomic ratio = 128
Lattice configuration refers to Figures 20 and 23

Volume Ratio of Materials H:U:A	4:1:0	4:1:$\frac{1}{2}$	4:1:1
Lattice Configuration	G	W	V
Size of Assembly (cube units)	11x11x11-	12x13x12	13x14x12-
Overall Density gm/cm^3	1.7	1.6	1.4
Density, gm U-235/cm^3 (ρ)	0.19	0.17	0.16
H-cubes	1032	1364	1512
U-cubes	258	341	378
Air Spaces (cube units)	0	170	378
Total "Cubes"	1290	1875	2268
Critical Mass U, kg	13.3	17.6	19.5
Critical Mass U-235, kg (M)	4.0	5.2	5.8
x*	--	2.93	2.09

* $M \propto (\rho)^{-x}$; values of x determined by comparison with the 4:1:0 assembly

TABLE 10

EFFECT OF DENSITY ON CRITICAL MASS

Experimental conditions:
1:2 ratio of H cubes to U cubes
Cubical assembly
Completely paraffin enclosed
H to U-235 atomic ratio = 16
Lattice configuration refers
to Figures 19 and 23.

Volume Ratio of Materials, H:U:A	1:2:0	1:2:$\frac{1}{2}$	1:2:1
Lattice Configuration	C	U	T
Size of Assembly, cube units	12x12x12	14x14x13	15x15x16
Overall Density, gm/cm^3	3.5	3.0	2.7
Density, gm U-235/cm^3 (ρ)	0.62	0.54	0.47
H-cubes	570	755	903
U-cubes	1140	1510	1807
Air spaces, cube units	0	377	904
Total "Cubes"	1710	2642	3614
Critical Mass U, kg	58.8	77.9	93.2
Critical Mass U-235, kg, (M)	17.5	23.2	27.8
x *	--	2.0 1.82	1.60

* $M \propto \rho^{-x}$; values of x obtained by comparison with the 1:2:0 array

The reliability of the value of the critical mass thus obtained depends on the nearness with which the sub-critical assemblies approach criticality.

The experiments on neutron multiplication performed and results obtained are described in the following paragraphs.

1. <u>Comparison of "Extrapolated" Value of Critical Mass With That Directly Measured</u>

Six sub-critical assemblies having H:U cube ratio of 1:4, were built in the range from 50% to 95% critical, and the values of neutron multiplication measured on the respective assemblies were used to obtain an "extrapolated" value of critical mass from the plot in Figure 11. The estimated value so obtained was about 2% higher than the directly measured value of the critical mass of an assembly under similar conditions. (The difference between the two directly measured values of the critical mass for this assembly appearing in Table 2 and on Figure 11 is due to the former having been corrected for the effect of the Al stacking shield.)

2. <u>Critical Mass of a Low-Moderation Assembly: H-cubes: U-cubes = 1:7</u>

The 50 kg of U-235 available in the U-cubes was not sufficient to produce criticality in an assembly containing only 4.6 H atoms per U-235 atom. Four sub-critical assemblies ranging from 56% to 75% of the subsequently estimated critical mass, were measured for neutron multiplication and the values obtained were used in Figure 12 to indicate an "extrapolated" critical mass. The value obtained, 64 kg of U-235, appears to be in good agreement with experimental values obtained for other conditions.

3. <u>Critical Mass of Unmoderated Assemblies</u>

Using the 3248 30% U-cubes available in these experiments (49.92 kg U-235) and the 1105 95% cubes used in the Los Alamos experiments (53.88 kg U-235) which are described in Part I, it was possible to build an assembly containing 103.8 kg U-235 at a density of 1.5 gm U-235/cc. The 95% cubes were uniformly inter-spread among the 30% cubes. This assembly was quite far from critical, even when completely enclosed in paraffin.

Neutron multiplication measurements were made on this assembly and on three similar assemblies containing less material. The data, presented in Figure 13, leave considerable uncertainty as to the critical mass of U-235 to be expected under these conditions. Data from the smaller assemblies fall on a fairly good straight line, which by linear extrapolation indicates a critical mass of 125 kg U-235. The data from the largest assembly, which should carry greater weight, because the assembly is nearer critical and because the data were corroborated by a second independent measurement, indicate that the extrapolation should not be linear, and that the critical mass is certainly much larger than

