

**H.C. PAXTON, J.T. THOMAS, DIXON CALLAHAN, AND E.B. JOHNSON,
"CRITICAL DIMENSIONS OF SYSTEMS CONTAINING U²³⁵, Pu²³⁹, AND U²³³,"
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TID-7028



**CRITICAL
DIMENSIONS
OF SYSTEMS
CONTAINING
 U^{235} , Pu^{239} ,
and U^{233}**

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Preface

This compilation of data has been possible only because of the efforts of many individuals in the laboratories where the data were generated and analyzed. To the credits given specifically to those persons by the bibliographic references, the authors add their appreciation. The authors are also grateful to John Daniels, Richard Lane, and Aubrey Thomas of the United Kingdom Atomic Energy Authority and to Christain Clouet d'Orval and Pierre Lecorche of the Centre d'Etudes Nucléaires of France for making available hitherto unpublished results from experiments performed in their countries.

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Oak Ridge, Tenn.
June 1964



Introduction

This report is a compilation of many critical data obtained from experiments performed in several laboratories since 1945. It supplements the Nuclear Safety Guide [Report TID-7016 (Rev. 1)]¹ and shows the bases of the recommendations that appear in the Guide. In addition, this compilation can extend the specifications of the Guide to conditions not explicitly covered. Although some of these data were collected in earlier summaries by Paxton and Graves^{3,3} and by Paxton,² the quantity of information and its rapid increase demand a more current and inclusive collection for effective application.

That the data are critical values must be emphasized. Since no safety factors are included, these values are not directly applicable to practical problems. Information for guidance in the safe design of equipment to handle the three common fissile materials has been published in the Nuclear Safety Guide and in the Handbook of Nuclear Safety.³ Several other authors have specified conditions for unique processes with a particular material.⁴

Experiments of several types which contribute results applicable to nuclear design and to safety problems have been described by Callihan.⁵ Of these experiments, critical measurements with materials of interest which are made chain-reacting in desired configurations yield information of greatest usefulness and accuracy. In instances where it is not possible to construct critical assemblies because of safety requirements of the equipment or the environment, the configuration of the material, an inadequate inventory, or the nature of the material itself, information may be obtained from assemblies which are not critical. These yield subcritical dimensions that are useful directly in many practical applications, and, in some cases, the measurements can be extrapolated to approximate critical dimensions. Critical parameters can also be derived from information gleaned from the spatial distribution of neutrons in a subcritical assembly. In these experiments, called exponential experiments, a bulk of the material of interest is placed proximate to a source of neutrons.

In still another method, known as replacement experiments, the nuclear properties of a sample of a material are compared to those of a "standard" material through observation of the effect on the reactivity of a critical assembly of the substitution of one sample for the other.

Many more fundamental, intensive properties of materials and of neutrons, such as cross sections and the neutron yield in the fission process, obtainable in a wide variety of experiments, contribute indirectly to the prediction of critical conditions through the application of nuclear reactor theory. These basic nuclear properties have been used in theoretical analyses to bridge some gaps in the data.

The compilation and correlation of data from many measurements in many laboratories require a certain amount of normalization or reduction to common terms. In some cases, for example, the effects of variations in geometry or in density on the results from different groups of experiments must be removed. The manner in which these alterations are made is described in the discussion of particular sets of data. Differences in normalization methods may have introduced slight apparent inconsistencies in a few of the values of critical dimensions.

A cursory examination of the contents of this compilation will reveal many areas in which critical data are insufficient to allow complete analyses of all problems in this specialized field of safety. Many of these deficiencies are due to the absence of data and emphasize the need for further experimentation. To a lesser degree the deficiencies are due to the problems of timing inherent in any attempted comprehensive survey. Frequent revisions of this compilation are necessary to present useful information as it becomes available.

Contents

Preface	iii
Introduction	v

PART I SINGLE UNITS

Relations for Conversion to Standard Conditions.....	3
Cylinder–Sphere Conversions	3
Core-density Conversions	4
U ²³⁵ -enriched Uranium Units, Reflected and Unreflected	9
Homogeneous Hydrogen-moderated Uranium at Various Enrichments	9
Heterogeneous Water-moderated Uranium at Various Enrichments.....	10
Metal–Solution Mixtures.....	11
Plutonium Units, Reflected and Unreflected.....	33
Homogeneous Moderated Plutonium	33
Heterogeneous Moderated Plutonium	35
U ²³³ Units, Reflected and Unreflected	41
Poisoned Solutions	46
U ²³⁵ Solutions.....	46
Soluble Poisons	46
Heterogeneous Poisons	46
Plutonium Solutions	47
U ²³³ Solutions	48
Fissile Units with Various Reflectors.....	55
Reflectors about Uranium and Plutonium Metal	55
Reflectors about Hydrogen-moderated Units	55
Systems with Nonhydrogenous Diluents	61
Complex Shapes.....	67
Annuli	67
Pipe Intersections	67

PART II MULTIPLE-UNIT ARRAYS

Homogeneously Moderated Units	75
Planar Arrays	75
Cylindrical Units of U(~90)	75
Slab Units of U(~90)	77
Combinations of Slab and Cylindrical Units of U(~90).....	77

Contents (Continued)

Units of U(<90)	77
Plutonium Solutions	78
Spatial Arrays	78
Metal Units	129
Uranium	129
Planar Arrays	129
Spatial Arrays	129
Plutonium	130
References	139

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Criticality Studies

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PART I
SINGLE UNITS

Relations for Conversion to Standard Conditions

Many of the data correlations that appear in this report required the correction of experimental information to certain "standard" conditions. Two of the more significant types of corrections, shape and density conversions, are considered immediately. Other types, such as the correction for variations in U²³⁵ enrichment, fit more naturally into later sections.

CYLINDER-SPHERE CONVERSIONS

Ratios of critical masses of cylinders (height, h, and diameter, d) to those of spheres appear vs. h/d in Fig. 1 for U(93)* solutions and in Fig. 2 for U(93.5) metal. The values for solutions are derived from measurements at Oak Ridge⁶⁻⁹ and those for metal are from the Los Alamos Scientific Laboratory (LASL).¹⁰⁻¹² A similar treatment of data for plutonium solutions [originated at the Hanford Atomic Products Operation (Hanford)¹³] and those for U²³³ solutions [originated at the Oak Ridge National Laboratory (ORNL)¹⁴] results in values that agree in form over a limited range of h/d.

For extrapolation of experimental critical dimensions to those of broad slabs and long cylinders, the following method is useful. The dimensions of critical cylinders of different size and of a critical sphere, all of the same composition, are related to each other through the expression for the geometric buckling, provided that appropriate values of the extrapolation distances are used. Effective values of the extrapolation distance that satisfy the relation

$$\left(\frac{2.405}{r_c + \delta_c} \right)^2 + \left(\frac{\pi}{h + 2\delta_c} \right)^2 = \left(\frac{\pi}{r_s + \delta_s} \right)^2$$

where r_c and h = the radius and height of the cylinder, respectively

r_s = the radius of the sphere

δ = the effective extrapolation distance appropriate to these dimensions

are given for U(93) solutions in Fig. 3 and for U(93.5) and δ -phase plutonium in Fig. 4. Values for both unreflected and water-reflected cylinders are included. The abscissa was chosen so that the value of δ_c at zero determines the thickness of an infinite slab [= $(\pi/B_s) - 2\delta_c$] and the value at unity determines the radius of an infinite cylinder [= $(2.405/B_s) - \delta_c$].

Another means of representing a family of critical cylinders, a method due to Schuske and Morfitt,¹⁵⁻¹⁷ is illustrated in Fig. 5 (Refs. 18, 20 to 22). This plot of the height vs. a critical lateral dimension of δ -phase plutonium assemblies appears to be a rectangular hyperbola whose asymptotes are the diameter of an infinite cylinder and the thickness of an infinite slab. That the

*The numbers in parentheses indicate the U²³⁵ content of the uranium expressed in weight percent.

values from Fig. 5 are somewhat smaller than the values for water-reflected δ -phase plutonium from Fig. 4 may be at least partially due to the greater effectiveness of the methacrylate plastic (Plexiglas) that was used as a reflector.^{1,2,23}

CORE-DENSITY CONVERSIONS

A change in the density of a fissile sphere by the ratio ρ/ρ_0 leads to a changed critical mass, m_c , that may be expressed as

$$m_c/m_{c0} = (\rho/\rho_0)^{-n}$$

where n is constant over a considerable range of density ratios. In fact, where density of both spherical core and reflector is changed by the same ratio and the ratio of reflector thickness to core radius is maintained, then $n = 2$ (the value for an unreflected sphere). Similarly, in the case of an infinite slab, the critical mass per unit area is necessarily independent of ρ (i.e., $n = 0$).

Where reflector characteristics remain constant, however, the value of n associated with the density change of a spherical core depends considerably upon the system. Combined LASL data for U(93.5) and δ -phase plutonium cores (see Fig. 6) seem to follow a unique relation between the density exponent and the degree of reflection.^{24,25} The scatter associated with subcritical plutonium measurements would mask any small differences between the two fissile materials.

The scant experimental values of n (as determined by the UKAEA Atomic Weapons Research Establishment at Aldermaston, ORNL, and LASL) for near-equilateral nonmetal cores are 1.43 for U(30)O₂-paraffin at $H/U^{235} = 8.26$ in an 8-in.-thick Perspex* (methacrylate plastic, $\rho = 1.19$ g/cm³) reflector;²³ 1.46 for U(30)O₂-paraffin at $H/U^{235} = 16.5$ in an 8-in.-thick Perspex reflector;²³ 1.57 for U(30)O₂-paraffin at $H/U^{235} = 16.5$ in an 8-in.-thick polyethylene reflector;²³ 1.88 for U(93)O₂(NO₃)₂ solution at $H/U^{235} = 230$ in a thick water reflector (possibly influenced by the manner in which voids were introduced);²⁶ and 1.57 for U(93)H₃C in an 8³/₄-in.-thick U(0.7) reflector.²⁷

The lack of experimental core-density exponents for solutions forces the use of computed values. Figure 7 shows such exponents calculated by the DSN method²⁸ using Hansen-Roach cross sections.²⁹ However, the lack of general experimental verification of these calculated density exponents should be kept in mind.

*A material that is equivalent to Plexiglas as a neutron reflector.

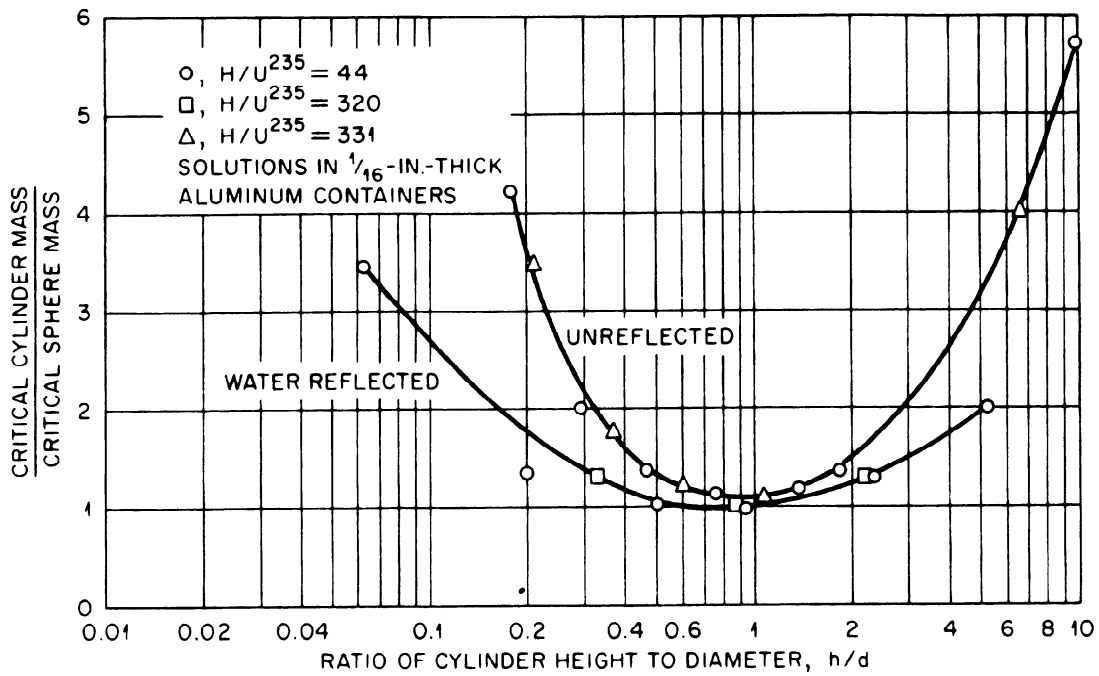


Fig. 1 – Ratio of cylindrical to spherical critical masses of $U(93)O_2F_2$ solutions.

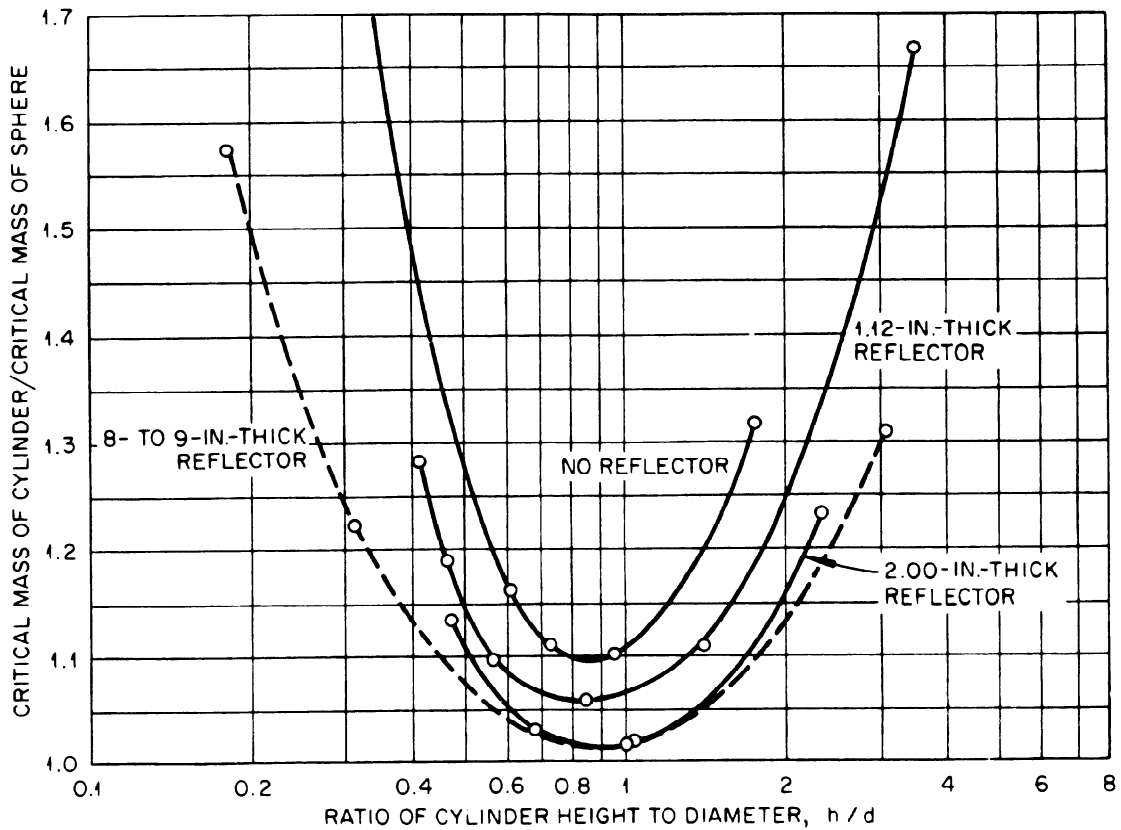


Fig. 2 – Ratio of cylindrical to spherical critical masses of $U(93.5)$ metal reflected by natural uranium.

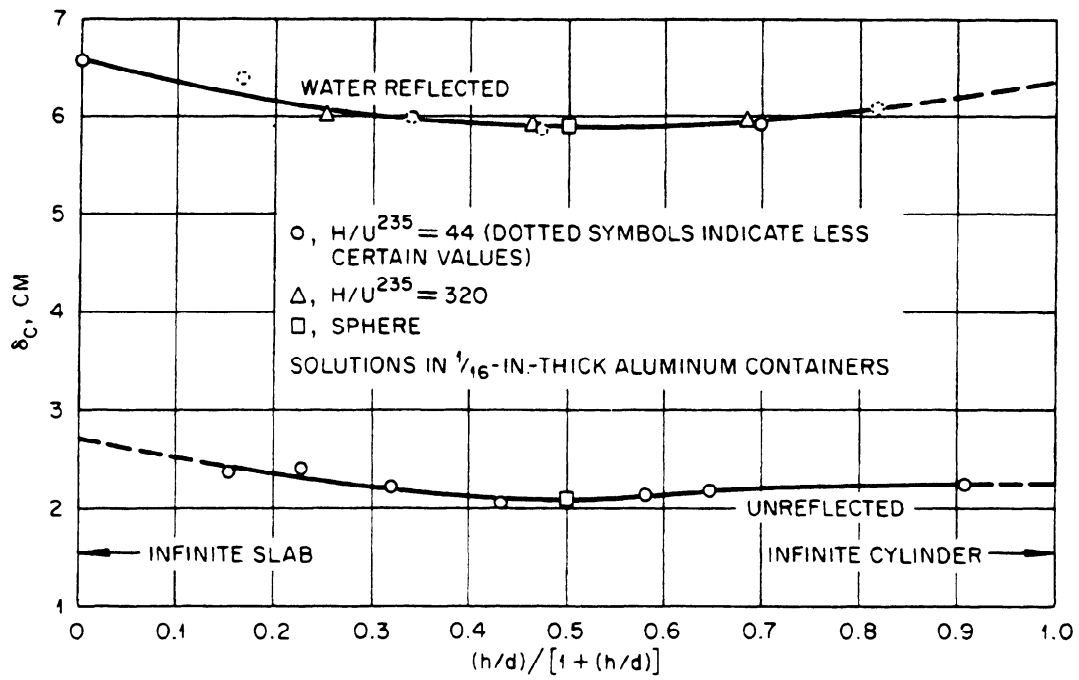


Fig. 3 – Effective extrapolation distances for cylinders of $U(93.2)O_2F_2$ solutions. Cylinder height and diameter are h and d , respectively.