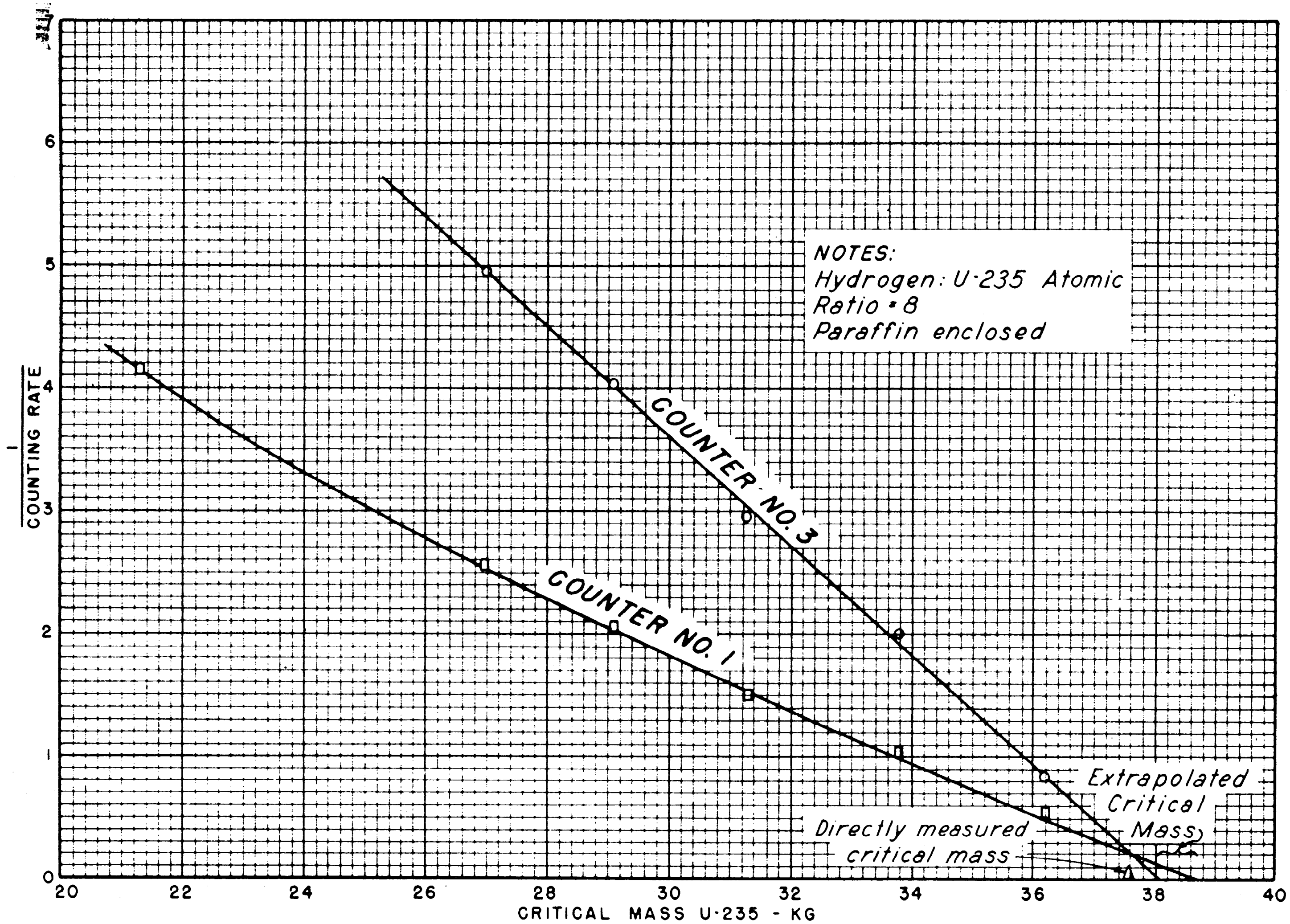

FIGURE 11

INVESTIGATION OF CRITICAL MASS DETERMINATION BY MULTIPLICATION METHOD

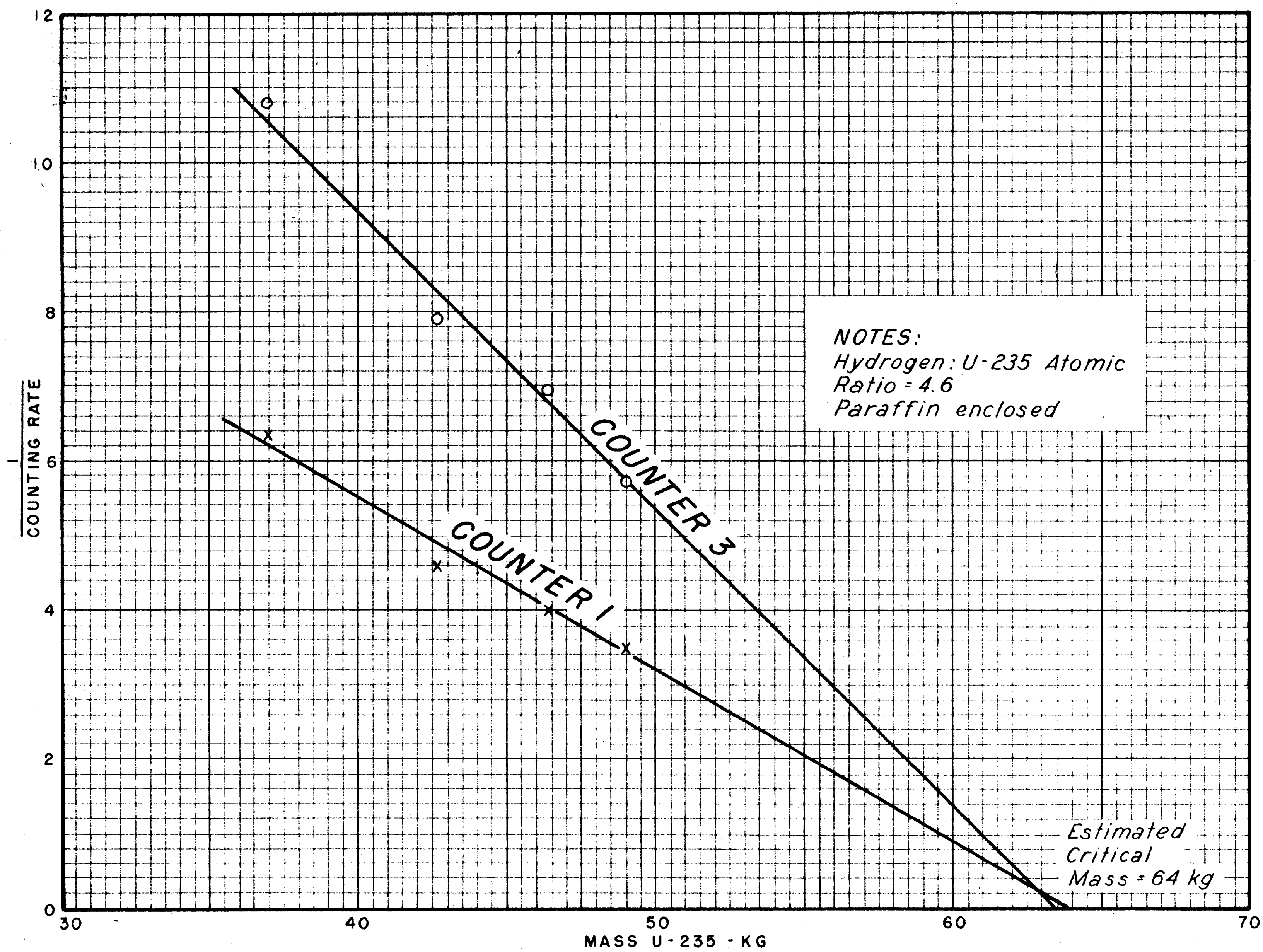

FIGURE 12

CRITICAL MASS ESTIMATION BY MULTIPLICATION METHOD

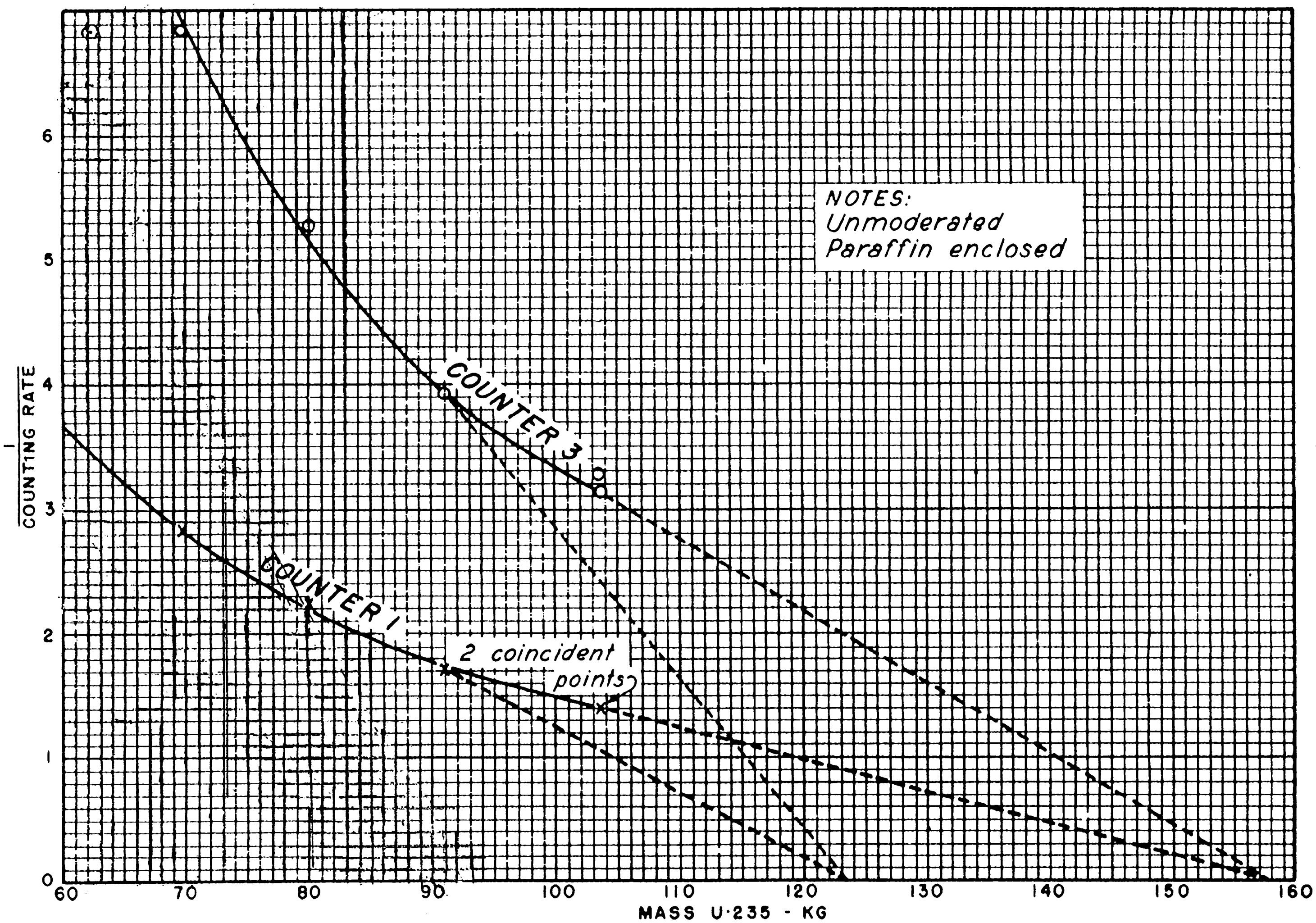

FIGURE 13

CRITICAL MASS ESTIMATION BY MULTIPLICATION METHOD

125 kg. Linear extrapolation of the data from the two largest assemblies only indicate the critical mass is around 160 kg. The true value, for the conditions stated, is certainly at least 125 kg, and is probably considerably larger.

G. EFFECT OF INTERACTION BETWEEN TWO SEPARATED ASSEMBLIES.

1. An Experiment on Arrays with no Reflector

The combined critical mass of two untamped arrays, separated by various distances, was determined in order to obtain information on the effect of interaction between the two arrays. This process of mutual exchange of neutrons between separated arrays is important in considering safe separations between stored containers of active material.

A single array with no reflector and of dimensions about 14x14x12.5 inches of H:U-235 atomic ratio equal to 32, was first assembled in the usual manner and adjusted to criticality. The two halves of this assembly, which had been separated by 1/16" of aluminum, were then moved apart and cubes were added to the respective halves in such a way as to preserve a cross-section of 14" x 14" for the facing sides of the two arrays. Sufficient quantities of materials were added so that criticality was achieved in successive trials at various separations up to 8". A graph of the total mass of U-235 in the system as a function of separation at criticality is given in Figure 14.

In another experiment, an alternative to maintaining a constant area of facing sides in the two arrays, two tests were made in which the respective halves were cubical in shape and are also reported in Figure 14.

The effect of cadmium shielding on interaction was tested on five assemblies which were built to criticality with a cadmium sheet inserted between the separated halves.

As the most important result obtained from these experiments, it was found that 90% of the critical mass of a single array could be collected into each of two components separated by as little as 8". Although one could collect amounts of material in each of two separated assemblies equal to that in a single critical assembly only with complete isolation of the two, it appears that interaction, in the range of concentrations studied here, is less effective than has previously been calculated.

A comparison with the approximate theory[3] used by the Y-12 Special Hazards Group in estimating safe separations of stored vessels or reactors is plotted on Figure 14. It was assumed that the critical dimensions of the halves are related to those of the single array by

$$K_1^2 = (1 - \omega)K_2^2 \qquad (1)$$

[3] Murray, R. and Schmidt, G. W., "Interaction of Solutions of Uranium Salt", A-7.390.20, Jan 2, 1947

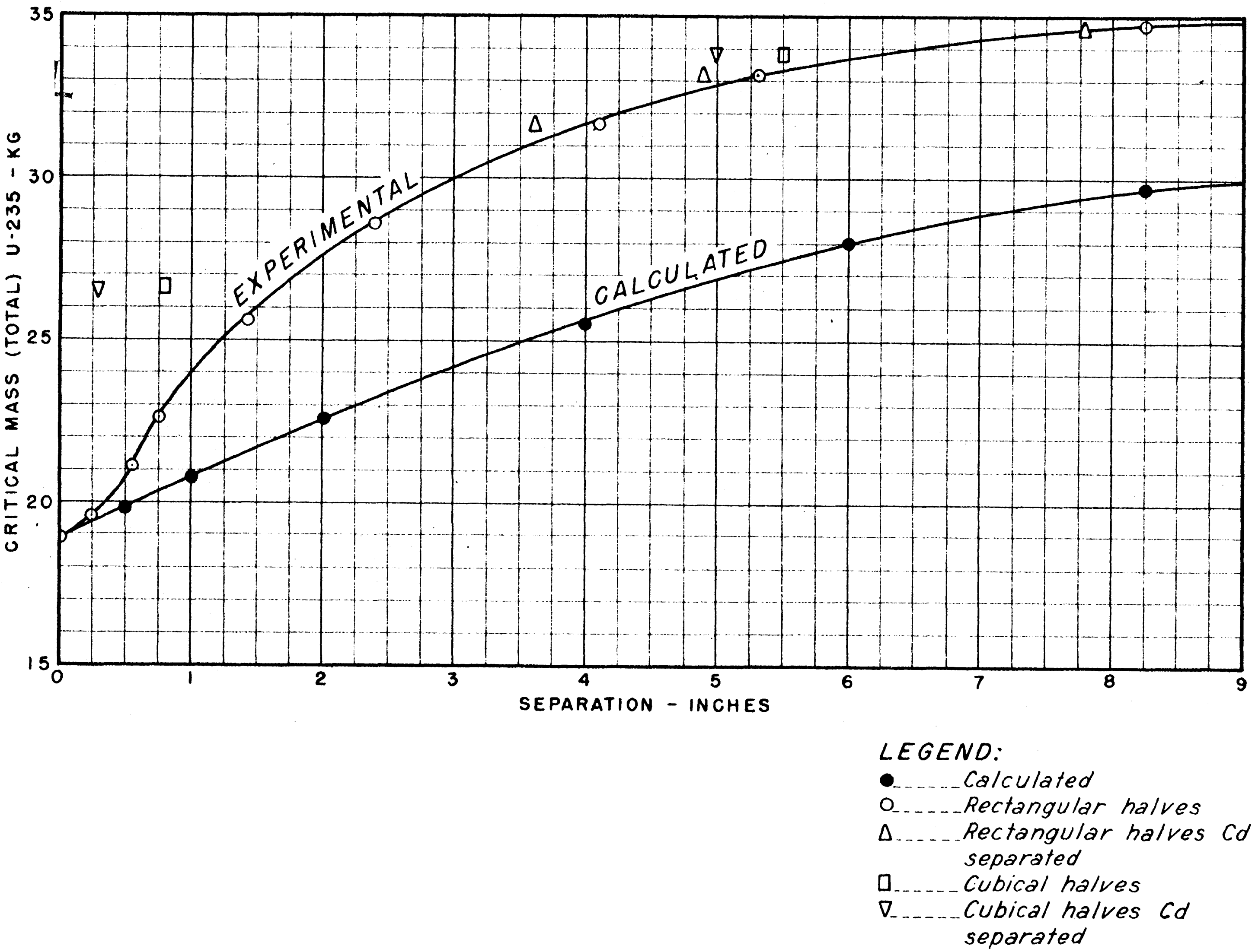

FIGURE 14

VARIATION OF CRITICAL MASS WITH SEPARATION OF PARTS where ω is the fractional solid angle subtended at one half by the other and where K is the negative of the "buckling", a geometry parameter familiar in pile design. (K_1 refers to the hypothetical array equivalent to the two separated halves and K_2 to the single critical assembly.)

The largest possible ω, that formed at the center of one face by the area of the adjacent face, was calculated for various separations. K_2 was determined from the relation

$$K^2 = \pi^2/a^2 + \pi^2/b^2 + \pi^2/c^2 \qquad (2)$$

where a, b and c are the dimensions of the single critical cube. A number of valuesof K_1, each characteristic of the system composed of the two parts separated a particular distance were then obtained from equation (1). From these values of K_1, and the fixed cross-sectional area, ab, values of the third dimension, c', of assemblies equivalent to the two separated halves were calculated by equation (2). The masses corresponding to these values of c' are those plotted in Figure 14.

It should be emphasized, however, that the exact agreement of the experimental and calculated curves at zero separation is a result of the normalization effected by the use of the single critical cube's dimensions in evaluating K_2.

It is clear that the effect of separation is larger than that assumed in these calculations. That is, the "degree of criticality" of separated assemblies is much less than calculations on the above basis would indicate.

The result, that the cadmium shielding allows a smaller separation for the same total mass (or requires a larger total mass at the same separation), was to be expected because some thermal neutrons are absorbed by the cadmium and thus prevented from entering the opposing half. That this effect is rather small was also expected. The percentage of the leaking neutrons that are thermal is predicted by the conservative untamped water boiler theory[4], to be considerably less than 1% for this H:U-235 atomic ratio of 32.

2. Experiments on Separated Arrays Partially Enclosed in Paraffin

When two halves of what was intended to become a single assembly of active material surrounded by paraffin reflector were built at some distance apart and then brought together, the exposed surface of the two facing sides of the respective halves were composed of an inner rectangle of active material concentrically surrounded by paraffin reflector. As the halves are brought together, the exchange of neutrons between the facing rectangles of material in the two halves is hindered only by the air-gap separating them. Obviously the exchange of neutrons across the gap plays an important role when criticality is achieved with a gap still remaining. One would expect this exchange to be related to the total solid angle "seen" by the respective rectangles of active material in each other. Calculations by

[4] Murray, R. and Schmidt, G. W., "Criticality in Untamped Uranium Solutions", A-7.390.13, February 4, 1947

Murray and Schmidt[5] show that the fractional solid angle between the faces is essentially a linear function of separation for distances up to one or two inches.