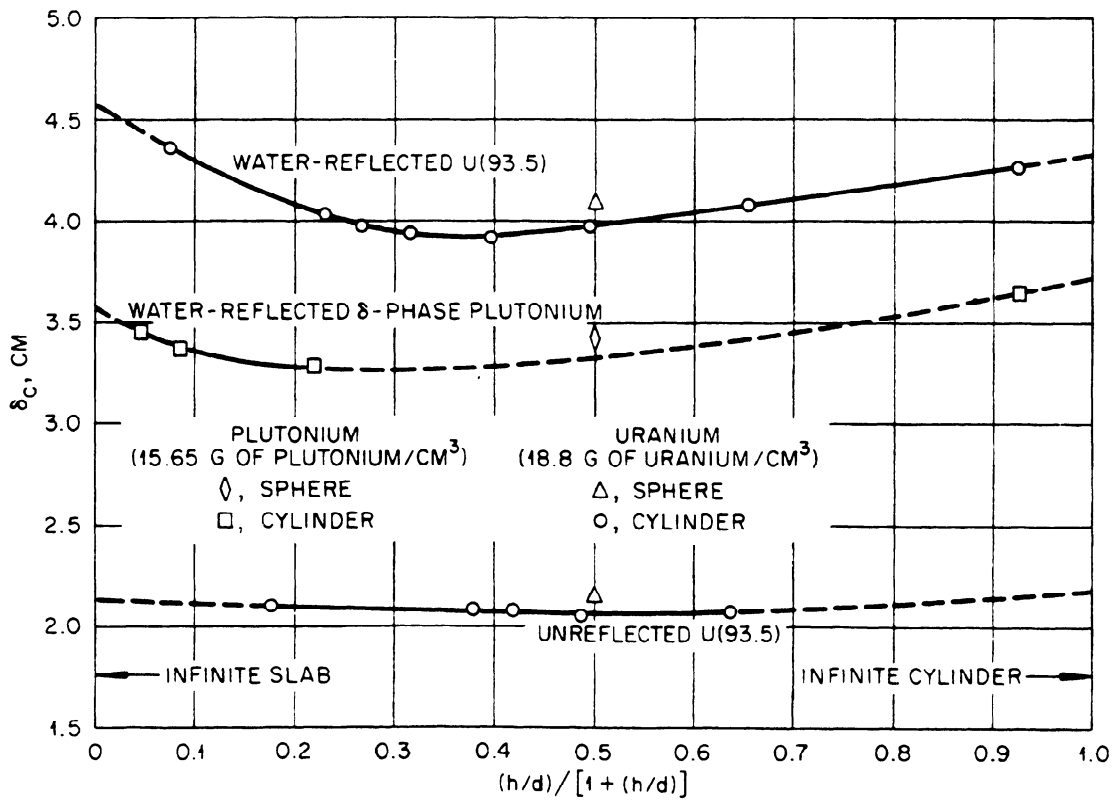


Fig. 4 – Effective extrapolation distances for cylinders of $U(93.5)$ and δ -phase plutonium metal. Cylinder height and diameter are h and d , respectively.

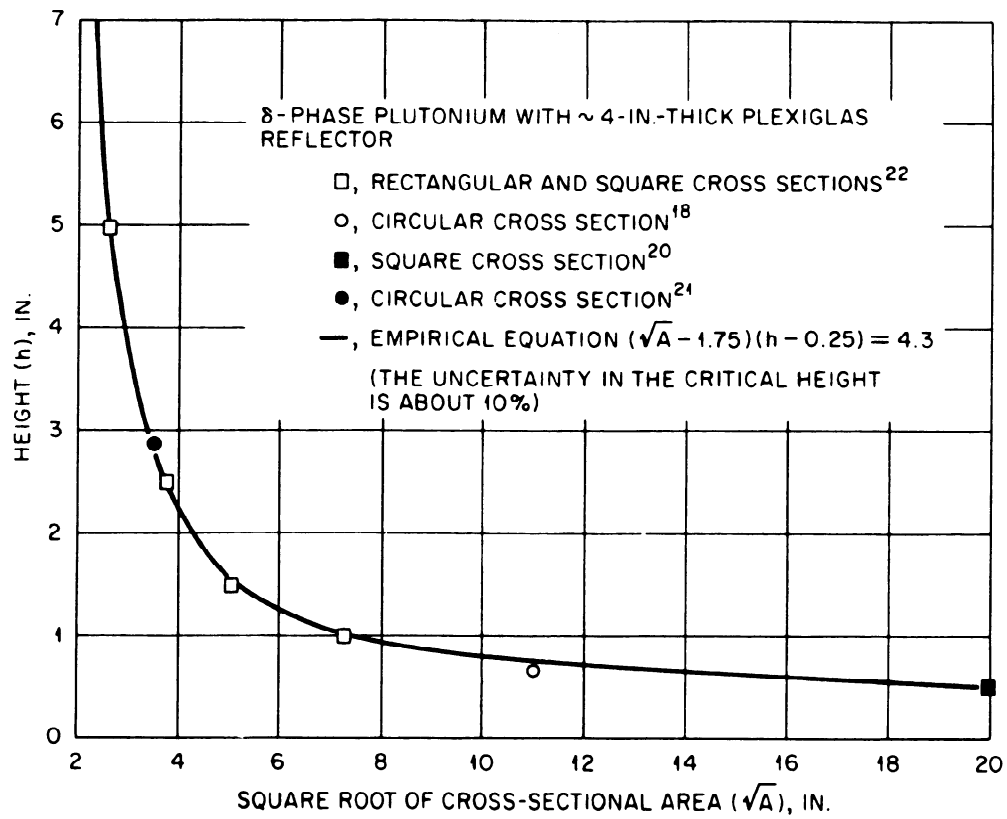


Fig. 5 – Dimensions of critical Plexiglas-reflected δ -phase plutonium cylinders.

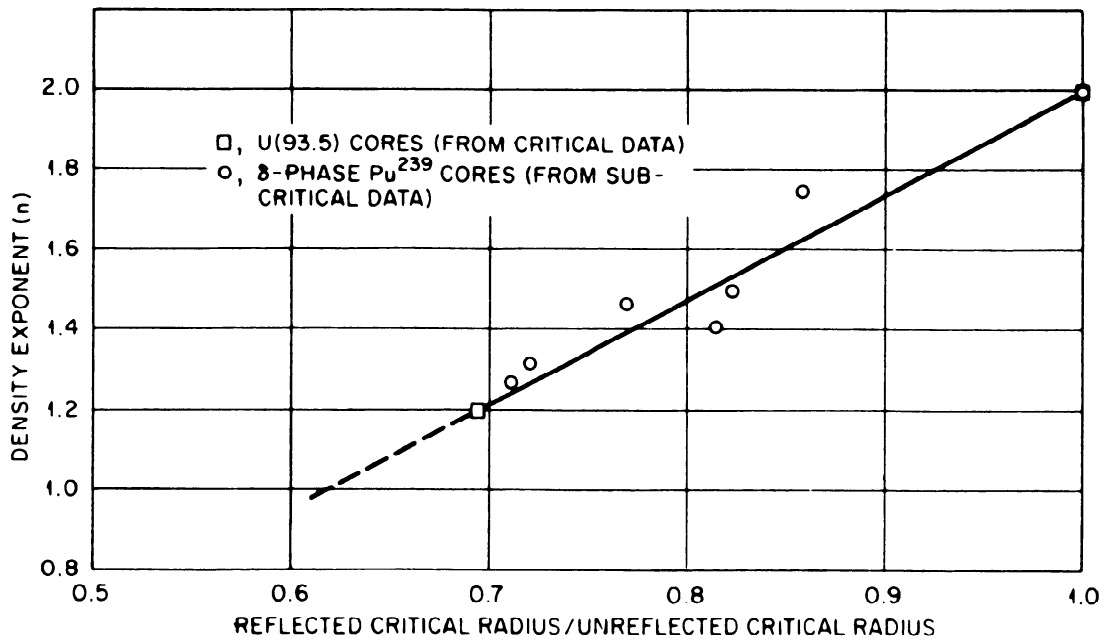


Fig. 6 – Density exponents of unmoderated cores in constant-density reflectors. [Critical mass \propto (core density) $^{-n}$.]

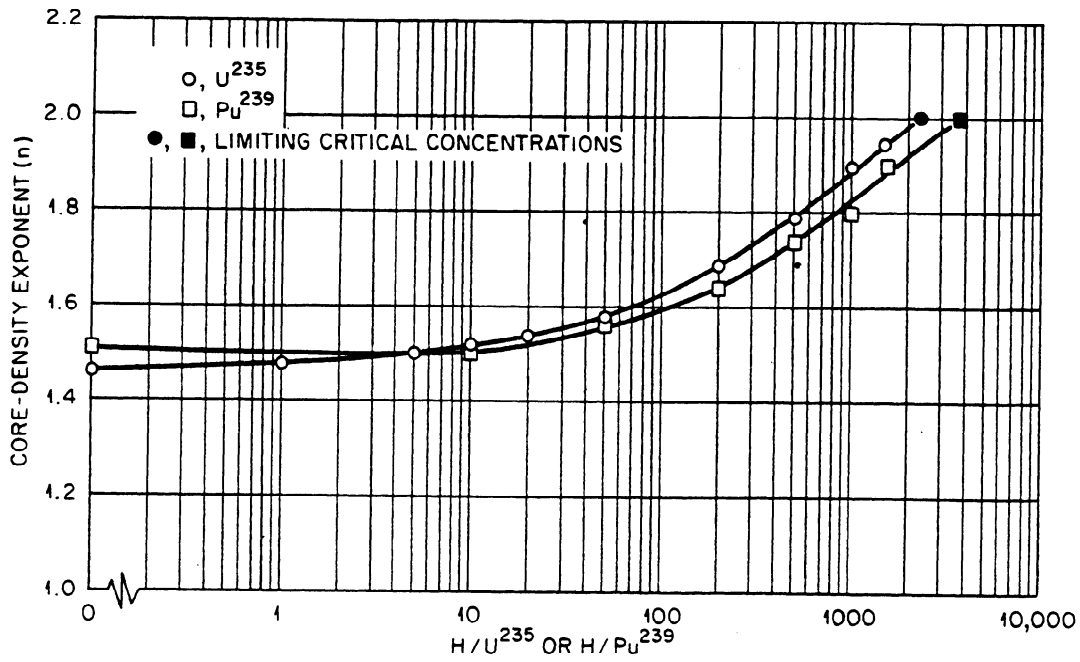


Fig. 7 - Computed core-density exponents for water-reflected spheres of homogeneous metal-water mixtures in constant-density reflectors. [Critical mass \propto (core density)⁻ⁿ.] Dashed symbols indicate limits of critical concentrations.

U^{235} -enriched Uranium Units, Reflected and Unreflected

HOMOGENEOUS HYDROGEN-MODERATED URANIUM AT VARIOUS ENRICHMENTS

Figures 8 and 9 represent critical masses and critical volumes of homogeneous water-moderated spheres of U(93.2), both bare (except for the thin-wall container) and water reflected. Estimates of corresponding diameters of infinite critical cylinders appear in Fig. 10, and estimates of thicknesses of infinite critical slabs appear in Fig. 11. The double-branched curves show how concentrated UO_2F_2 solutions depart from ideal metal-water mixtures. Densities of the fissile isotope in the metal-water mixtures to which the lower branches apply are greater than actually found in practice; hence critical values are lower limits. For convenience the assumed relations between the density of U^{235} and the atomic ratio H/U^{235} are given in Table 1 and in Fig. 12.

Sources of experimental data and the nature of conversions to the conditions of Figs. 8 to 11 are given briefly. The portions of these figures for the H/U^{235} range greater than 20 are based on ORNL critical-solution measurements of a variety of cylinders,⁶⁻⁸ some spheres or cubes,^{10,30} and a slab.^{7,9} Extrapolation of critical solutions concentration data to zero buckling gives 12.30 ± 0.10 g of U^{235} /liter as the limiting critical concentration.⁷² Westinghouse Electric Corp., Bettis Atomic Power Division,³² and Hanford, from measurements with the Physical Constants Testing Reactor (PCTR),³¹ have substantiated this value. Conversions to spheres for Figs. 8 and 9, to infinite cylinders for Fig. 10, and to infinite slabs for Fig. 11 assume equibuckling relations with the use of the effective extrapolation distances of Fig. 3. The points at $H/U^{235} = 13.5$ and 3.2 (from LASL uranium "hydride" measurements²⁷) required major compositional corrections that were obtained from DSN calculations. The unreflected point at $H/U^{235} = 13.5$ has added uncertainty because the value was from a subcritical experiment. The U^{235} metal points were based on LASL values^{10,33,34} and ORNL slab data³⁵ (with DSN correction from Plexiglas to water reflector), supported by University of California Radiation Laboratory at Livermore (Livermore) measurements.^{36,37} Shape conversions for the metal were made by using extrapolation distances from Fig. 4.

Core-density corrections for water-reflected spheres were made by using the computed relations of Fig. 7 and, for metal, by using the experimental points of Fig. 6. In regions of scanty or uncertain data, the curves of Figs. 8 to 11 are guided in form by the results of DSN calculations³⁸ but they are scaled conservatively.

Figures 13 to 16 give critical data for homogeneous hydrogen-moderated units of uranium enriched to different degrees. There has been no adjustment of the parameters to a single standard chemical composition, and the various experimental relations between U^{235} density and H/U^{235} appear in Fig. 12. Shapes were converted in the same way as those for Figs. 8 to 11. The sources of the data in the figures are given in their captions. The two sets of curves (Figs. 8 to 11 and Figs. 13 to 16) represent generalizations of data from many experiments by methods that perhaps differ slightly. These differences may have introduced apparent inconsistencies in the values of the critical dimensions. Table 2 gives data from Harwell⁴¹ for U(44.6), from Dounreay^{42,106} for U(92.14) and U(30), and from Aldermaston²³ for U(30.14), and a value from LASL⁴³ for U(14.7).

Hanford measurements in the PCTR* give 1.034 ± 0.010 as the limiting critical U^{235} enrichment for homogeneous hydrogen-moderated systems.⁴⁵

Critical masses of unmoderated U(93.5) metal vs. U^{235} enrichment are given in Fig. 17. The curves, based on LASL critical and exponential data,⁴⁶⁻⁴⁹ follow the approximate relation: U^{235} critical mass = constant (enrichment)^{-0.7} for enrichments above about 25 wt. % U^{235} . The exponential experiments indicate that unmoderated uranium cannot become critical if the U^{235} content is below 5 or 6 wt. %.

HETEROGENEOUS WATER-MODERATED URANIUM AT VARIOUS ENRICHMENTS

A large variety of measurements on slightly enriched uranium metal or oxide lattices in water were summarized by Kouts et al.⁵⁰ at the Geneva Conferences of 1955 and 1958. Of general interest are more recent measurements from Hanford⁵¹ and from Savannah River⁵² with U(3.0) metal rods of each of five diameters latticed at various spacings to establish minimum values of the spherical critical mass and maximum values of the buckling. Hanford is also the source of more limited measurements on rods of U(2.00) metal.⁵³ Minimum critical mass and maximum buckling as a function of rod diameter are given in Fig. 18 for water lattices of rods of three U^{235} contents. It may be noted that the minimum critical masses are smaller than for the corresponding homogeneous systems (compare with Fig. 21) and that different rod sizes give the smallest critical mass and the largest buckling.

Unlike slightly enriched uranium, water lattices of U(93.5) metal give minimum critical masses that are greater than for the homogeneous system. The LASL measurements were on 1-in. cubes, $\frac{1}{2}$ -in. cubes, and $\frac{1}{8}$ -in.-diameter rods.⁵⁴ Figure 19 shows critical mass as a function of the volume-to-surface ratio of an element in the lattice. As may be derived from Fig. 20, surface-to-surface spacings that correspond to minima in critical mass vary from 0.7 in. for the 1-in. cubes to 0.6 in. for the $\frac{1}{8}$ -in. rods.

Minimum critical mass of heterogeneous uranium in water as a function of enrichment is inter-compared with homogeneous cases in Fig. 21. Similar presentations of minima for critical volumes, for estimated diameters of infinite cylinders, and for thicknesses of infinite slabs appear in Figs. 22, 23, and 24, respectively.

The influence on critical mass of spacing between pool type reactor (Oak Ridge Reactor and Bulk Shielding Reactor) fuel elements in water has been investigated at ORNL.⁵⁵ Each element consisted of 24-in.-long plates containing U(93.2)-aluminum alloy which were supported in a frame or box 3 in. square. Most elements contained 168 g of U^{235} , for which $H/U^{235} = \sim 370$ when closely packed in water, but others contained 140 and 200 g of U^{235} . Observations may be summarized as follows:

1. Flooded bundles of square cross section (one element long) had critical masses of 2.6 kg of U^{235} with $\frac{1}{2}$ -in. spacing between fuel sections (essentially independent of the mass of U^{235} per element within the range of 140 to 200 g), and 5.2 kg of U^{235} with $1\frac{1}{4}$ -in. spacing. With $2\frac{1}{4}$ -in. spacing, it appears that an infinite number of elements would not be critical.
2. There was no appreciable neutron multiplication in a flooded 11- by 12-element bundle in which 0.02-in.-thick cadmium sheet separated rows (in one direction). The same was true when water and cadmium were removed and the array was surrounded by 12-in.-thick paraffin.
3. An infinite number of elements arranged side by side on a plane in water will be subcritical. Two such layers of 26 elements each (twenty-four 200-g and twenty-eight 168-g elements) were subcritical when submerged at optimum spacing.

*The validity of experimental results from the PCTR has been strengthened by the favorable comparison of values of k_{∞} of a particular material determined by the PCTR and by critical experiments at ORNL.⁴⁴

METAL-SOLUTION MIXTURES

All experimental information about another practical type of inhomogeneous system, combinations of fissile metal and fissile solution, is derived from subcritical measurements at Rocky Flats.⁵⁶

Other experiments related critical concentrations of U^{235} in solution to the critical thickness of a U(93) metal slab within the solution.^{57,58} Figure 25 gives results for 5- by 8-in. slabs in a 9.45-in.-diameter 16-in.-high solution, and Fig. 26 gives results for 10- by 16-in. slabs in a 30-in.-diameter 28-in.-high solution.

Table 1 - DENSITY OF X VS. H/X ATOMIC RATIO
 [X \equiv U^{235} as U(93.2), U^{233} as U^{233} (98 wt.%), or Pu^{239}]

H/X	U^{235} density, g/cm ³		U^{233} density, g/cm ³		Pu^{239} density, g/cm ³		
	Metal-H ₂ O	Solution*	Metal-H ₂ O	Solution*	α -Pu - H ₂ O	δ -Pu - H ₂ O	Solution*
0	17.53		18.28		19.6	15.65	
1	10.48		10.71		11.27	9.85	
2	7.48		7.57		7.91	7.18	
3	5.81		5.86		6.09	5.66	
5	4.02		4.03		4.18	3.96	
10	2.27		2.27		2.34	2.27	
20	1.21	1.01	1.208	1.00	1.242	1.222	1.027
30	0.83	0.75	0.823	0.744	0.846	0.837	0.76
50	0.51	0.48	0.503	0.483	0.517	0.513	0.49
100	0.257	0.255	0.255	0.253	0.262	0.261	0.257
200	0.129	0.129	0.128	0.126	0.132		0.131
300	0.0865	0.086	0.0858	0.0853	0.088		0.087
500		0.052		0.0516			0.053
1000		0.0260		0.0258			0.0264
1500		0.0174		0.0172			0.0176
2000		0.0130		0.0129			0.0132
3000							0.0088

* UO_2F_2 solution and the analog for plutonium.

Table 2 – CRITICAL MASSES AND VOLUMES OF HYDROGEN-MODERATED SPHERES, HEMISPHERES,
AND CUBES OF URANIUM AT SEVERAL ENRICHMENTS

H/U ²³⁵	U ²³⁵ density, g/liter	Water reflected		Unreflected	
		Volume, liters	Mass, kg of U ²³⁵	Volume, liters	Mass, kg of U ²³⁵
U(92.14)O ₂ F ₂ , water solution, ¹⁰⁶ spheres					
365	70.5			22.27	1.570
635	40.7			34.91	1.421
U(44,6)O ₂ F ₂ , water solution, ⁴¹ sphere dimensions transformed from cylinders					
258	97.4	11.5	1.12	17.6	1.71
493	51.8	19.5	1.01		
678	37.9	29.5	1.12		
U(30.45)O ₂ F ₂ , water solution, ¹⁰⁶ spheres					
76	287.5			22.27	6.403
218	113.0			22.27	2.517
351	71.6	14.85	1.063		
534	47.76			34.91	1.667
573	44.5	22.27	0.991		
783	32.75	34.91	1.143		
1037	24.8			91.34	2.265
1193	21.6	91.34	1.973		
U(30.3)O ₂ F ₂ , water solution, ⁴² sphere dimensions transformed from cylinders					
76.7	288	11.3	3.26	19.5	5.62
106	220	11.6	2.54	20.0	4.38
167	146	11.6	1.70	20.0	2.93
257	97.8	13.0	1.28	22.1	2.16
378	67.5	16.1	1.08	26.3	1.77
439	58.4	17.1	1.00	27.7	1.62
657	39.4	27.8	1.10	42.1	1.66
815	31.7	38.1	1.24	55.5	1.76
U(30.45)O ₂ F ₂ , water solution, ¹⁰⁶ hemispheres					
277	90.0	17.44	1.571		
488	52.0			45.67	2.375
728	35.2	45.67	1.608		
U(14.7)O ₂ SO ₄ , water solution, ⁴³ spheres					
	~81	14.8	~1.2		
U(30.14)O ₂ -CH ₂ compacts, ²³ cube dimensions transformed from parallelepipeds					
Perspex reflected					
8.14	1570	13.9	21.8	31 ± 2	49 ± 3
8.14	1190*	27.6	32.9		
16.3	1130	9.37	10.59	26.5	30.0
16.3	845*	19.37	16.40		

Table 2 - (continued)

H/U ²³⁵	U ²³⁵ density, g/liter	Water reflected		Unreflected	
		Volume, liters	Mass, kg of U ²³⁵	Volume, liters	Mass, kg of U ²³⁵
U(30.14)O ₂ -CH ₂ compacts, ²³ cube dimensions transformed from parallelepipeds					
Perspex reflected					
39.2	668	6.66	4.45	18.40	12.29
81.3	332	5.83	1.94	15.34	5.09
81.3	248*	12.1 [†]	3.00 [†]	38.3 [†]	9.51 [†]
81.4	244 [‡]	9.58 [†]	2.38 [†]	25.6 [†]	6.24 [†]
Polythene reflected					
8.14	1190*	32.4	38.6		
16.3	1130	10.21	11.54	26.5	30.0
16.3	845*	22.3	18.9		
39.2	668	7.21	4.82	18.40	12.29
81.3	332	6.32	2.10	15.34	5.09
81.3	248*	13.7	3.39	38.3 [†]	9.51 [†]
81.4	244 [‡]	10.58	2.58	25.6 [†]	6.24 [†]

*Small holes were drilled in the compacts to reduce the density to these values.

[†] Estimated from replacement measurements on one face.

[‡] Extra graphite was added to change the effective composition of the wax to CH.

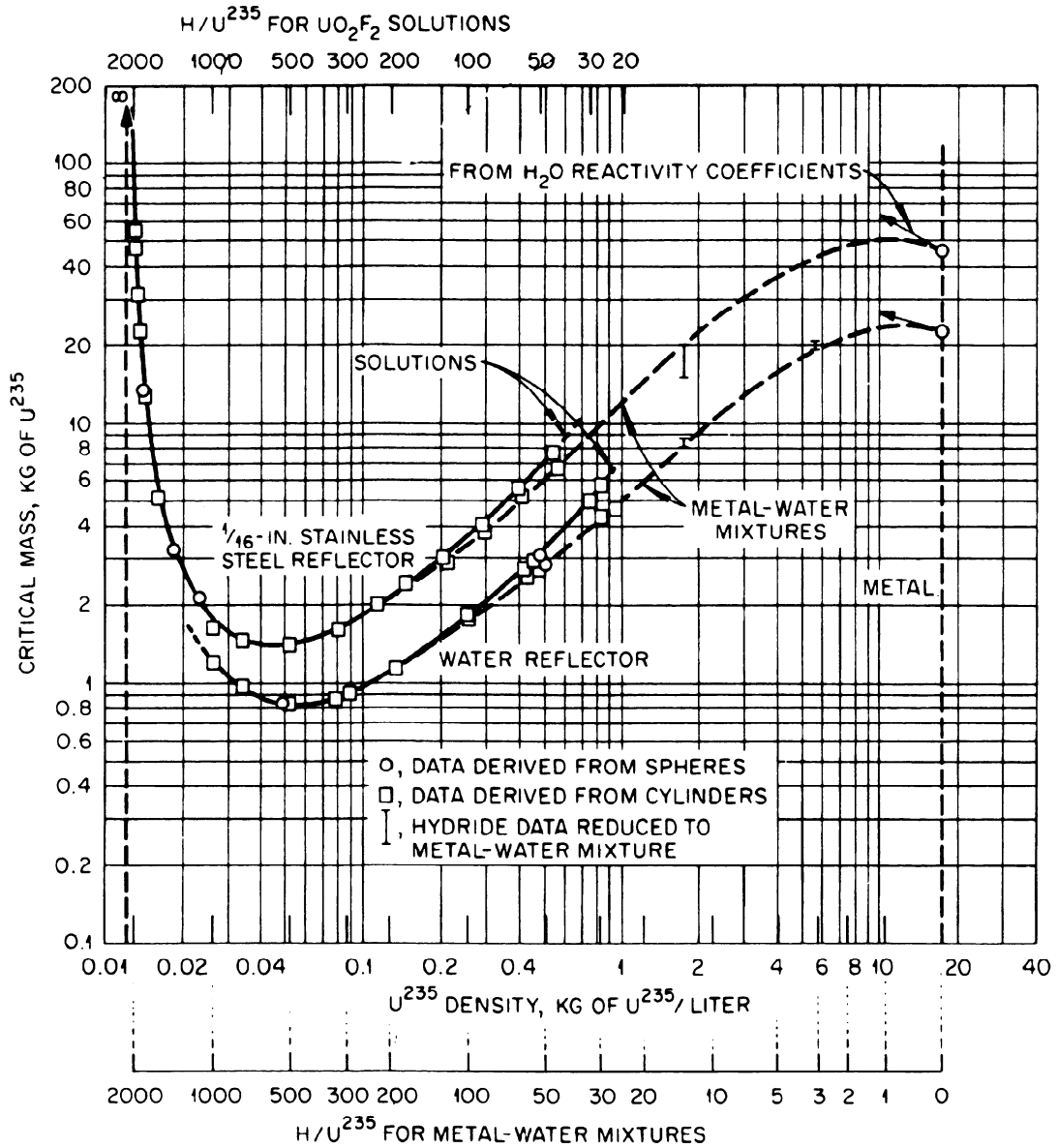


Fig. 8 – Critical masses of homogeneous water-moderated $U(93.2)$ spheres.

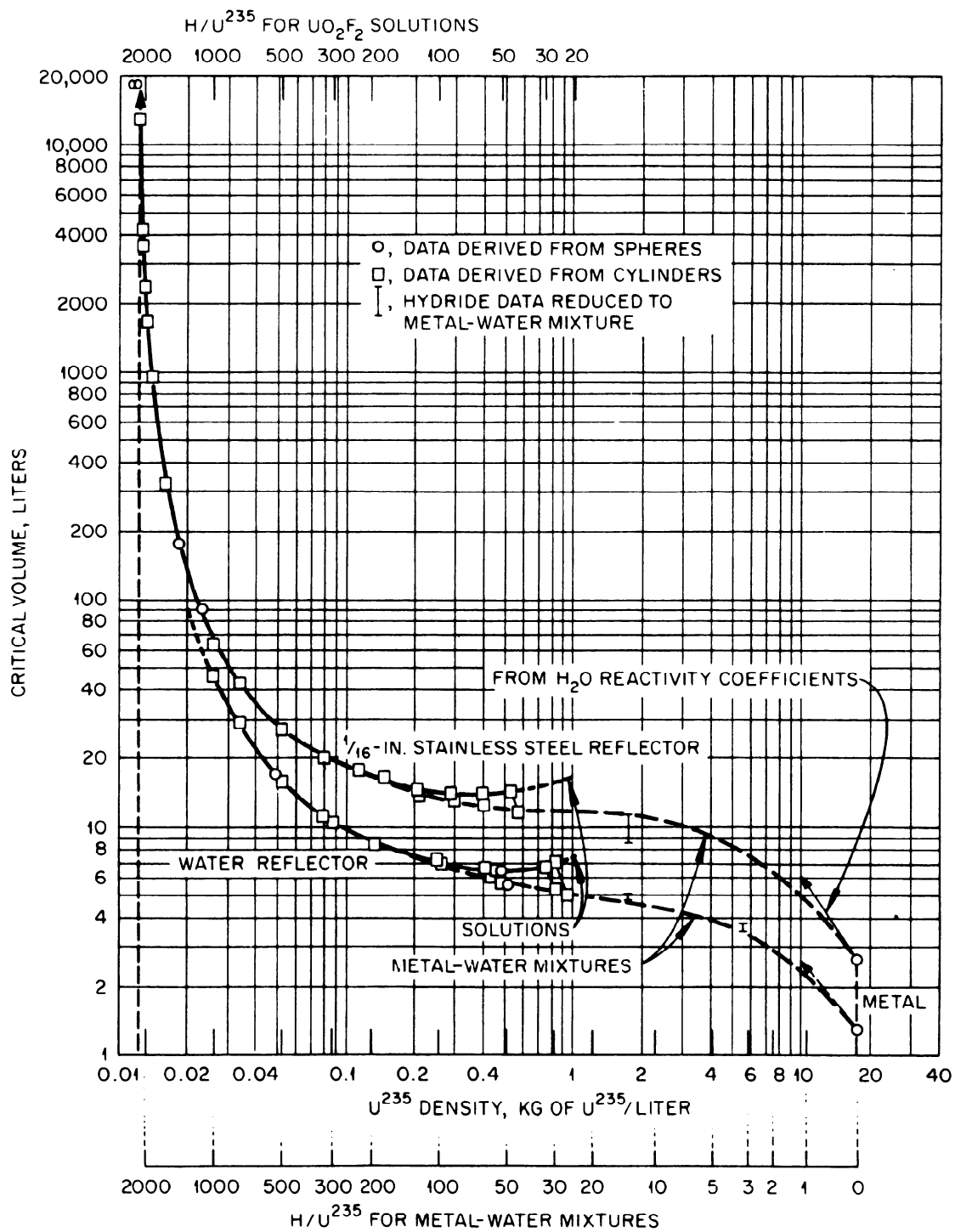


Fig. 9 - Critical volumes of homogeneous water-moderated U(93.2) spheres.

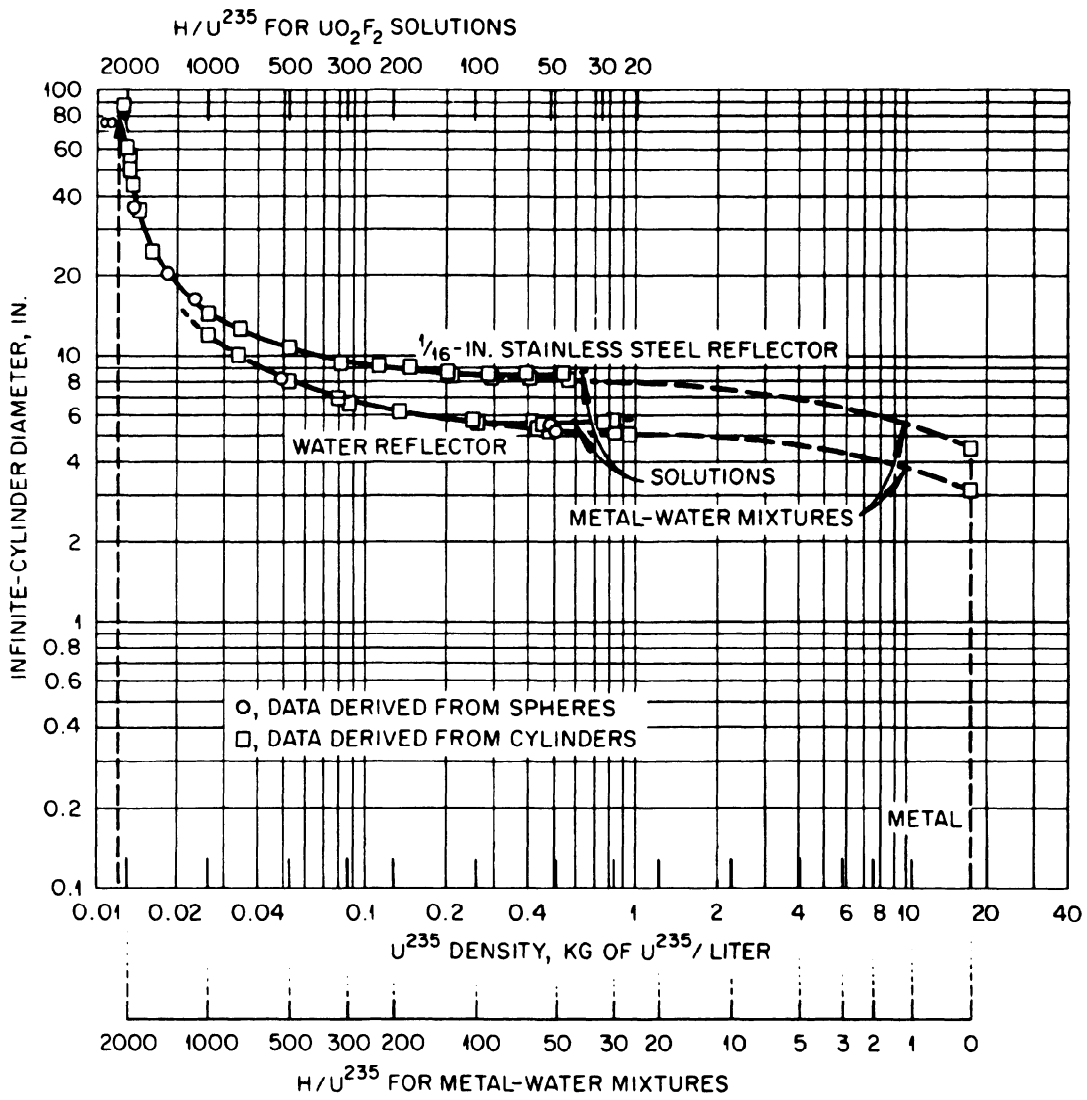


Fig. 10 – Estimated critical diameters of infinite cylinders of homogeneous water-moderated $U(93.2)$.