Experiments were performed to measure the effect of separation between the two parts of an assembly on the total amount of active material in the assembly required to produce criticality. It was found that the critical amount of material required was very nearly linearly related to the separation between the halves, for separations up to two or three inches as shown, for example, in Figure 15, the increase in critical mass being about 20% for each inch of separation in this range of separation. For separations larger than about three inches, the relation between critical mass and separation deviated from linearity as shown by the curve in Figure 14.

In the actual performance of the experiments advangage was taken of this practically linear relation at small separation of the parts of the assembly. Instead of minutely adjusting the mass of the active material until it was critical with the two parts just in contact, the experiment was done in the following manner. The mass was first adjusted to be critical at a separation of a few tenths of an inch and that distance measured. A small change in the mass was then made and a new critical separation measured. This was repeated and a plot of the data extrapolated to zero separation for the desired result.

H. EFFECT OF REMOVING SMALL AMOUNTS OF REFLECTOR FROM PARAFFIN ENCLOSED ASSEMBLIES

In building paraffin enclosed critical assemblies of various dimensions, it was sometimes difficult to avoid small crevices between the pieces of paraffin, especially at the corners and top of the assembly. To study the effect this might have on the critical mass, a determination was made on one assembly having an H:U-235 ratio equal to 16, of the increase in mass required for criticality as successively larger amounts of paraffin reflector were removed. It was found that the percentage increase in mass required for criticality was linearly proportional to the percentage of surface not covered by reflector, the mass increasing about 1.2% for each percent of reflector removed. This was true up to 10% of the total reflector, beyond which the experiment was not carried. The results are shown in Figure 16.

I. EFFECT OF ISOTOPIC CONCENTRATION ON CRITICAL MASS

Any change of the isotopic composition of the active material used in these experiments with a given degree of moderation, will cause, at the same time, a change in the overall density of the assembly, a change in the U-235 density and in the homogeneity of mixing

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[5] Murray, R. and Schmidt, G. W., "Methods of Calculating Solid Angles", A-7.390.16, November 25, 1946.