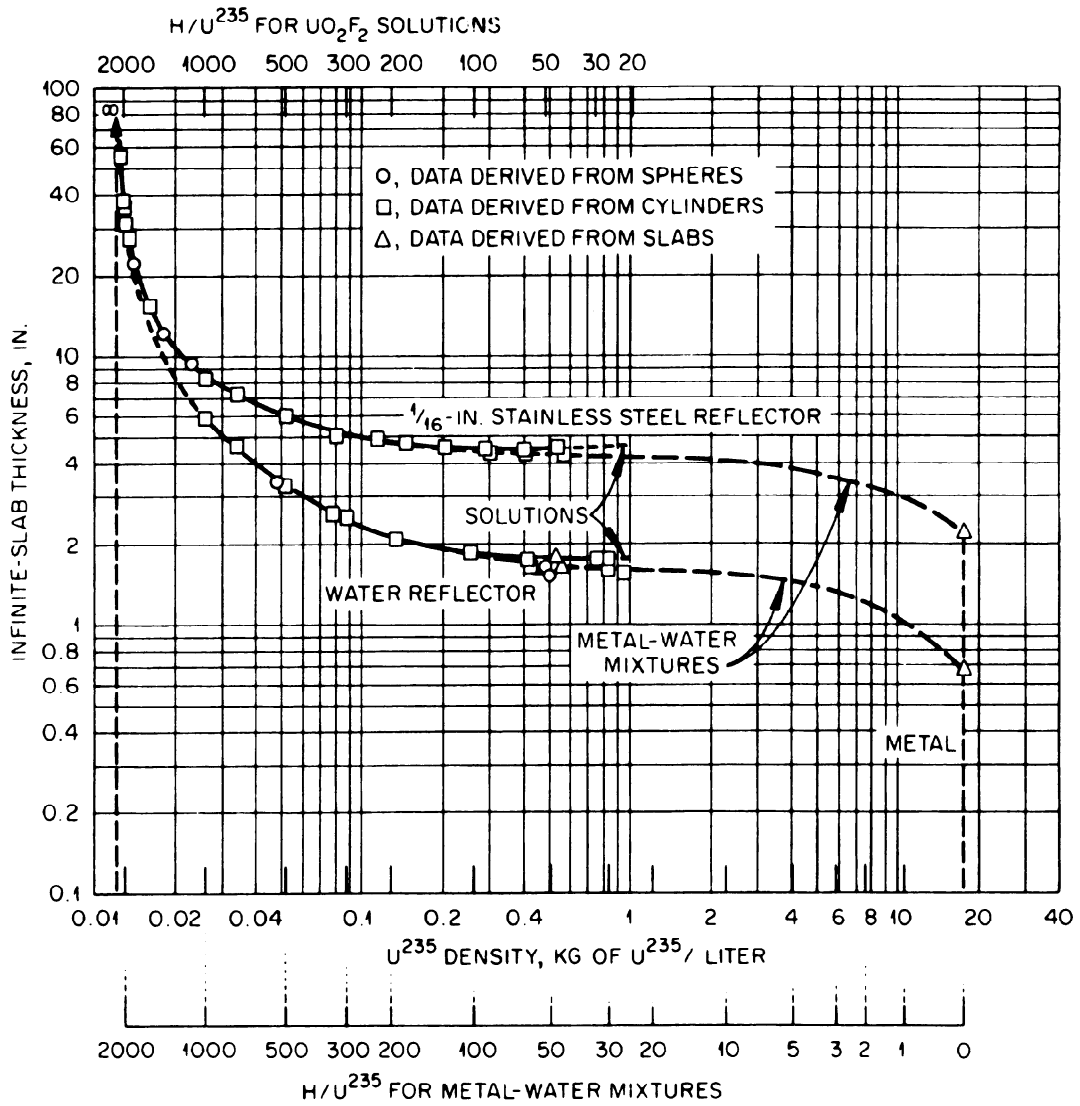


Fig. 11— Estimated critical thicknesses of infinite slabs of homogeneous water-moderated U(93.2).

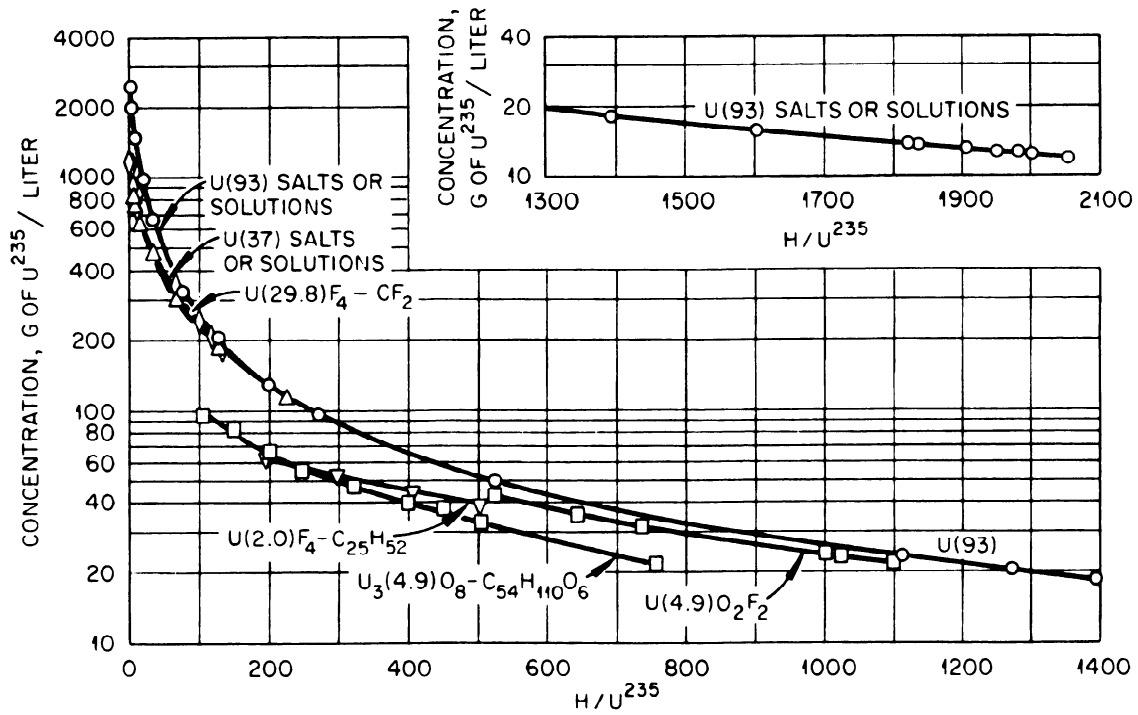


Fig. 12 - Concentration as a function of H/U^{235} atomic ratio for various U^{235} enrichments.

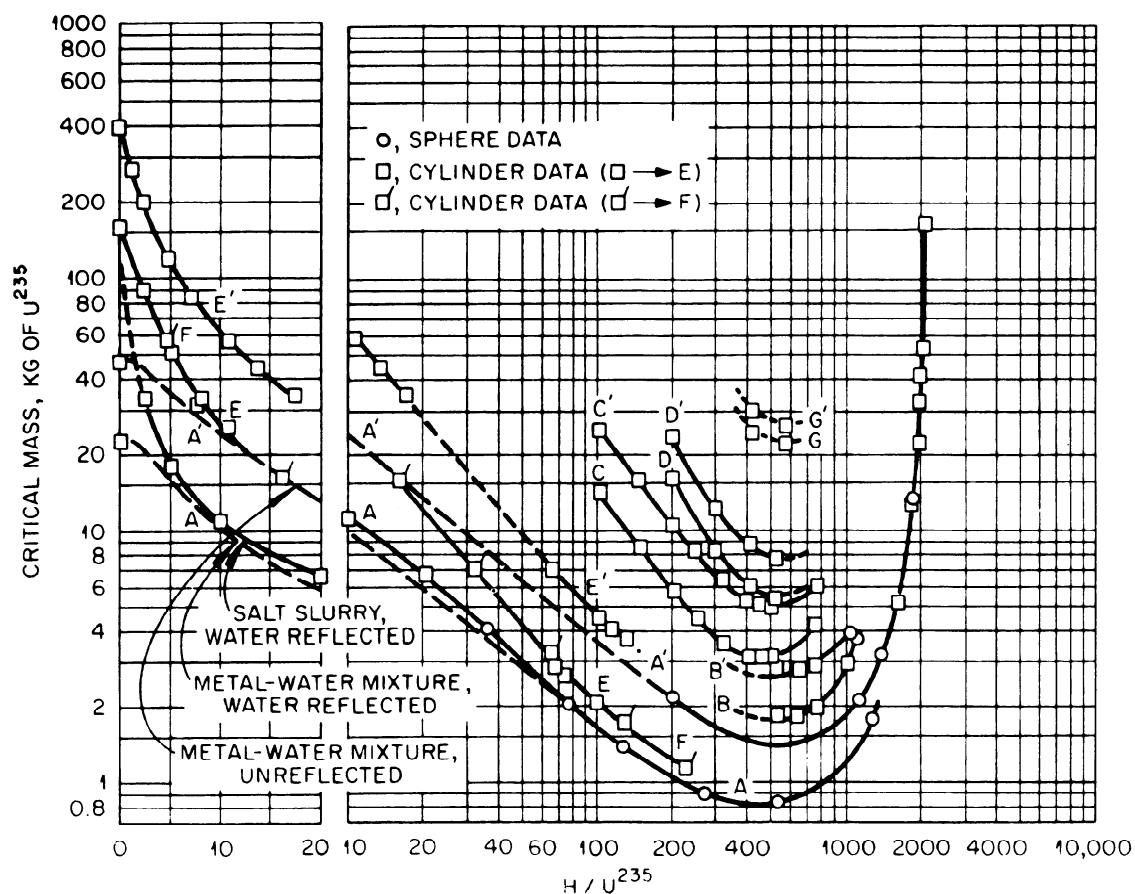


Fig. 13 – Critical mass of U^{235} -enriched uranium in spherical geometry as a function of H/U^{235} atomic ratio.

Curve A: $U(93)O_2F_2$ solutions and $U(95)F_4-CF_2-CH_2$, water reflected, Refs. 6, 8, 30, 34, and 72.

Curve B: $U(4.9)O_2F_2$ solutions, water reflected, Ref. 40.

Curve C: $U_3(4.9)O_8-C_{57}H_{110}O_6$, water reflected, Ref. 40.

Curve D: $U(2.0)F_4-C_{25}H_{52}$, water reflected, Refs. 103 and 104.

Curve E: $U(37)O_2F_2$ solutions and $U(37)F_4-CF_2-C_5H_8O_2$, water reflected, Refs. 40 and 105.

Curve F: $U(29.8)F_4-CF_2-CH_2$, paraffin reflected, Ref. 39.

Curve G: $U(1.42)F_4-C_{40}H_{81}$, water reflected, Ref. 106.

The prime letters indicate unreflected values.

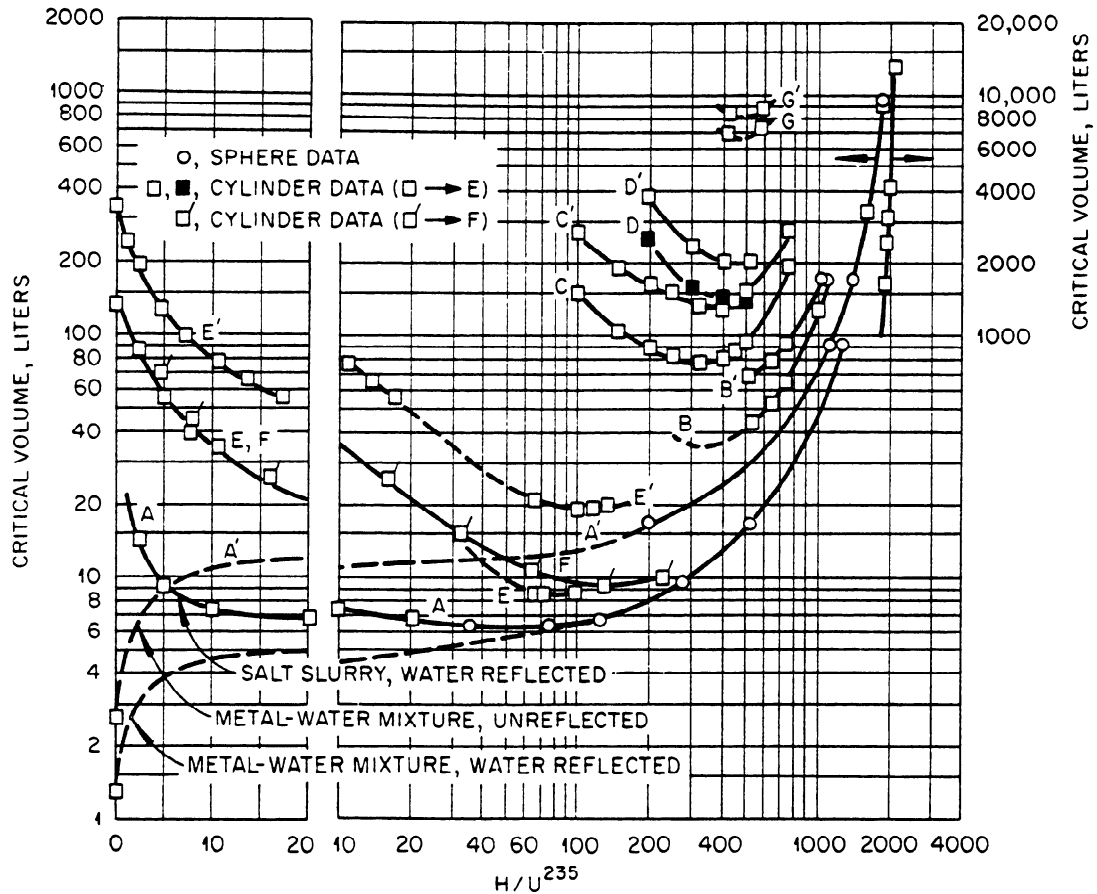


Fig. 14 - Critical volume of U^{235} -enriched uranium in spherical geometry as a function of H/U^{235} atomic ratio.

Curve A: $U(93)O_2F_2$ solutions and $U(95)F_4-CF_2-CH_2$, water reflected, Refs. 6, 8, 30, 34, and 72.

Curve B: $U(4.9)O_2F_2$ solutions, water reflected, Ref. 40.

Curve C: $U_3(4.9)O_8-C_{57}H_{110}O_6$, water reflected, Ref. 40.

Curve D: $U(2.0)F_4-C_{25}H_{52}$, water reflected, Refs. 103 and 104.

Curve E: $U(37)O_2F_2$ solutions and $U(37)F_4-CF_2-C_5H_8O_2$, water reflected, Refs. 40 and 105.

Curve F: $U(29.8)F_4-CF_2-CH_2$, paraffin reflected, Ref. 39.

Curve G: $U(1.42)F_4-C_{40}H_{81}$, water reflected, Ref. 106.

The prime letters indicate unreflected values.

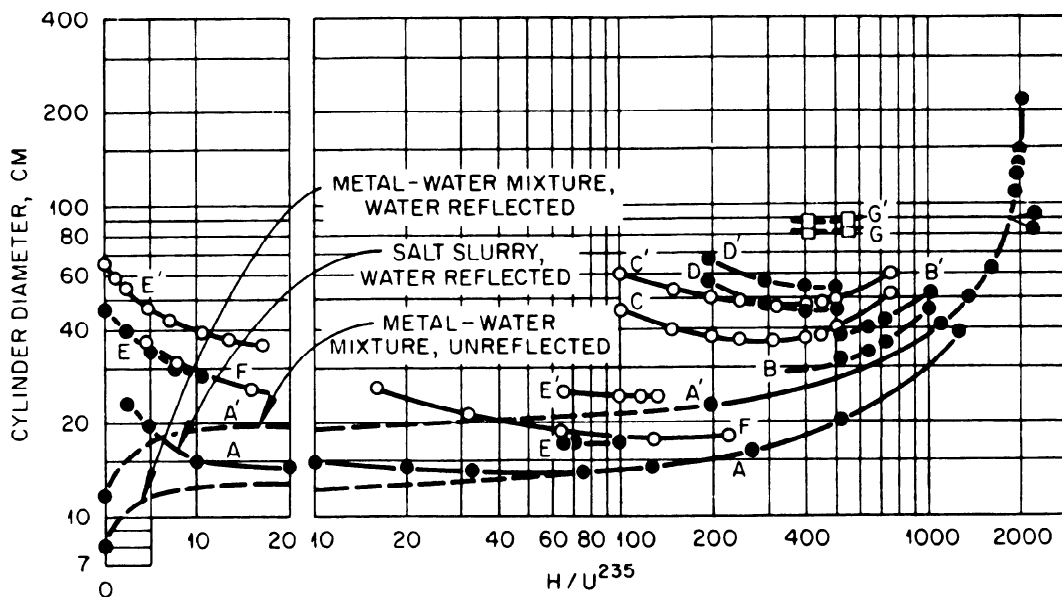


Fig. 15 - Critical diameter of infinite cylinders as a function of H/U^{235} atomic ratio.

Curve A: $U(93)O_2F_2$ solutions and $U(95)F_4-CF_2-CH_2$, water reflected, Refs. 6, 8, 30, 34, and 72.

Curve B: $U(4.9)O_2F_2$ solutions, water reflected, Ref. 40.

Curve C: $U_3(4.9)O_8-C_{57}H_{110}O_6$, water reflected, Ref. 40.

Curve D: $U(2.0)F_4-C_{25}H_{52}$, water reflected, Refs. 103 and 104.

Curve E: $U(37)O_2F_2$ solutions and $U(37)F_4-CF_2-C_5H_8O_2$, water reflected, Refs. 40 and 105.

Curve F: $U(29.8)F_4-CF_2-CH_2$, paraffin reflected, Ref. 39.

Curve G: $U(1.42)F_4-C_{40}H_{81}$, water reflected, Ref. 106.

The prime letters indicate unreflected values.

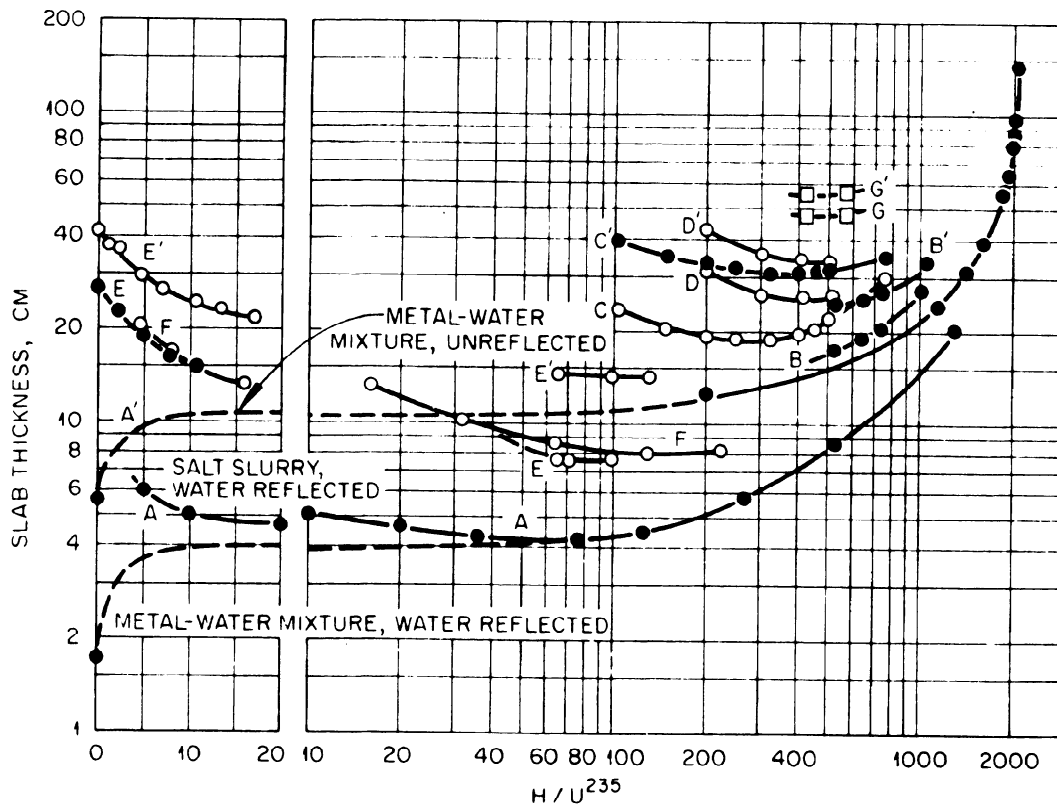


Fig. 16 - Critical thickness of infinite slabs as a function of H/U^{235} atomic ratio.