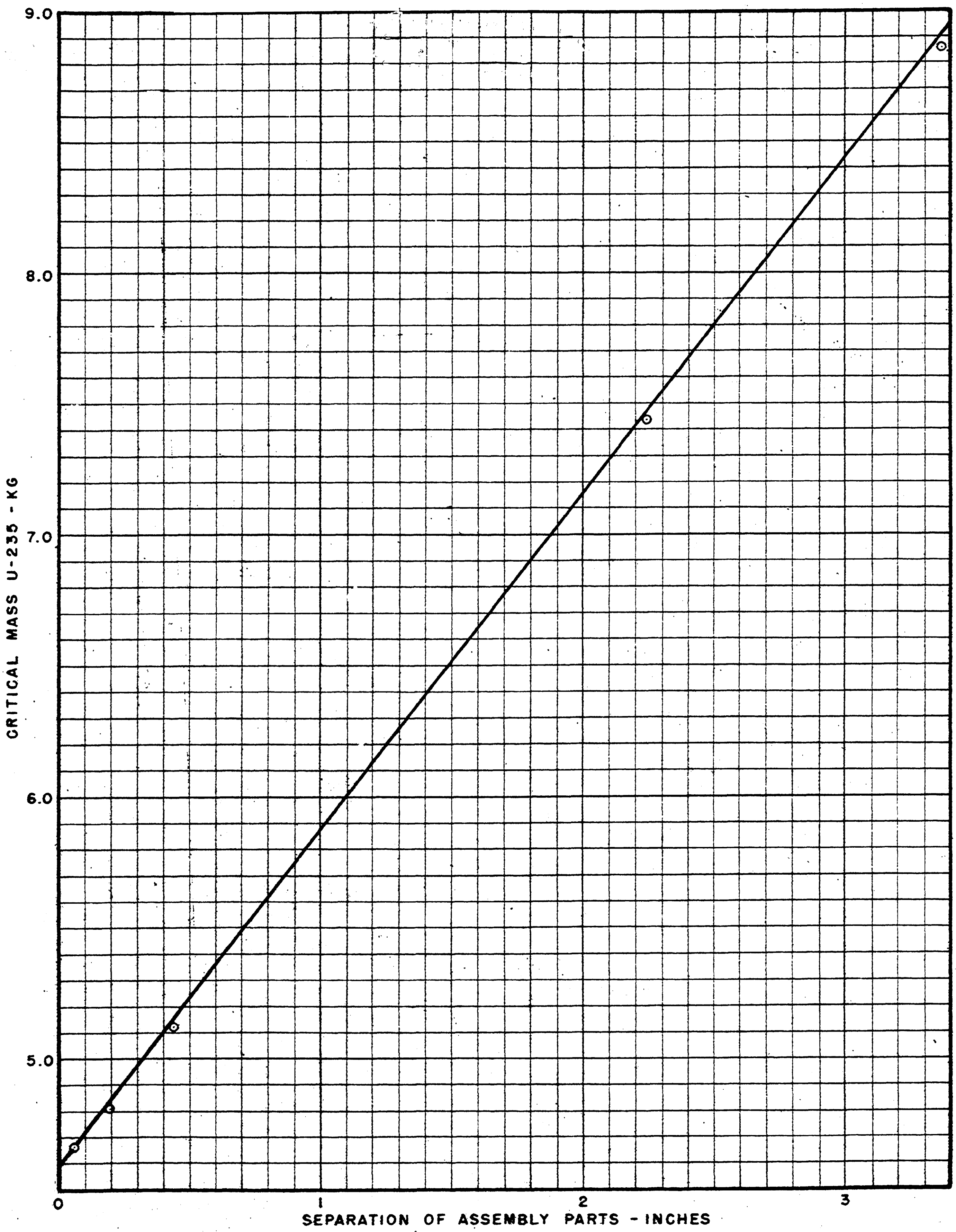


FIGURE 15

VARIATION OF CRITICAL MASS
WITH SEPARATION OF PARTS

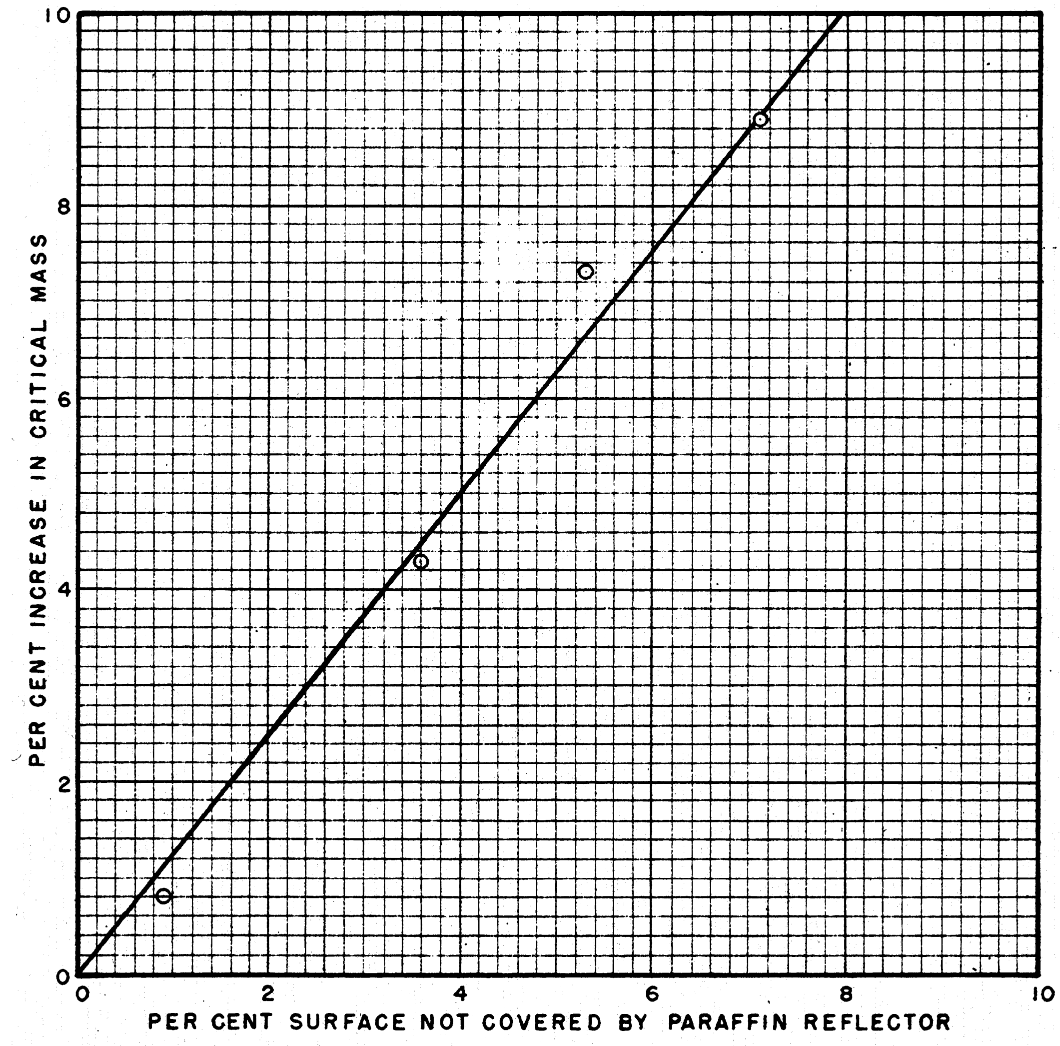


FIGURE 16

VARIATION OF CRITICAL MASS
WITH INCOMPLETE REFLECTOR

of the active material and the moderator. For example, consider two assemblies having an H:U-235 atomic ratio of 60, one build with uranium having a U-235 isotopic content of 30% and the other of 95%. The overall density of the first would be 2.2 gm/cm^3, that of the second 1.5 gm/cm^3. The first would have a U-235 density of 0.31 gm/cm^3 and the second 0.43 gm/cm^3. The 30% material would have been stacked in an H:U cube ratio of 2:1 and the 95% in a 6:1 pattern.

In this comparison of the results of the experiments with those described in Part I using material of 95% assay, no attempt has been made to normalize the densities. However, the effect of the inhomogeneity has been considered by Greuling in those assemblies where the neutrons are mostly thermal. In Figure 17 have been plotted the values of the critical mass of the two isotopically different materials as a function of the degree of moderation. In addition to the experimentally observed data, the values calculated for the case of uniform mixtures of the hydrogen and U-235 are given. It is evident, as is expected, that the presence of U-238 affects the critical mass most decidedly in the region of low moderation, i.e., where the proportion of fast neutrons is large. In regions of high moderation, when corrections are made for the effects of inhomogeneous mixing, the critical mass depends on the amount of U-235 present and is very little affected by presence of U-238.

VI. SUMMARY OF RESULTS

Tabulation of the results obtained in these experiments on uranium material of 30% U-235 isotopic concentration is best presented in graphical form. In Figure 18 where the values of the critical masses obtained are shown as a function of the H:U-235 atomic ratio, as much description as possible of the experimental conditions is given beside each plotted point. Curves have been drawn through the experimental data obtained from unenclosed assemblies and from these enclosed in paraffin. A comparison of the latter with the values calculated for homogeneous mixtures of the hydrogen and the U-235 is shown in the two lower curves. The values for assemblies of low moderation are quite uncertain. The critical masses obtained under various other conditions as indicated are given.

VII. ACKNOWLEDGEMENTS

The assistance of S. D. Snyder, W. G. Kirby, J. K. Fox, W. T. Daniel, Thos. McKay and Harold Geller in the preparation of the experimental materials is gratefully acknowledged. Credit is due J. K. Lykins, W. T. Leland, and Dr. R. C. Smith for the instrumentation used in the control of these experiments.

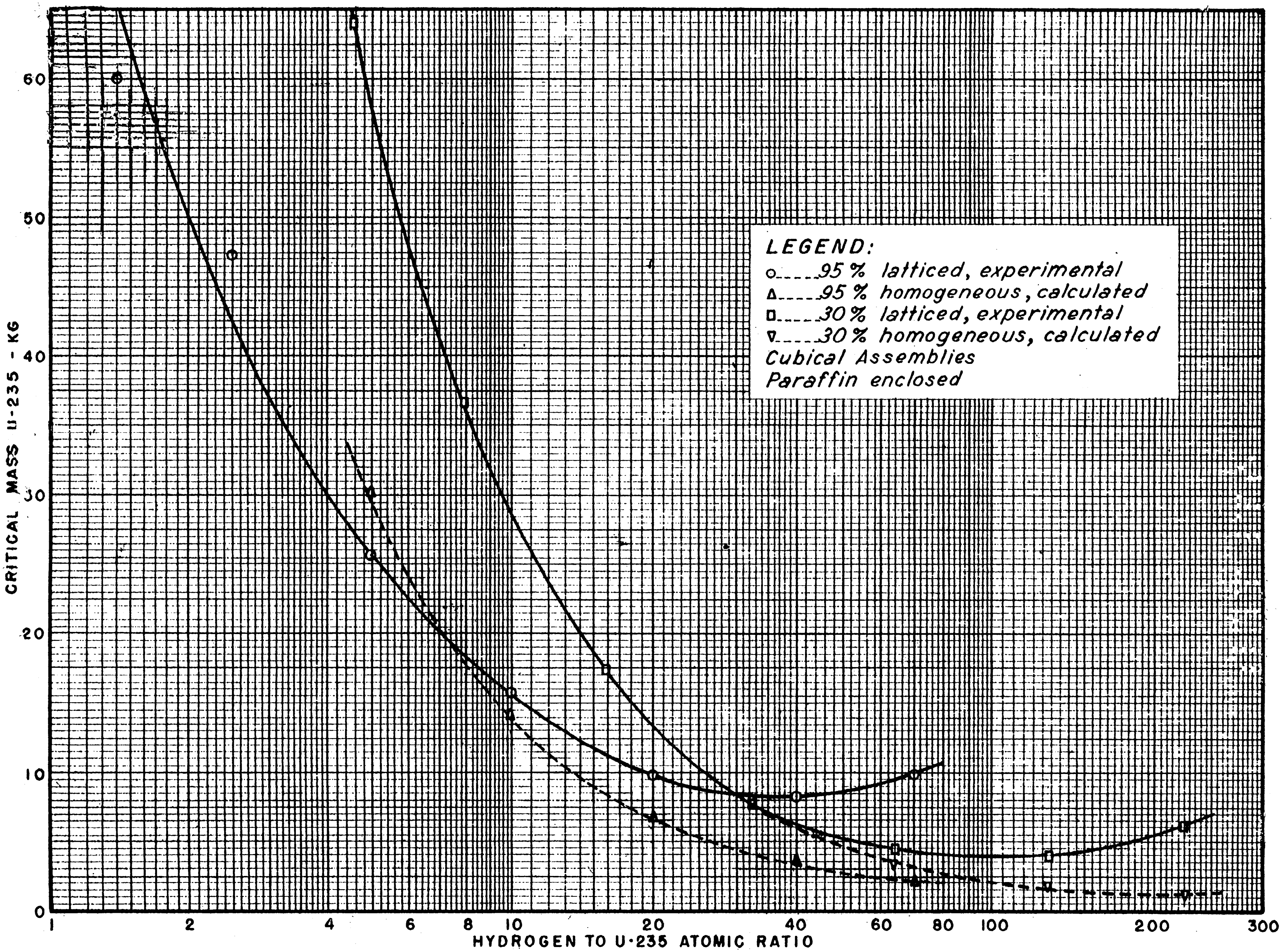


FIGURE 17

VARIATION OF CRITICAL MASS WITH ISOTOPIC CONTENT

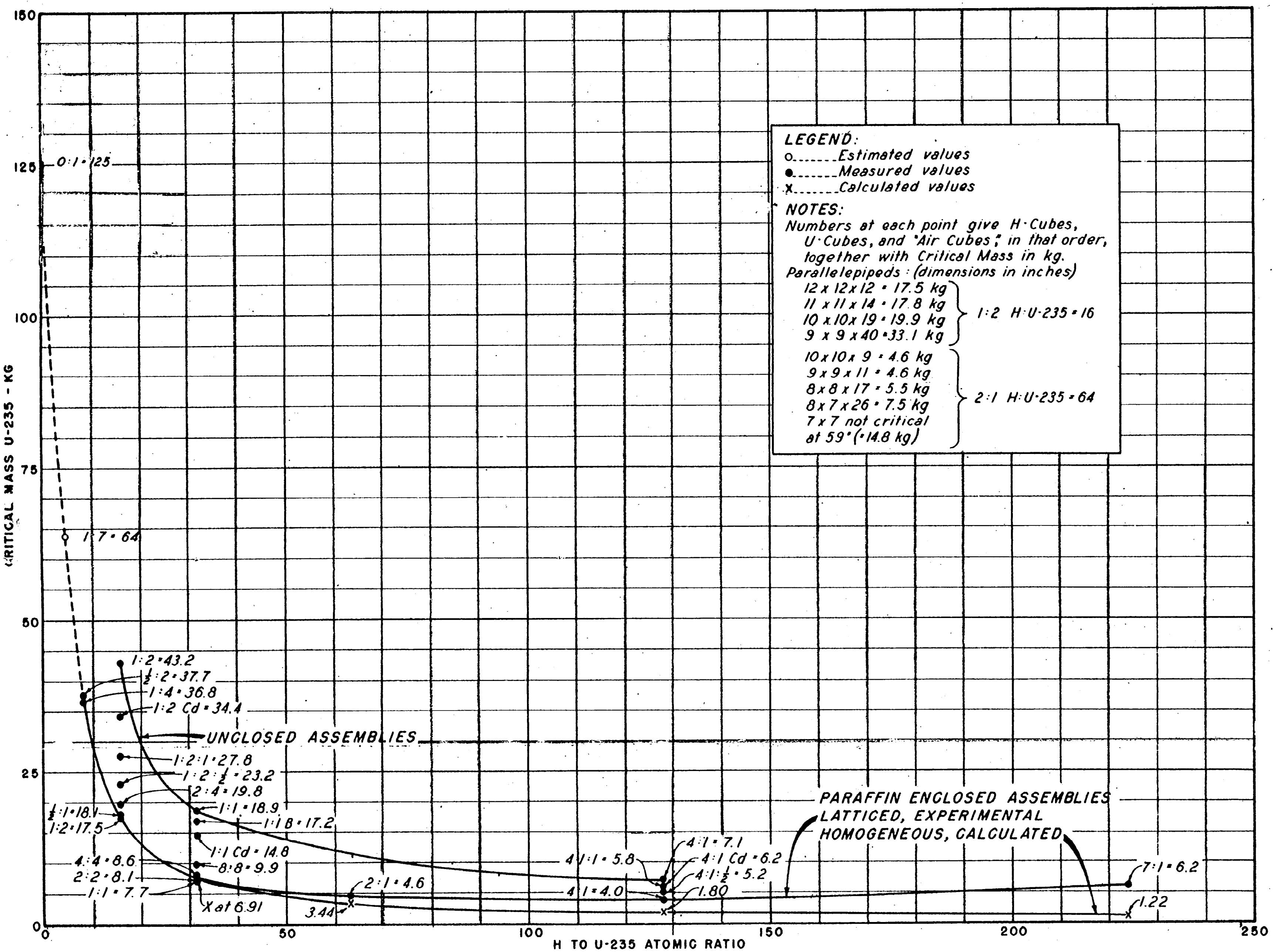


FIGURE 18

SUMMARY OF CRITICAL MASSES OF U-235
OBTAINED UNDER CONDITIONS INDICATED

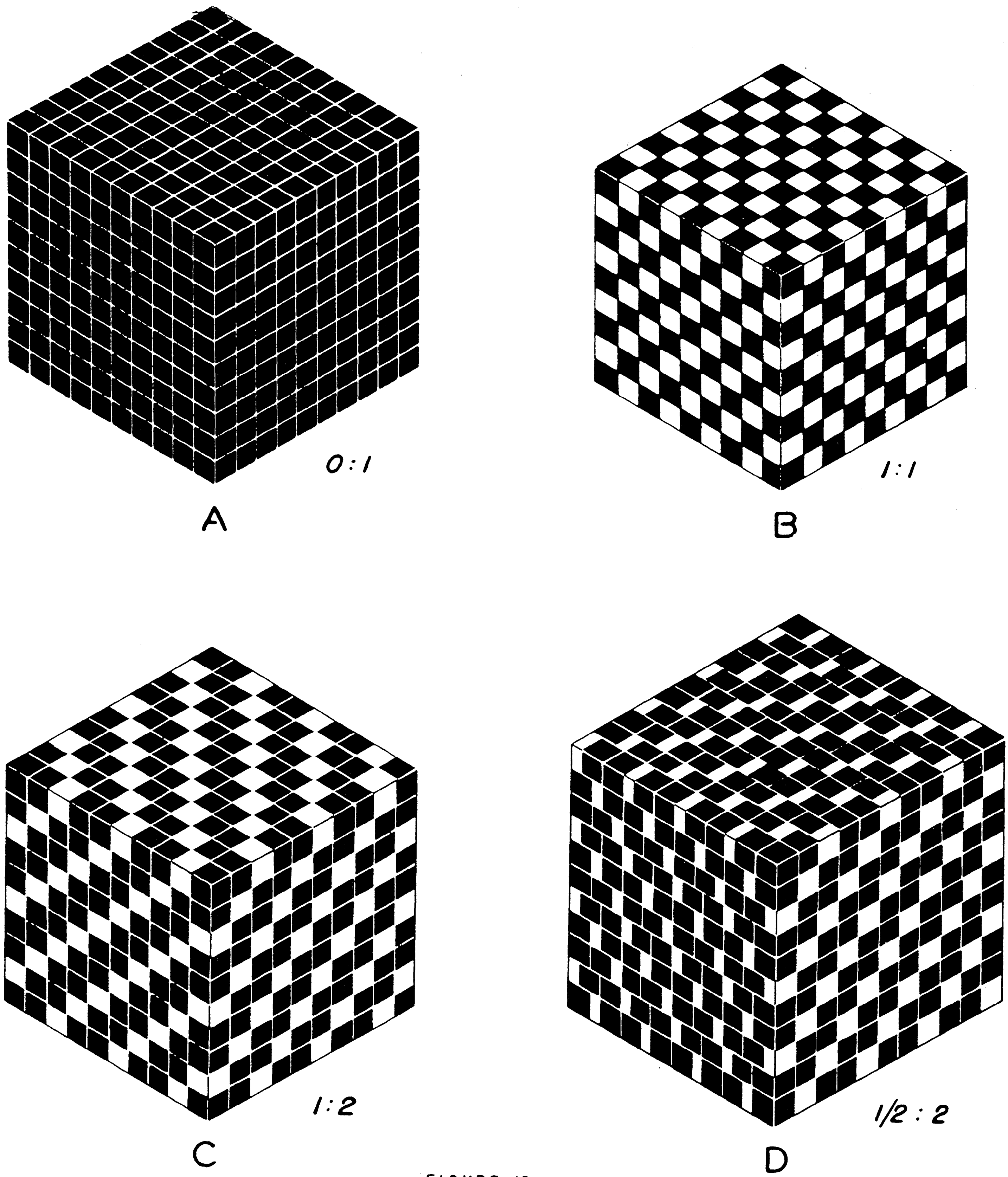
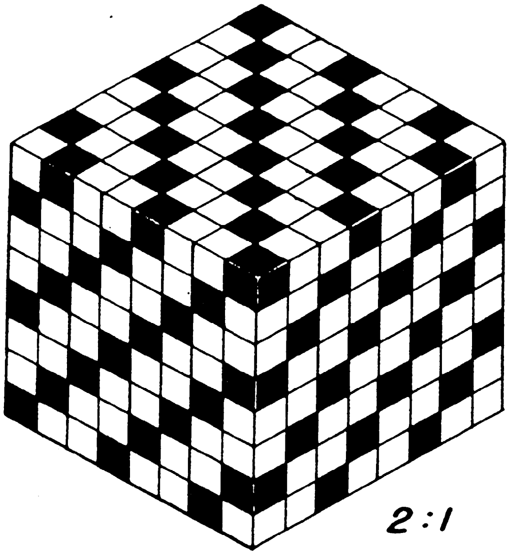