Curve A: $U(93)O_2F_2$ solutions and $U(95)F_4-CF_2-CH_2$, water reflected, Refs. 6, 8, 30, 34, and 72.

Curve B: $U(4.9)O_2F_2$ solutions, water reflected, Ref. 40.

Curve C: $U_3(4.9)O_8-C_{57}H_{110}O_6$, water reflected, Ref. 40.

Curve D: $U(2.0)F_4-C_{25}H_{52}$, water reflected, Refs. 103 and 104.

Curve E: $U(37)O_2F_2$ solutions and $U(37)F_4-CF_2-C_5H_8O_2$, water reflected, Refs. 40 and 105.

Curve F: $U(29.8)F_4-CF_2-CH_2$, paraffin reflected, Ref. 39.

Curve G: $U(1.42)F_4-C_{40}H_{81}$, water reflected, Ref. 106.

The prime letters indicate unreflected values.

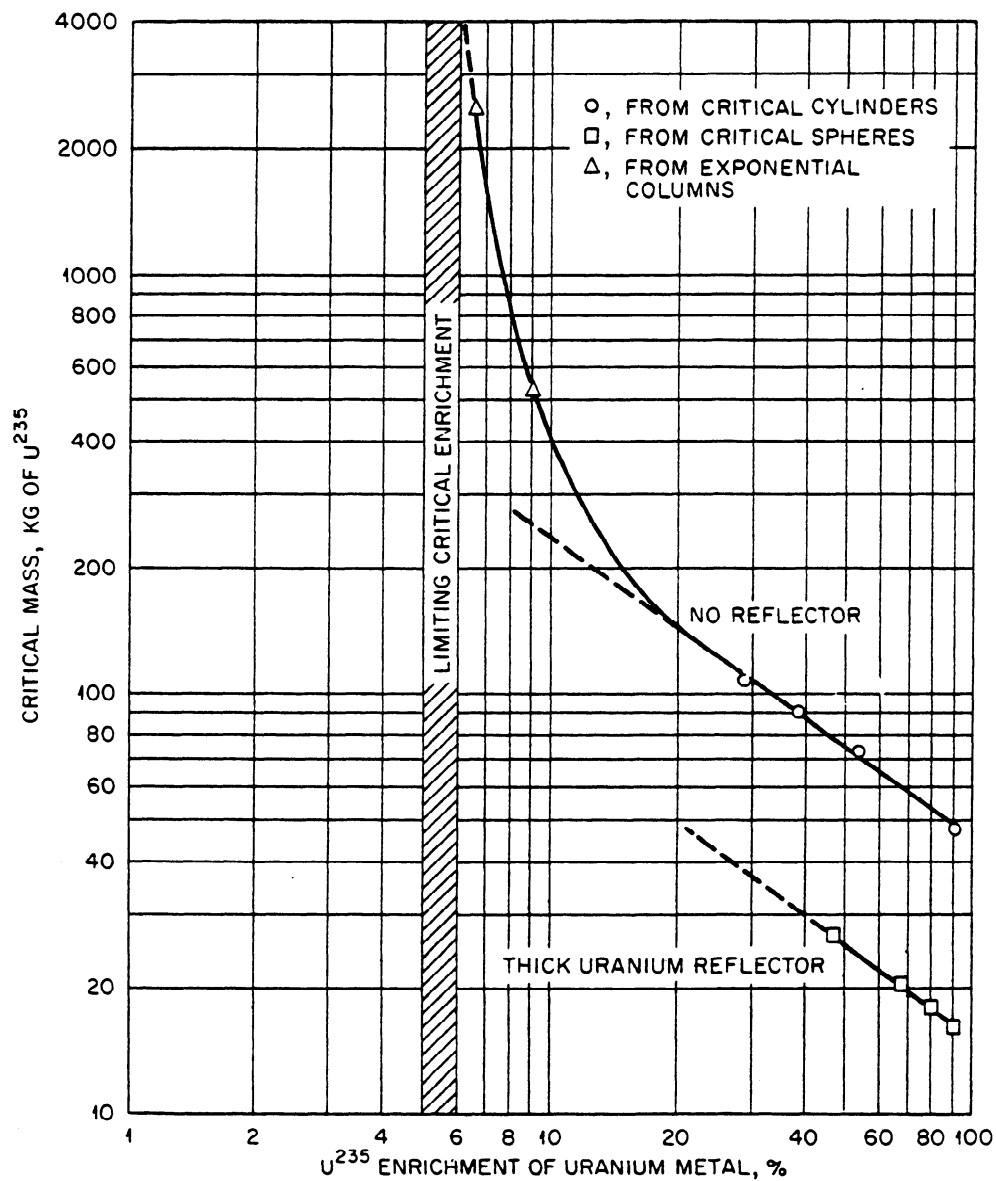


Fig. 17 - Critical mass vs. U^{235} enrichment of uranium metal. The shaded strip represents the range of uncertainty in the value of U^{235} concentration below which uranium metal cannot be made critical.

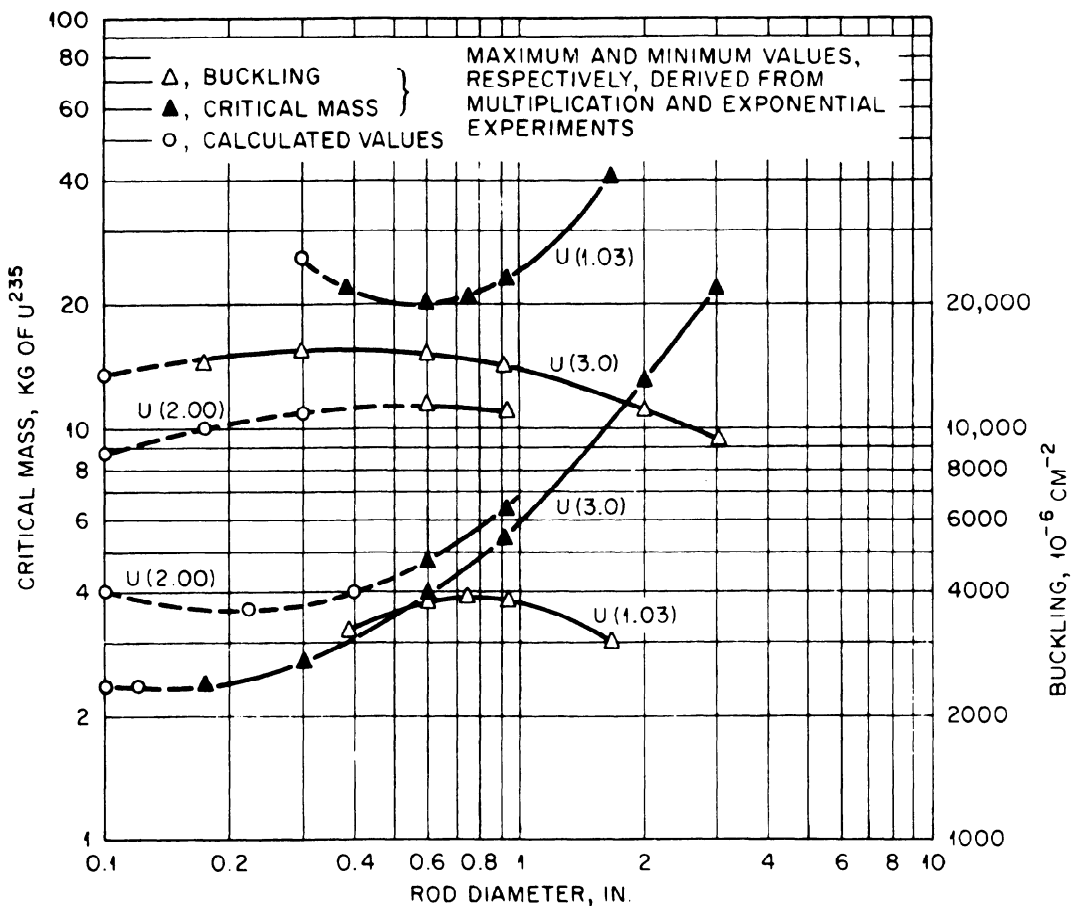


Fig. 18 – Minimum water-reflected critical mass and maximum buckling as a function of rod diameter for lattices of U(1.03), U(2.0), and U(3.0) in water.

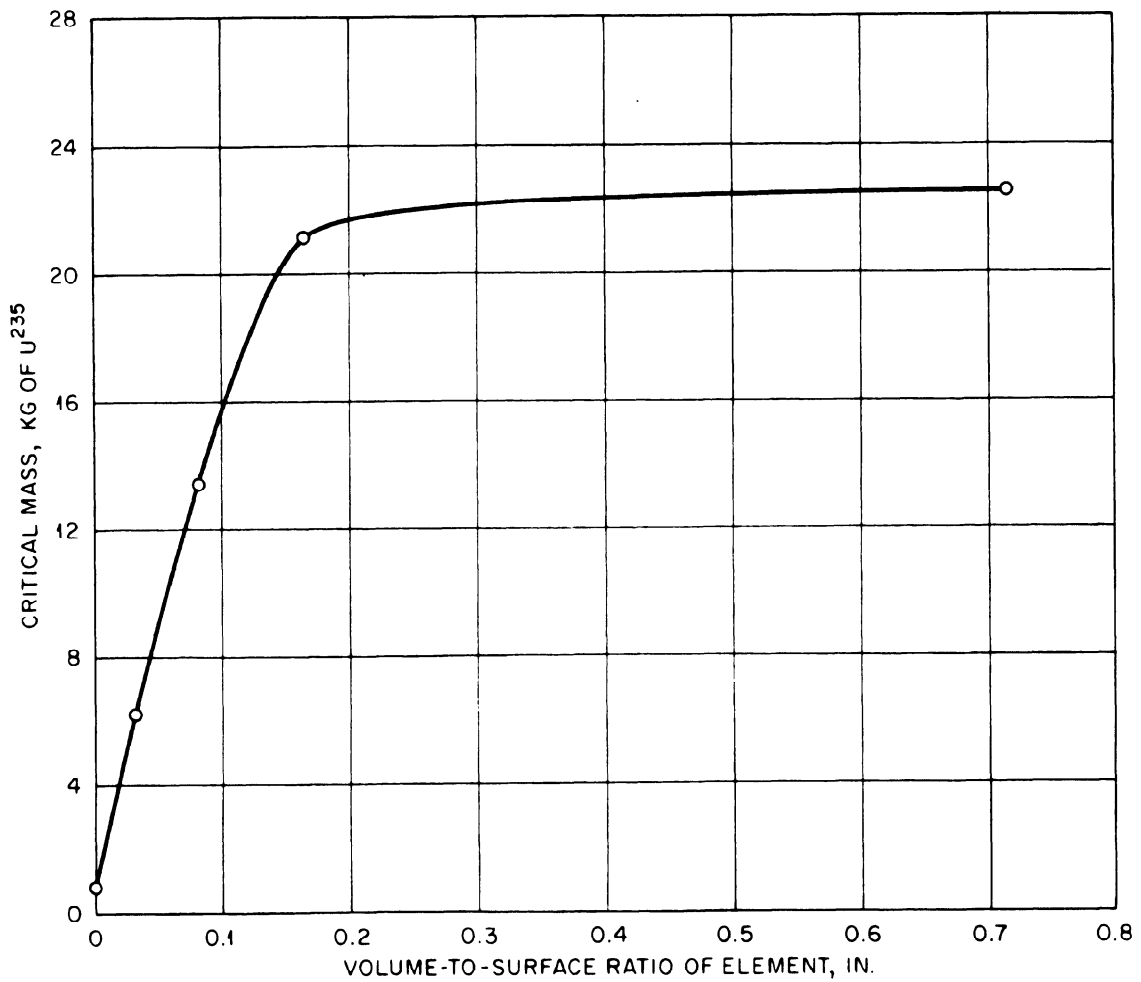


Fig. 19 – Minimum critical mass of submerged U(94) metal lattices as a function of volume-to-surface ratio of a fissile element.

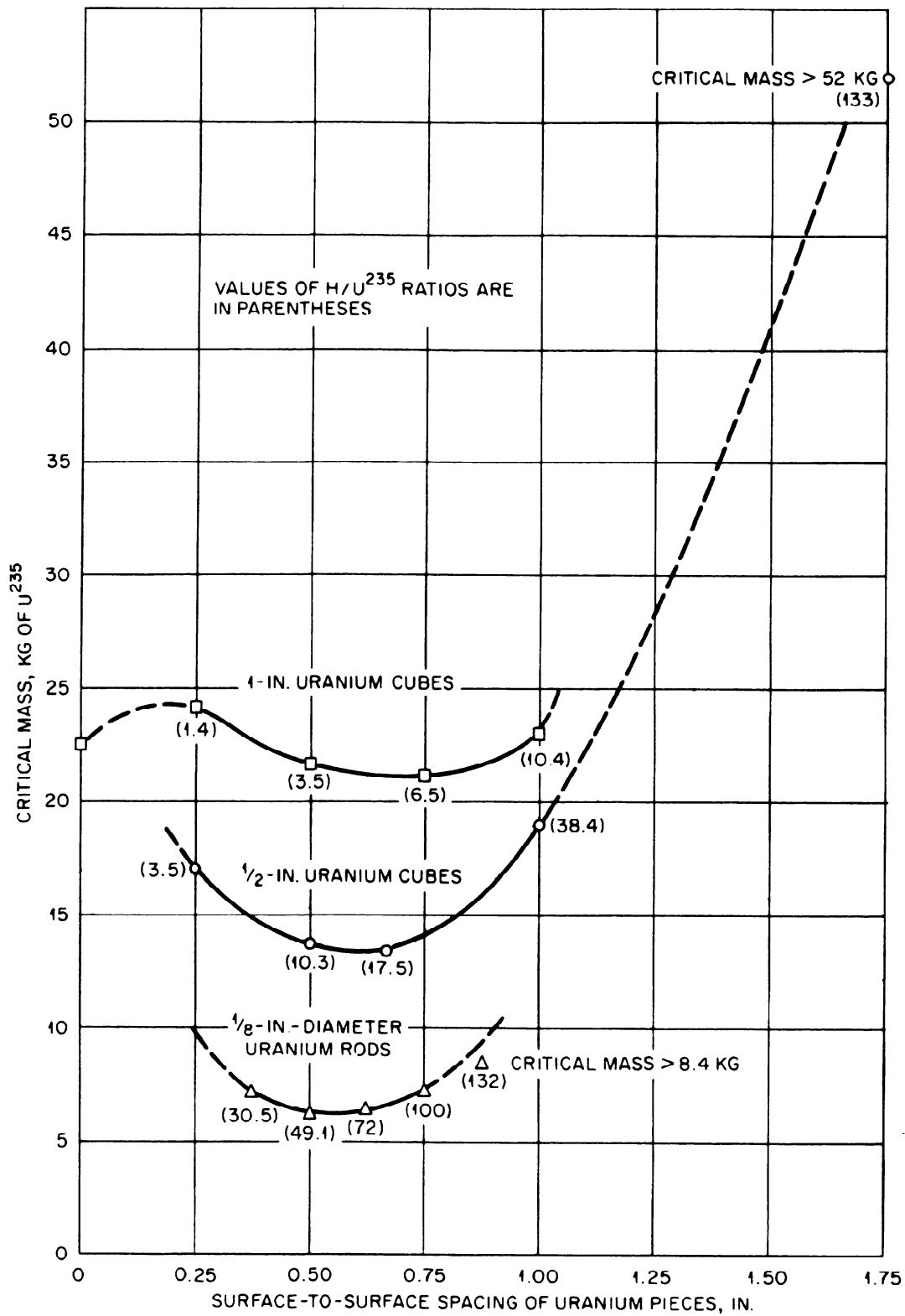


Fig. 20 – Critical mass of submerged U(94) metal lattices as a function of lattice spacing.

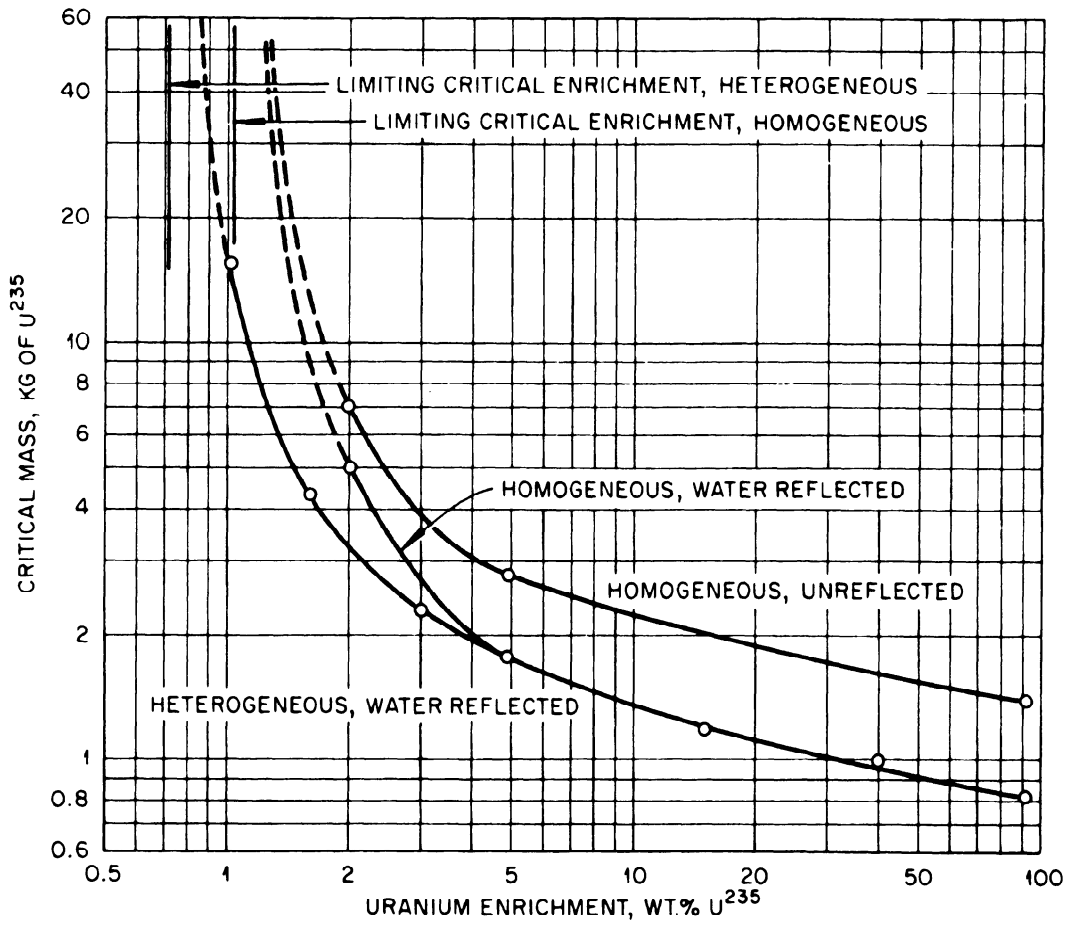


Fig. 21 - Minimum critical mass as a function of U^{235} enrichment in hydrogen-moderated systems.