FIGURE 19

LATTICE CONFIGURATIONS OF
H CUBES (WHITE) AND U CUBES (BLACK)



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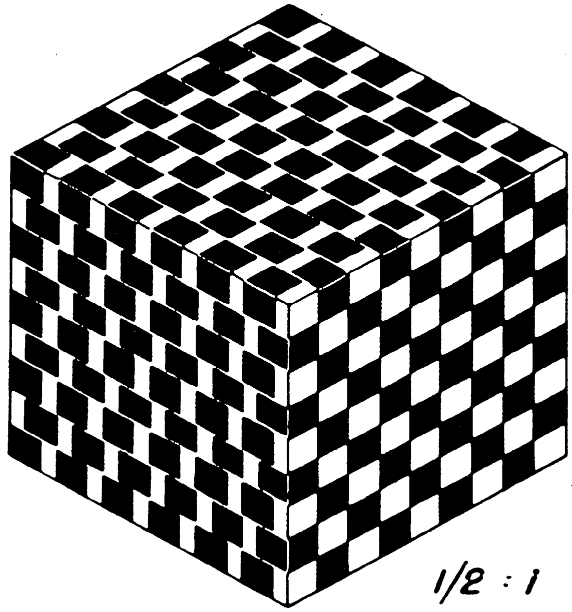

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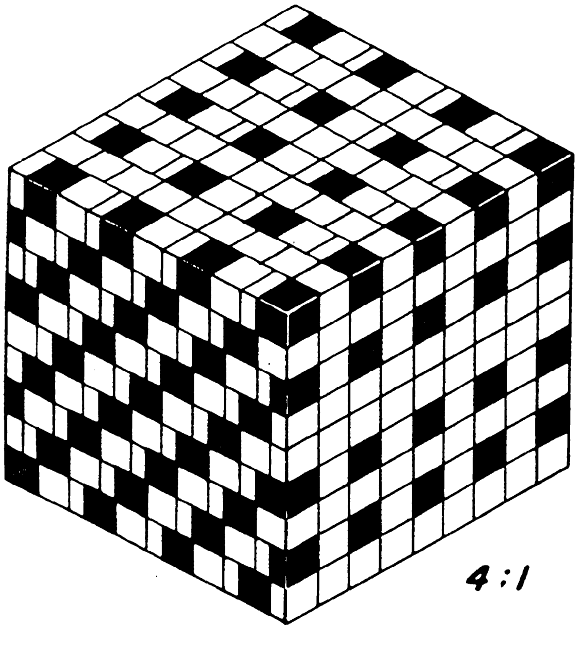

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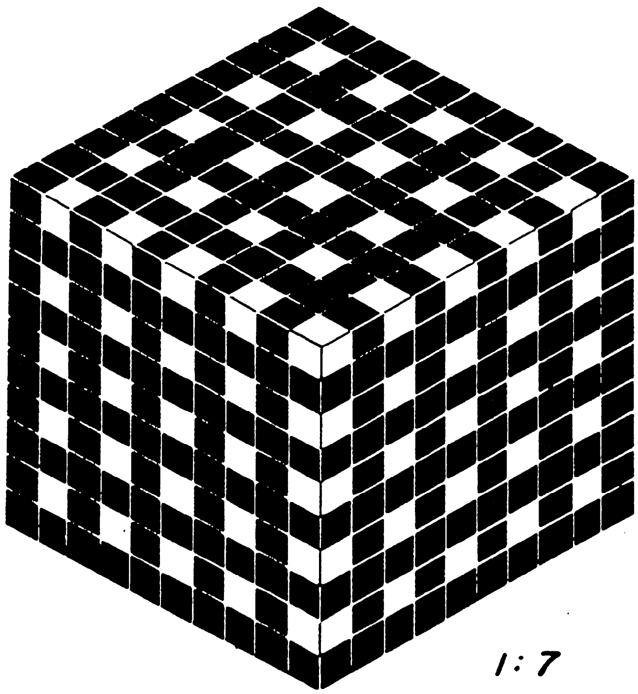

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FIGURE 20

LATTICE CONFIGURATIONS OF
H CUBES (WHITE) AND U CUBES (BLACK)

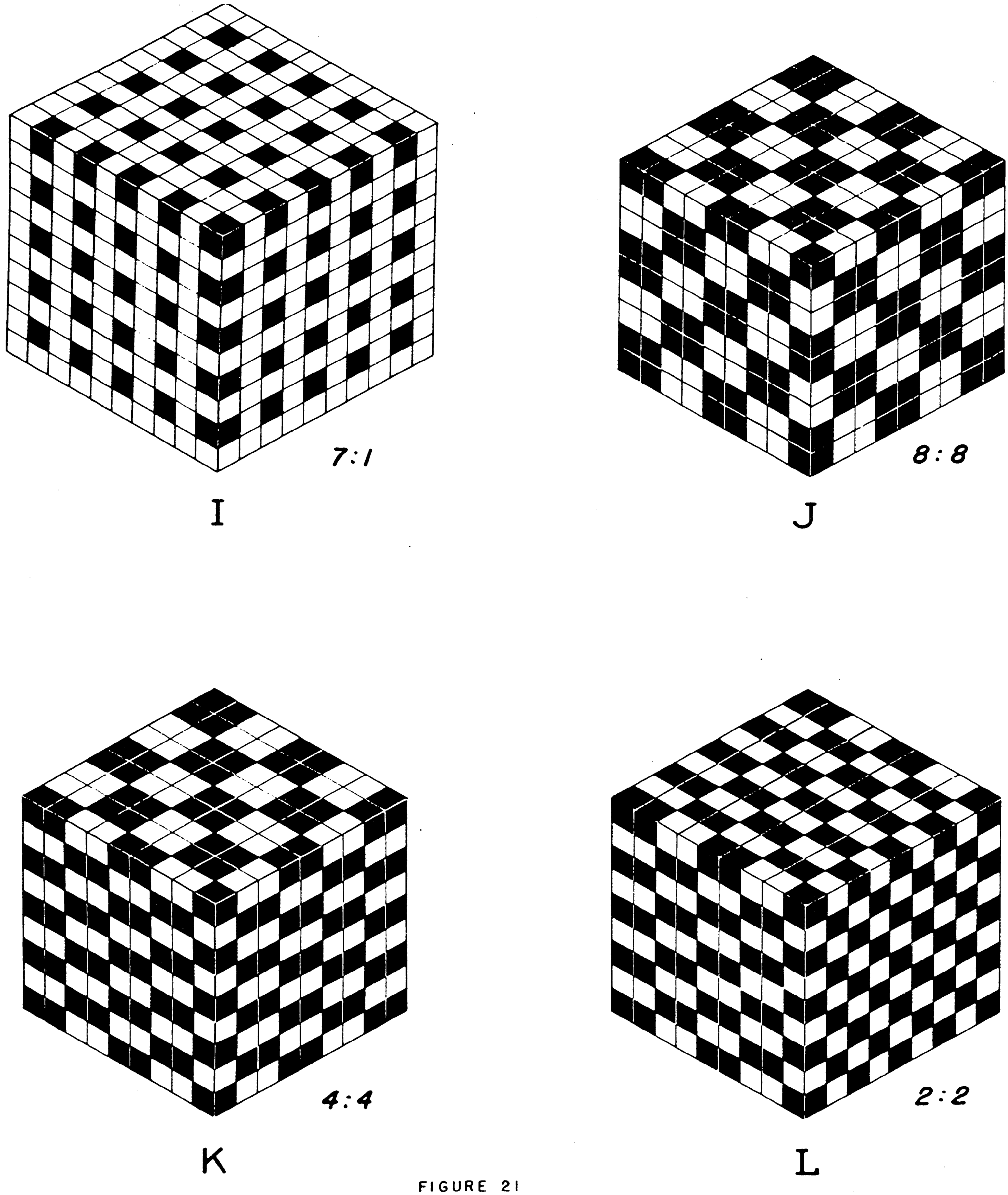

FIGURE 21

LATTICE CONFIGURATIONS OF
H CUBES (WHITE) AND U CUBES (BLACK)