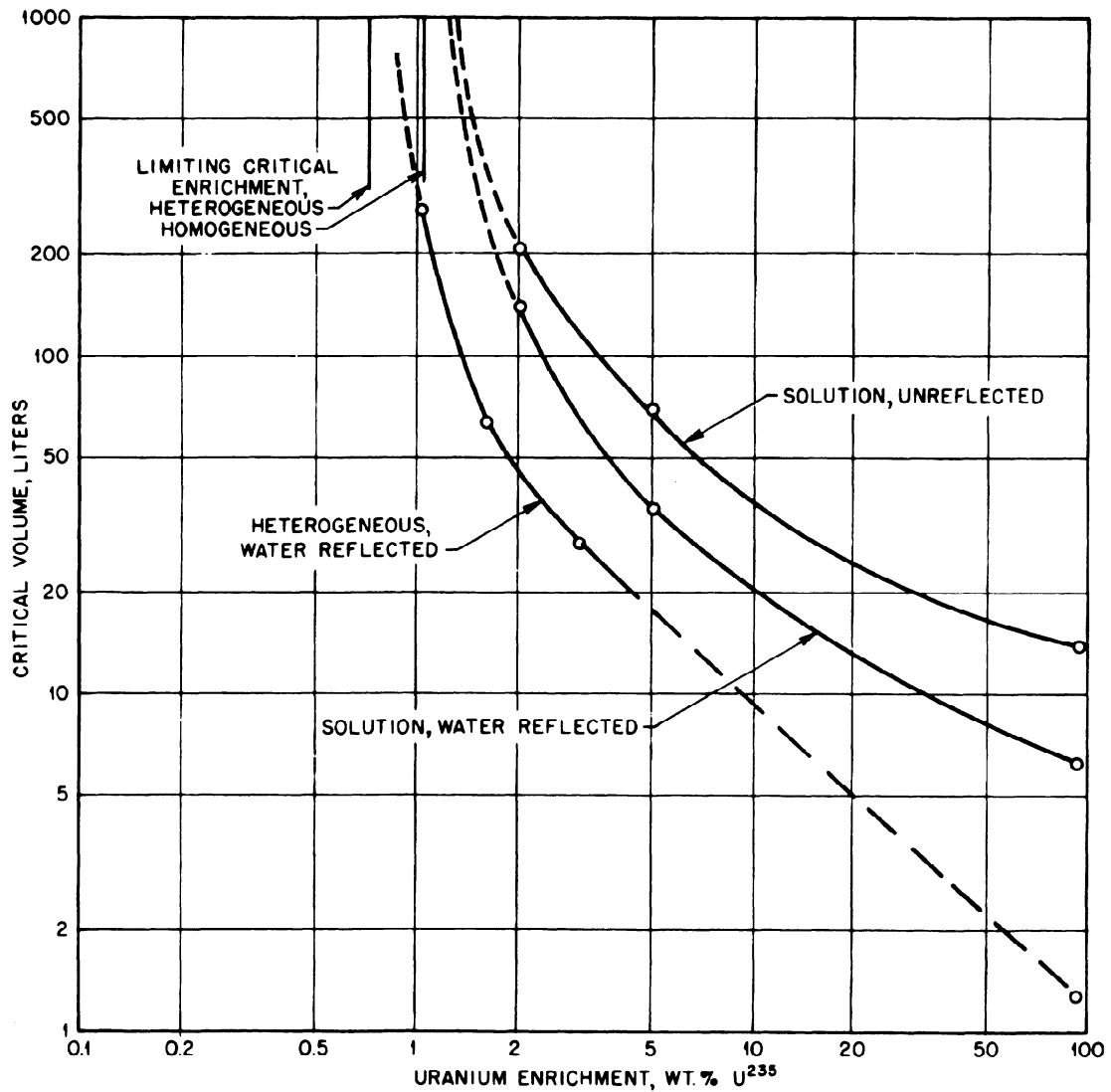


Fig. 22 - Minimum critical volume as a function of U^{235} enrichment in hydrogen-moderated systems.

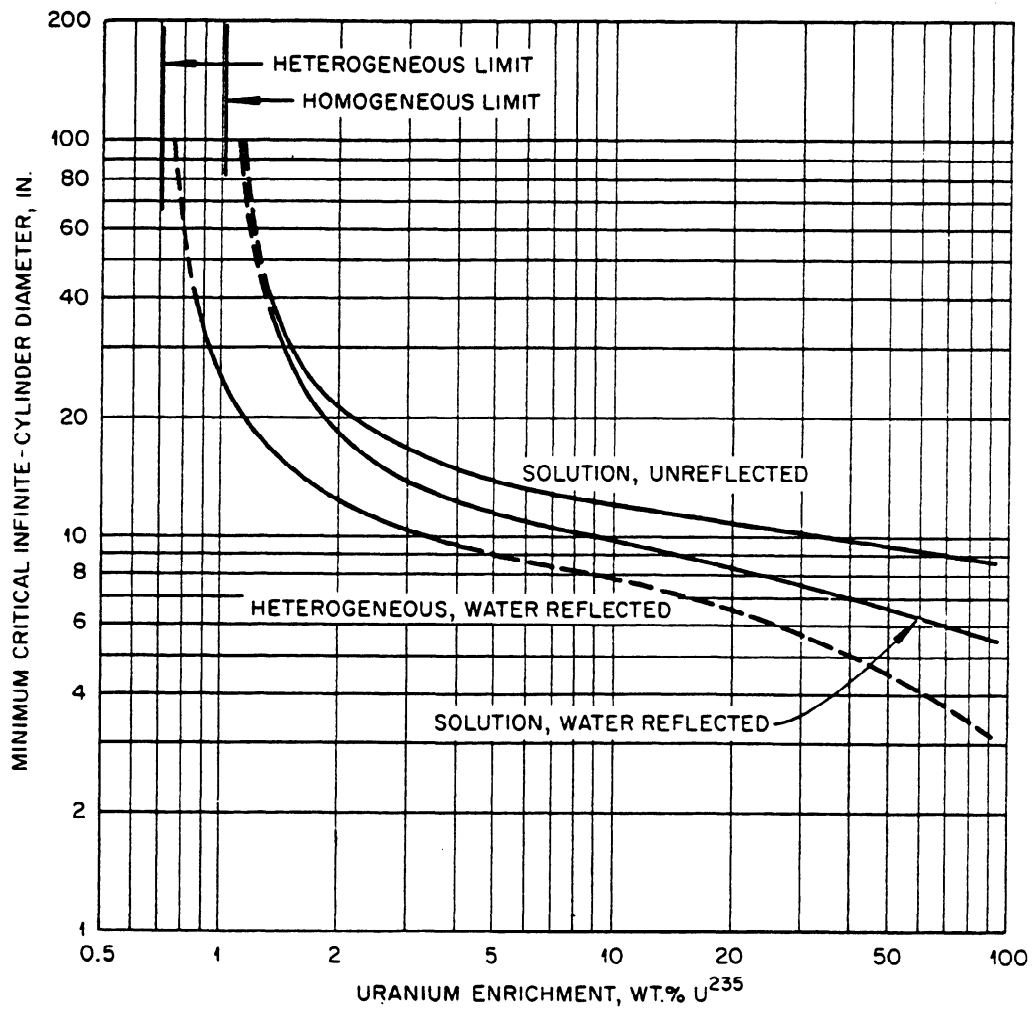


Fig. 23 - Minimum critical cylinder diameter as a function of U^{235} enrichment in hydrogen-moderated systems.

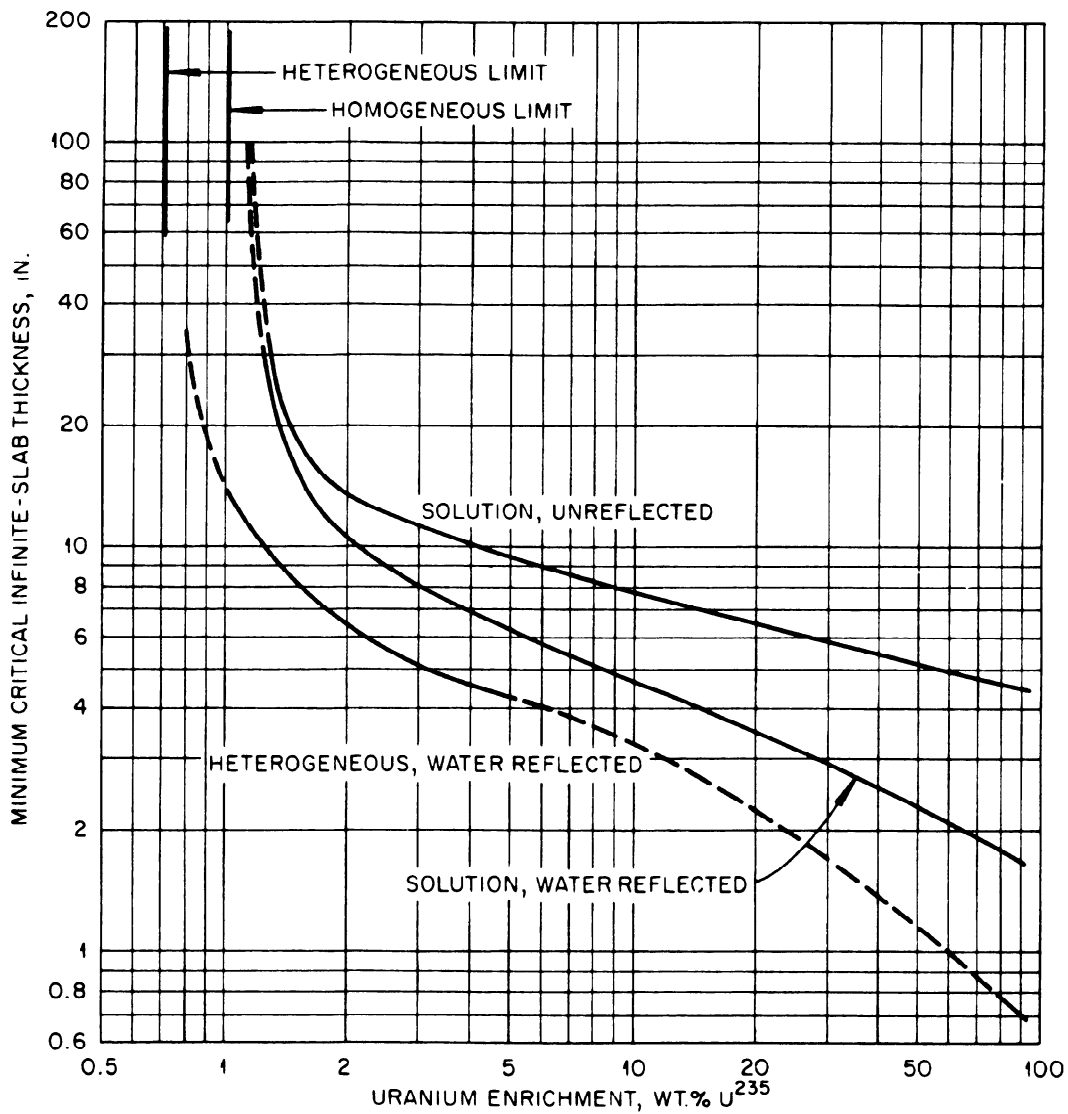


Fig. 24 – Minimum critical slab thickness as a function of U^{235} enrichment in hydrogen-moderated systems.

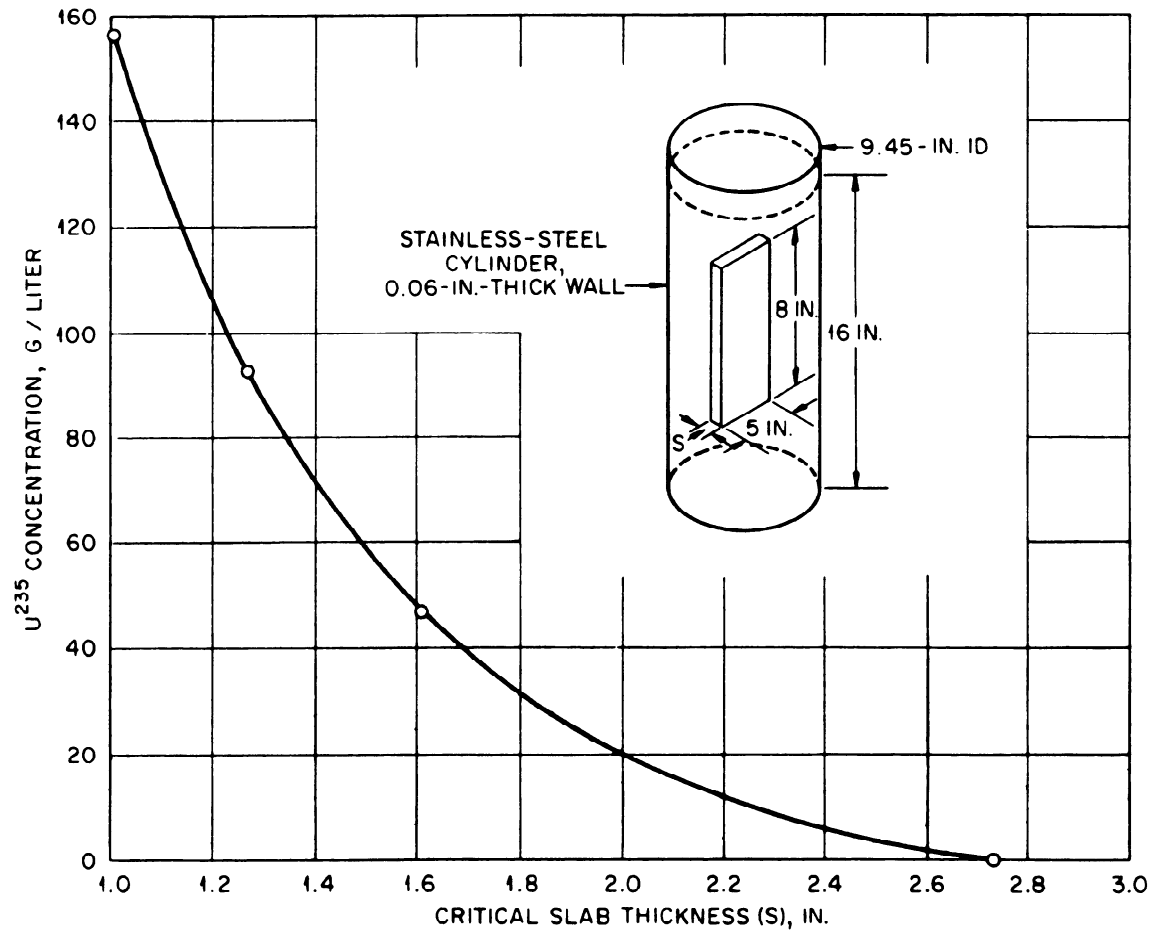


Fig. 25 – Critical concentration of 9.45-in.-dia $U(93)O_2(NO_3)_2$ solution surrounding a 5- by 8-in. slab of U(93) metal.

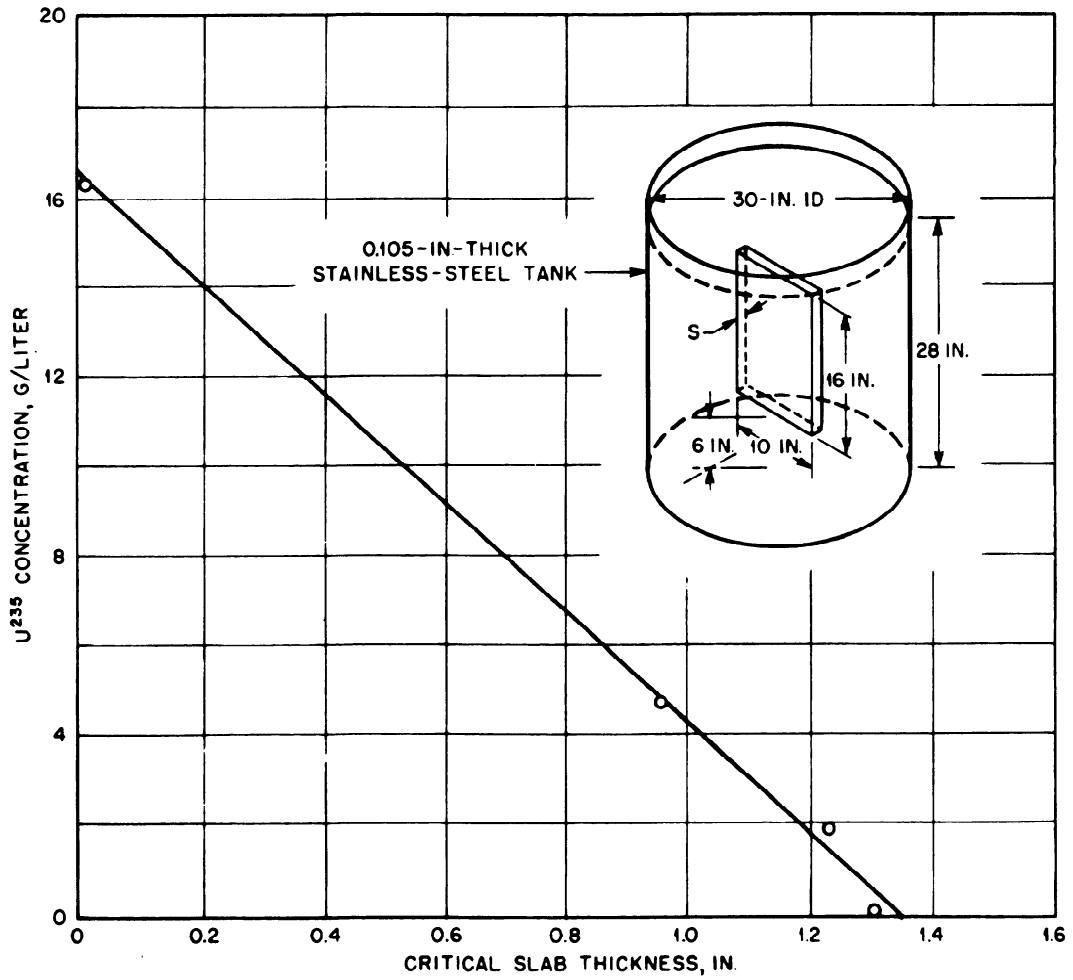


Fig. 26 - Critical concentration of 30-in.-dia $U(93)O_2(NO_3)_2$ surrounding a 10- by 16-in. slab of $U(93)$ metal.

Plutonium Units, Reflected and Unreflected

HOMOGENEOUS MODERATED PLUTONIUM

Critical parameters for homogeneous water-moderated Pu^{239} , obtained in the same manner as those in Figs. 8 to 11, are presented in Figs. 27 to 30. The branches of curves labeled "solution" are adjusted to apply to the fictitious " $\text{Pu}^{239}\text{O}_2\text{F}_2$ " in water; thus these branches give limits that are smaller than those of attainable solutions. A measurement in the PCTR at Hanford³¹ established 8.0 ± 0.3 g of Pu^{239} /liter as the limiting critical concentration in aqueous solution. Assumed relations between Pu^{239} density and H/Pu^{239} appear in Table 1. Relations between these quantities as measured for $\text{Pu}(\text{NO}_3)_4$ solutions are given in Fig. 31 (Ref. 59).

The Hanford critical data¹³ at $\text{H}/\text{Pu} > 250$ shown in Fig. 43 have empirical corrections for Pu^{240} and nitrate. The results are supported by Harwell measurements on partially reflected solutions.⁶⁰ Compositional corrections from DSN computations were applied to the Rocky Flats values from subcritical experiments^{18,61,62} within the range $1 < \text{H}/\text{Pu} < 120$ and to the Aldermaston value⁶³ at $\text{H}/\text{Pu} \sim 50$. Plutonium-metal data are from LASL^{33,64,65} and Livermore,^{21,67} supplemented by Rocky Flats^{19,20} results for slabs (with DSN conversions from Plexiglas to water reflection). Figure 32 gives the corrections computed by Goodwin⁶⁸ and by Roach⁶⁹ which were assumed in adjusting the critical data for the effects of Pu^{240} content.³⁸ It should be emphasized that there is little experimental confirmation of these curves.

Limited experimental information from Dounreay¹⁰⁶ for homogeneous mixtures of PuO_2 -polyethylene surrounded by a polyethylene reflector is given in Table 3.

Saclay^{97,107} is the source of the critical data summarized in Table 4 for cylinders of aqueous solutions of $\text{Pu}(\text{NO}_3)_4$ surrounded by light water and by other reflectors.

Table 3 - CRITICAL DIMENSIONS OF HOMOGENEOUS PuO_2 -POLYETHYLENE COMPACTS WITH POLYETHYLENE REFLECTOR

Plutonium concentration		Critical dimensions, in.	Critical mass, kg of Pu*
H/Pu	g of Pu*/cm ³		
~5,6	2.88	6 × 6 × 11.5	19.6
~5,6	2.88	8 × 8 × 5.95	18.0
~7	2.88	6 × 6 × 9.8	16.7
~7	2.88	7 × 7 × 6.8	15.7

*3.24% Pu^{240} , 0.16% Pu^{241} .

Table 4 - CRITICAL DIMENSIONS OF CYLINDRICAL VOLUMES OF AQUEOUS SOLUTIONS OF $\text{Pu}(\text{NO}_3)_4$
 [Pu^{240} content, $\sim 1.5\%$; reflector thickness, 15.7 in. on lateral surface only; containers, stainless steel
 (wall thickness, 0.16 in. for 16.2 in. diameter and 0.12 in. for others)]

Plutonium concentration		Density, g/cm^3	Total NO_3 , g/liter	Critical dimensions		
H/Pu	g of Pu/liter			Height, in.	Volume, liter	Mass, g of Pu
Cylinder diameter, 11.6 in., water reflector						
561	44.7	1.132	174	12.28 ± 0.02	21.27	951 ± 12
612	41.0	1.126	170	13.05 ± 0.02	22.61	927 ± 12
662	38.0	1.118	162	13.92 ± 0.02	24.11	916 ± 12
714	35.25	1.115	161	14.94 ± 0.02	25.92	914 ± 12
775	32.6	1.111	157	16.20 ± 0.02	28.09	916 ± 12
823	30.7	1.109	157	17.64 ± 0.02	30.62	940 ± 12
877	28.9	1.106	153	19.51 ± 0.02	33.9	980 ± 13
Cylinder diameter, 11.6 in., concrete reflector						
535	47.0	1.134	170	12.07 ± 0.02	20.88	981 ± 15
673	37.5	1.123	166	13.80 ± 0.02	23.91	897 ± 13
773	32.8	1.113	155	15.37 ± 0.02	26.67	875 ± 12
904	28.2	1.102	142	17.89 ± 0.02	31.08	877 ± 12
941	27.0	1.107	155	19.29 ± 0.02	33.53	905 ± 12
Cylinder diameter, 11.6 in., wood reflector						
578	43.6	1.125	162	13.25 ± 0.02	22.96	1001 ± 14
640	39.4	1.118	158	14.35 ± 0.02	24.90	981 ± 14
714	35.4	1.111	153	15.79 ± 0.02	27.33	968 ± 13
744	34.0	1.109	152	16.20 ± 0.02	28.10	955 ± 13
813	31.2	1.103	145	18.35 ± 0.02	31.87	994 ± 13
883	28.7	1.105	151	20.45 ± 0.02	35.58	1021 ± 13
Cylinder diameter, 12.8 in., water reflector						
541	45.54	1.133	177	10.34 ± 0.02	20.53	935 ± 9
646	38.17	1.113	176	11.42 ± 0.02	22.71	867 ± 8
750	33.28	1.118	168	12.88 ± 0.02	25.73	856 ± 8
830	30.07	1.111	161	14.01 ± 0.02	28.13	846 ± 7
892	27.97	1.105	152	15.14 ± 0.02	30.45	851 ± 7
976	25.67	1.110	146	17.22 ± 0.02	34.73	892 ± 7
1070	23.45	1.094	145	20.30 ± 0.02	41.72	964 ± 7
Cylinder diameter, 16.2 in., water reflector						
	57.4		*	8.71 ± 0.02	29.39	1687
	54.32		*	8.73 ± 0.02	29.45	1599
	51.83		*	8.86 ± 0.02	29.9	1550
	46.96		*	9.13 ± 0.02	30.82	1447
	41.16		*	9.68 ± 0.02	32.68	1347
	36.7		*	10.25 ± 0.02	34.62	1270
	30.9		*	11.42 ± 0.02	38.6	1193
	26.65		*	13.07 ± 0.02	44.22	1178
	22.6		*	15.65 ± 0.02	53.03	1198
	19.7		*	19.92 ± 0.02	67.62	1332
	18.8		*	23.46 ± 0.02	79.78	1500

*The free nitric acid content was $\sim 2N$.

HETEROGENEOUS MODERATED PLUTONIUM

The single set of measurements on lattices of plutonium fuel rods in water was performed at Hanford with 0.51-in.-diameter rods of aluminum-5 wt. % plutonium.⁷⁰ Critical mass and buckling as functions of H/Pu appear in Fig. 33.

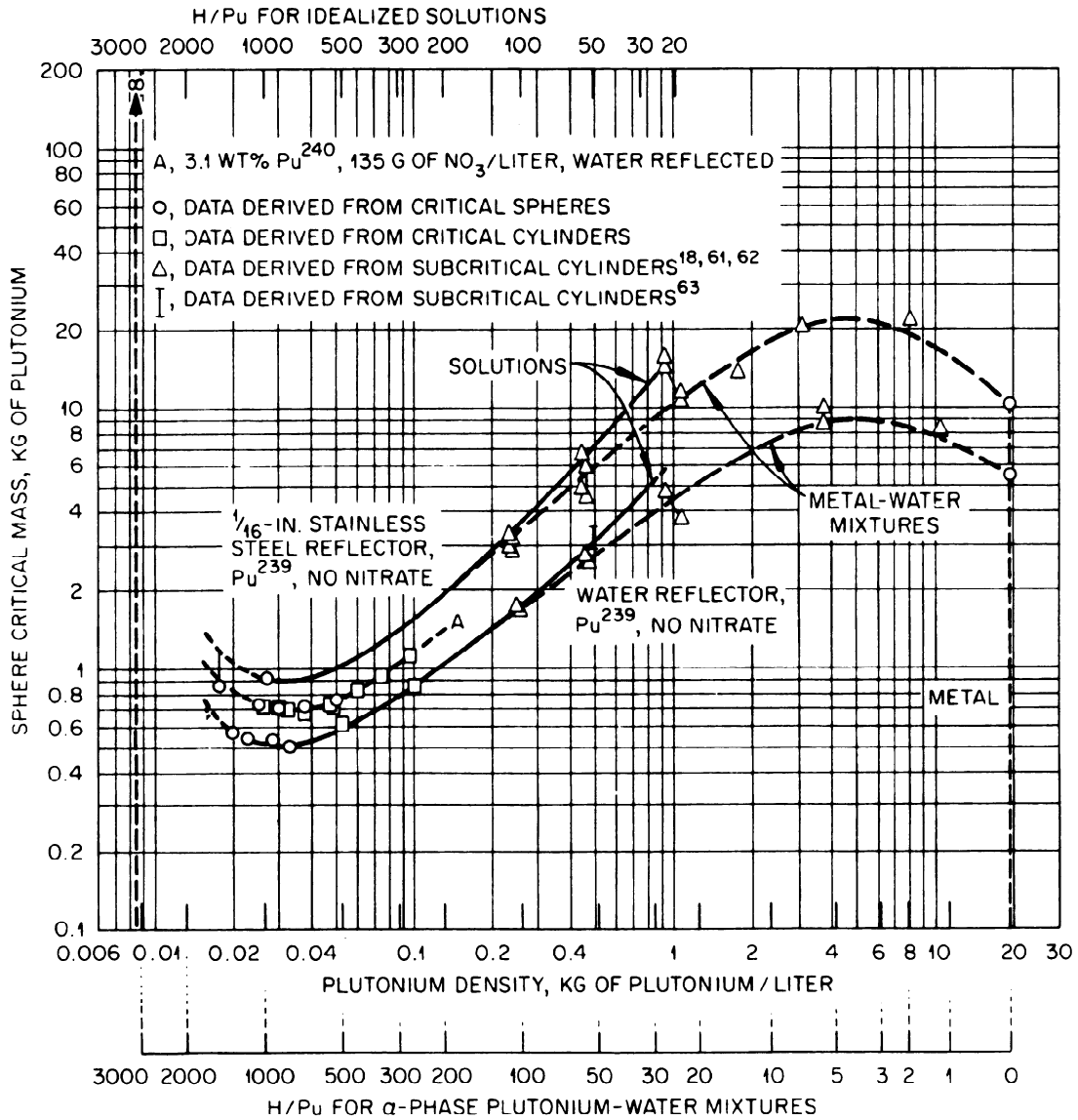


Fig. 27 – Critical masses of homogeneous water-moderated plutonium spheres.

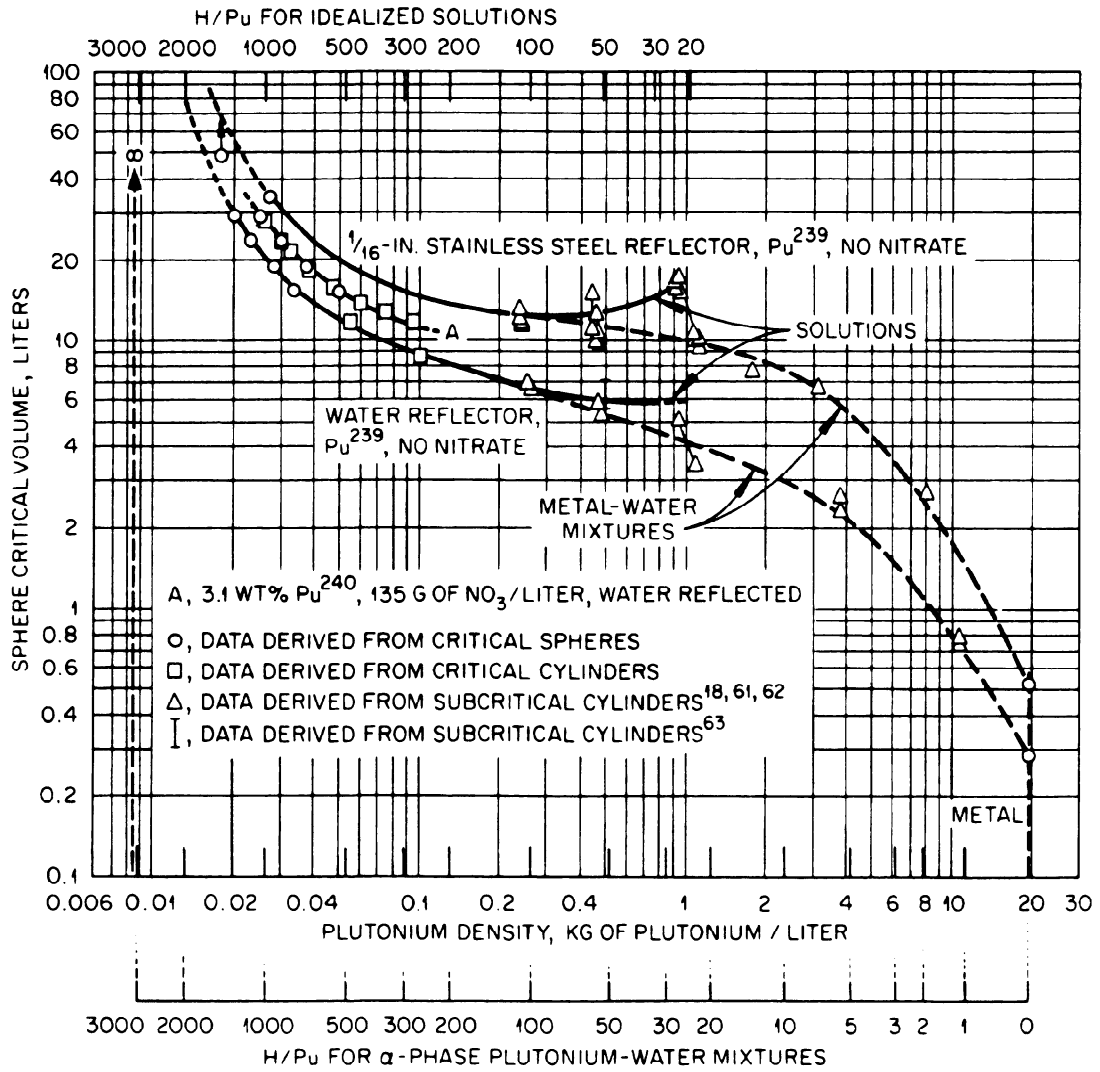


Fig. 28 – Critical volumes of homogeneous water-moderated plutonium spheres.

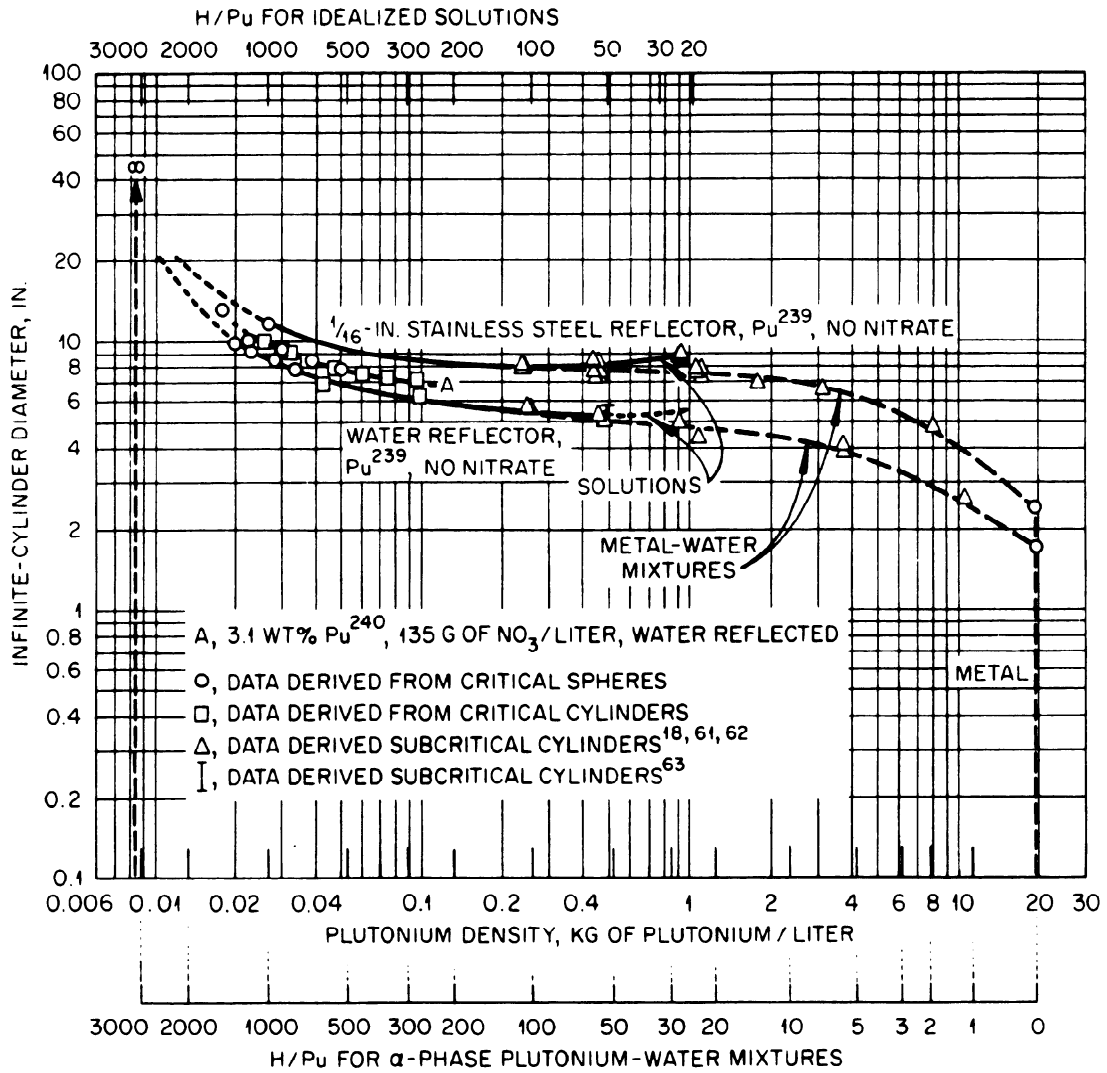


Fig. 29 – Estimated critical diameters of infinite cylinders of homogeneous water-moderated plutonium.