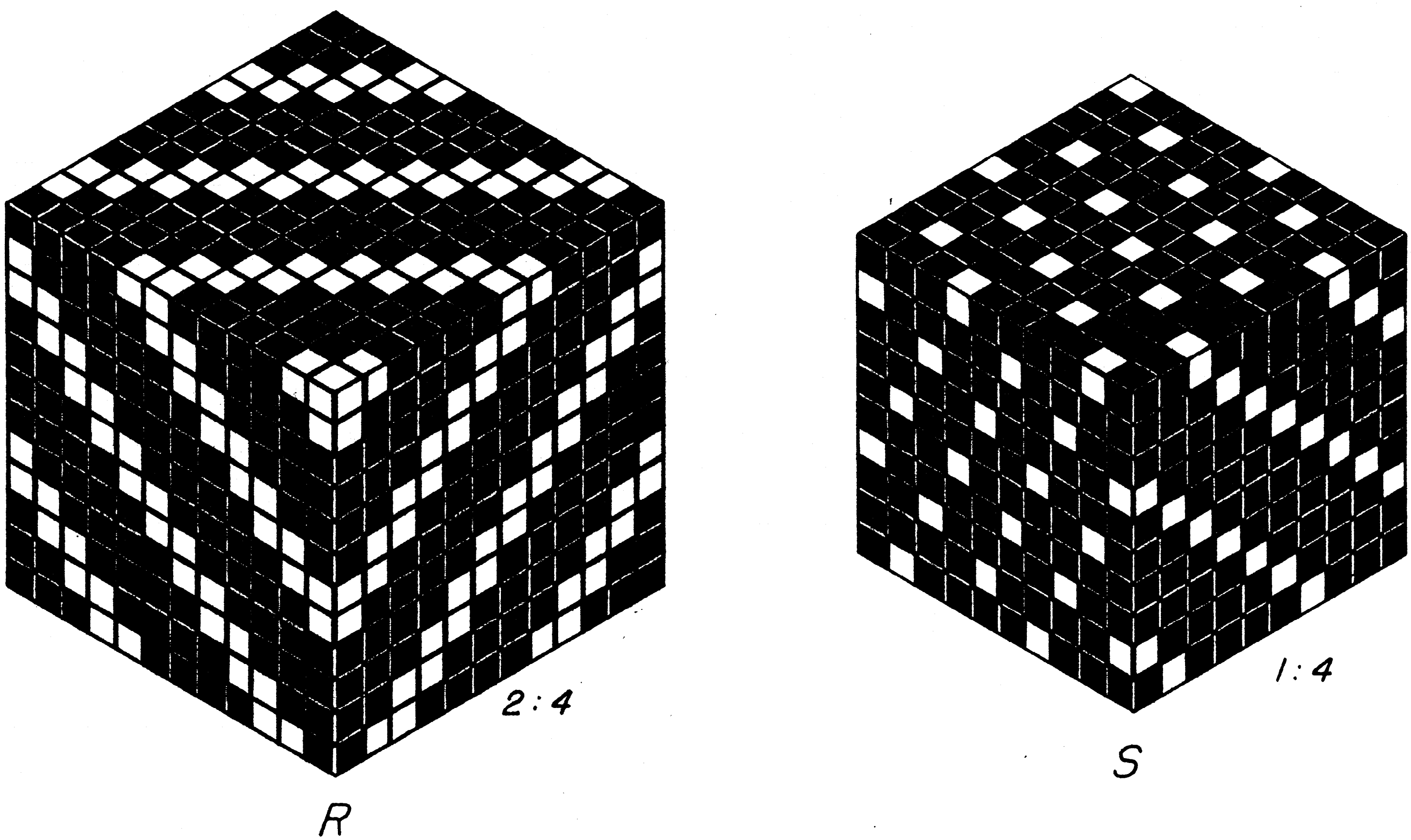

FIGURE 22

LATTICE CONFIGURATIONS OF
H CUBES (WHITE) AND U CUBES (BLACK)

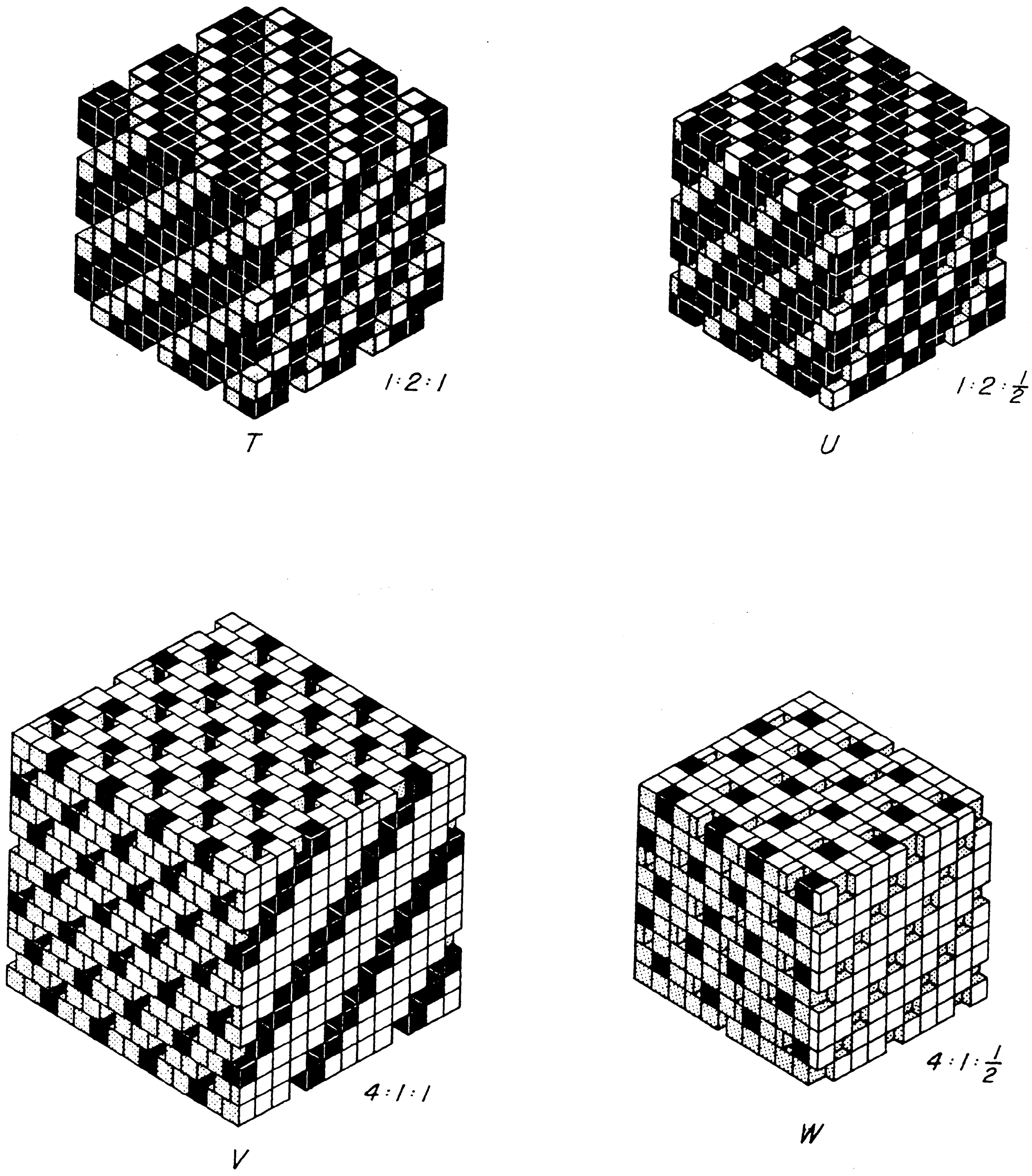

FIGURE 23

LATTICE CONFIGURATIONS OF H CUBES (WHITE),
U CUBES (BLACK), AND AIR SPACES (BLANK)