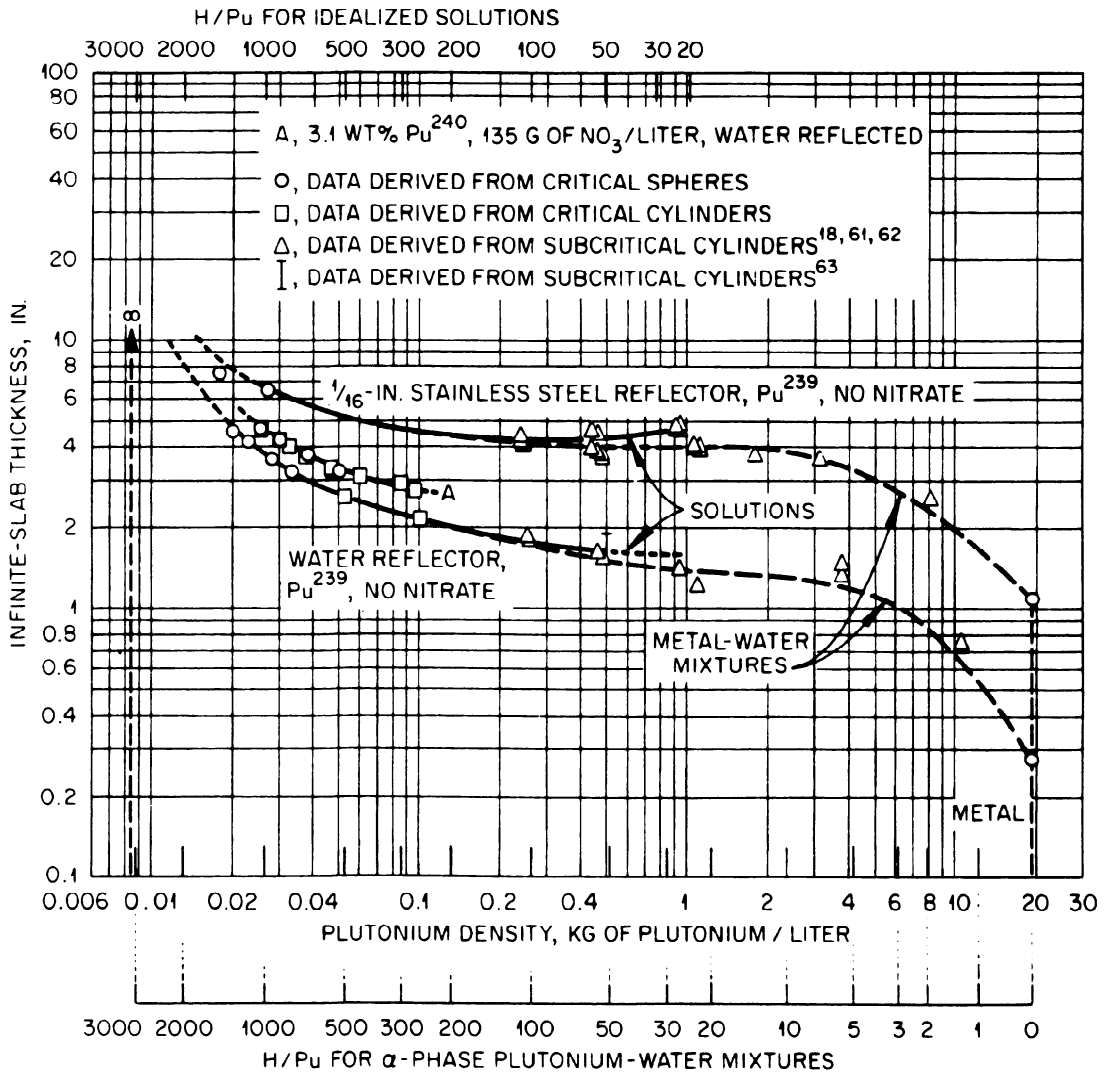


Fig. 30 – Estimated critical thicknesses of infinite slabs of homogeneous water-moderated plutonium.

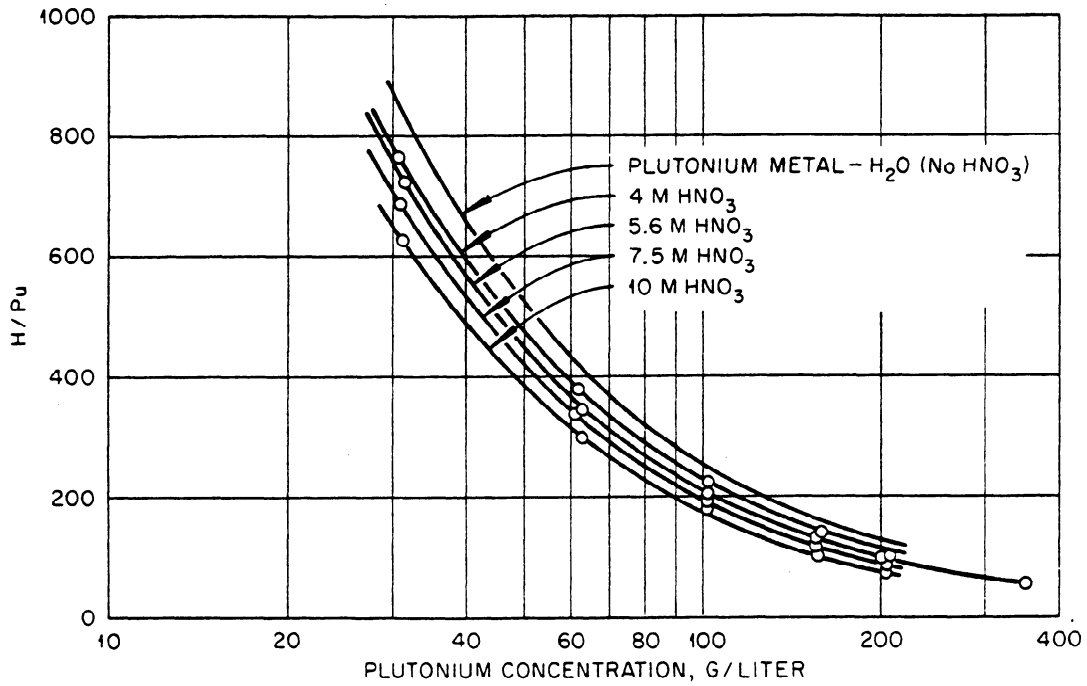


Fig. 31 - H/Pu atomic ratio as a function of plutonium concentration for $Pu(NO_3)_4$ solutions containing 4M, 5.6M, 7.5M, and 10M HNO_3 , and for idealized plutonium-water mixtures.

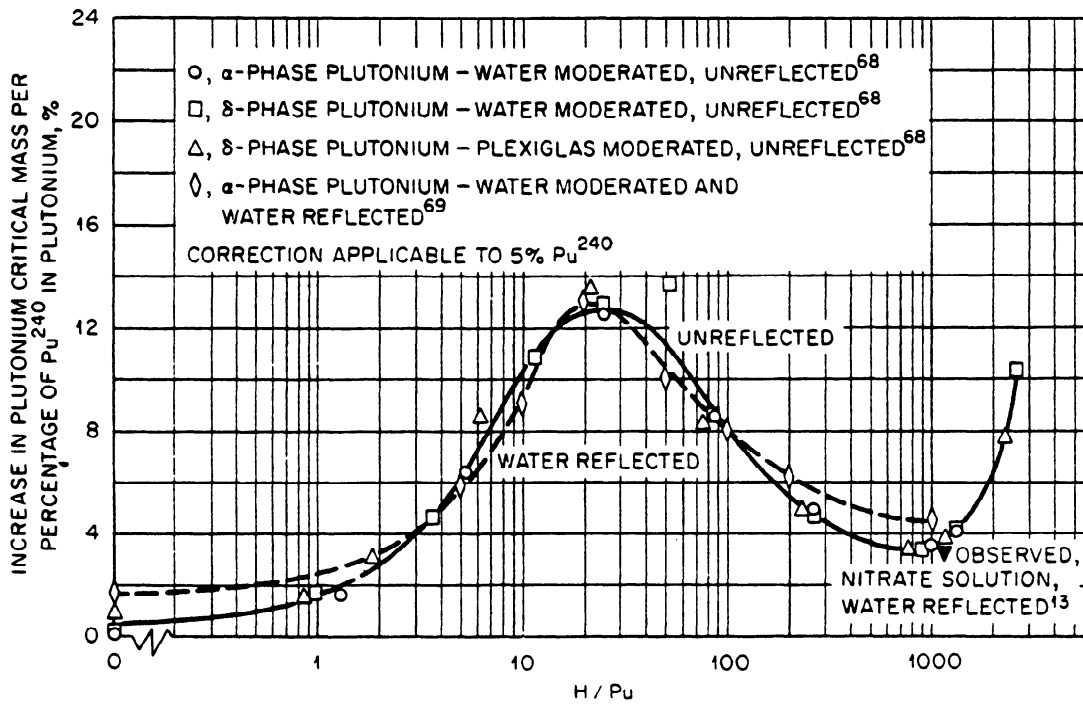


Fig. 32 - Computed effect of Pu^{240} on critical mass of hydrogen-moderated plutonium spheres.

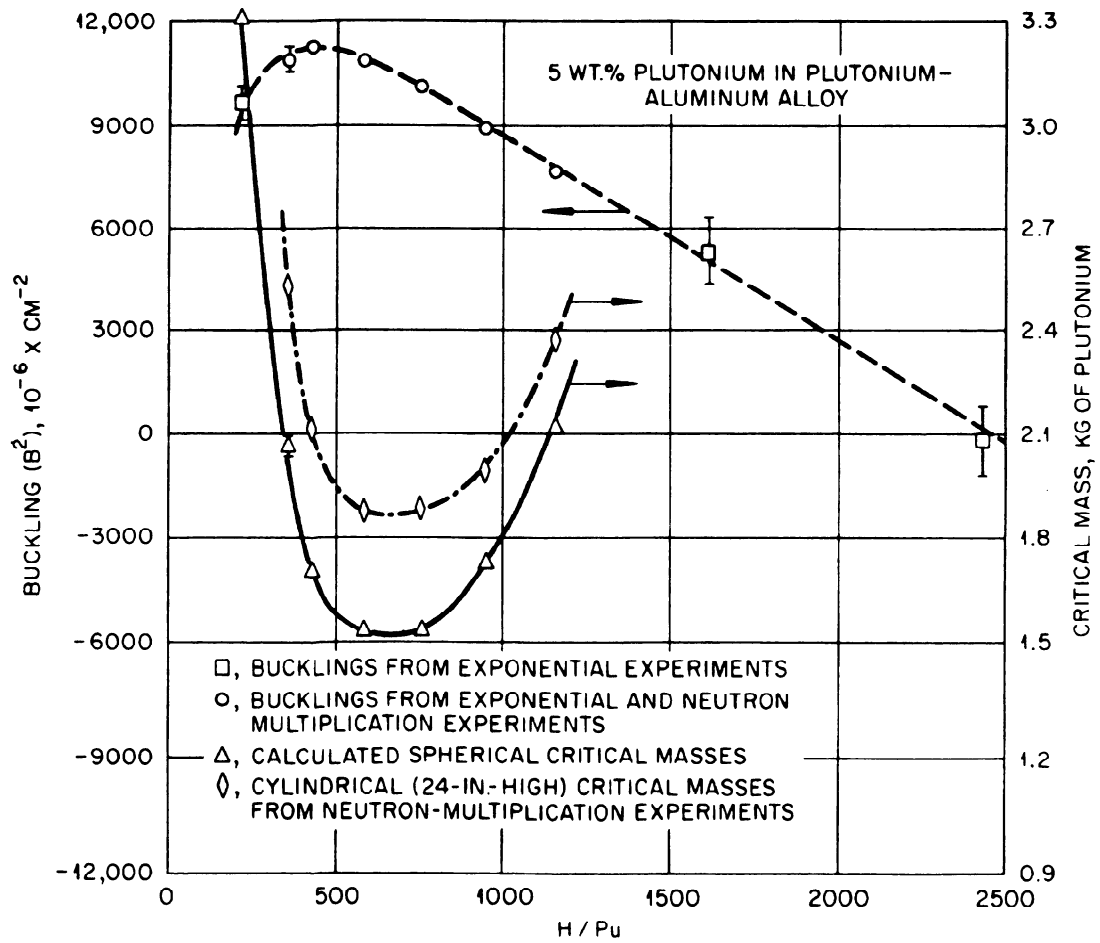


Fig. 33 – Buckling and critical mass of water-moderated and -reflected lattices of plutonium-aluminum-alloy rods.

U^{233} Units, Reflected and Unreflected

Figures 34 to 37 present critical data for homogeneous water-moderated U^{233} which are similar to the representations for U^{235} and Pu^{239} . All values for U^{233} solutions are from ORNL.^{14,72} Extrapolation of values of critical solution concentrations to zero buckling gives 11.25 ± 0.10 g of U^{233} /liter as the limiting critical concentration.

Densities of U^{233} in the UO_2F_2 solutions and for metal-water mixtures appear as functions of H/U^{233} in Table 1. Critical data for U^{233} metal (from LASL) exist only for spheres,^{65,71} and the water-reflected critical mass is derived indirectly from the correlations of Fig. 45. Shape conversions required extrapolation distances for U^{233} which were scaled from those of U^{235} by matching the reflector savings for U^{233} and U^{235} spheres.

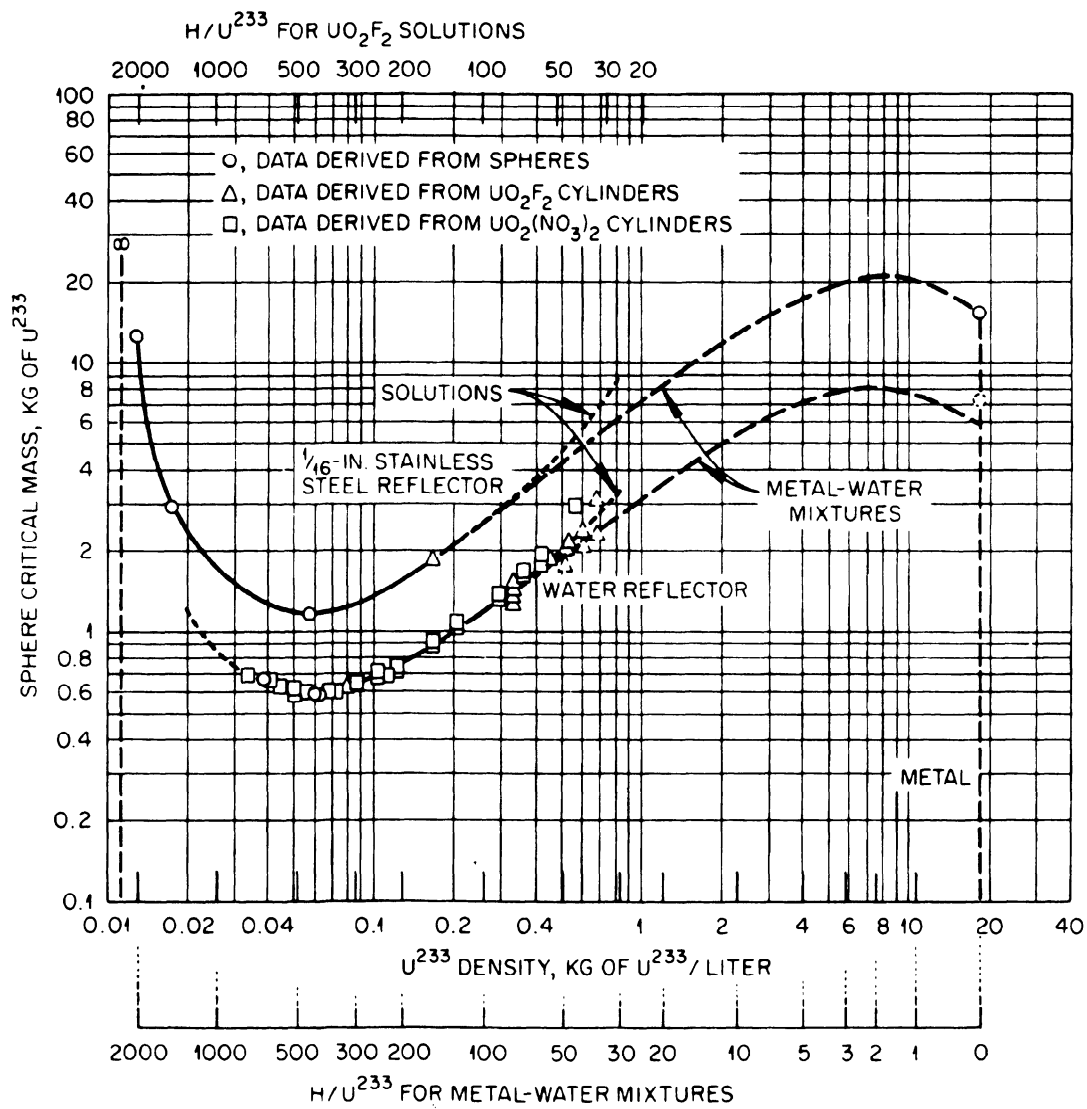


Fig. 34 – Critical masses of homogeneous water-moderated U^{233} spheres. Dashed symbols represent less certain values.

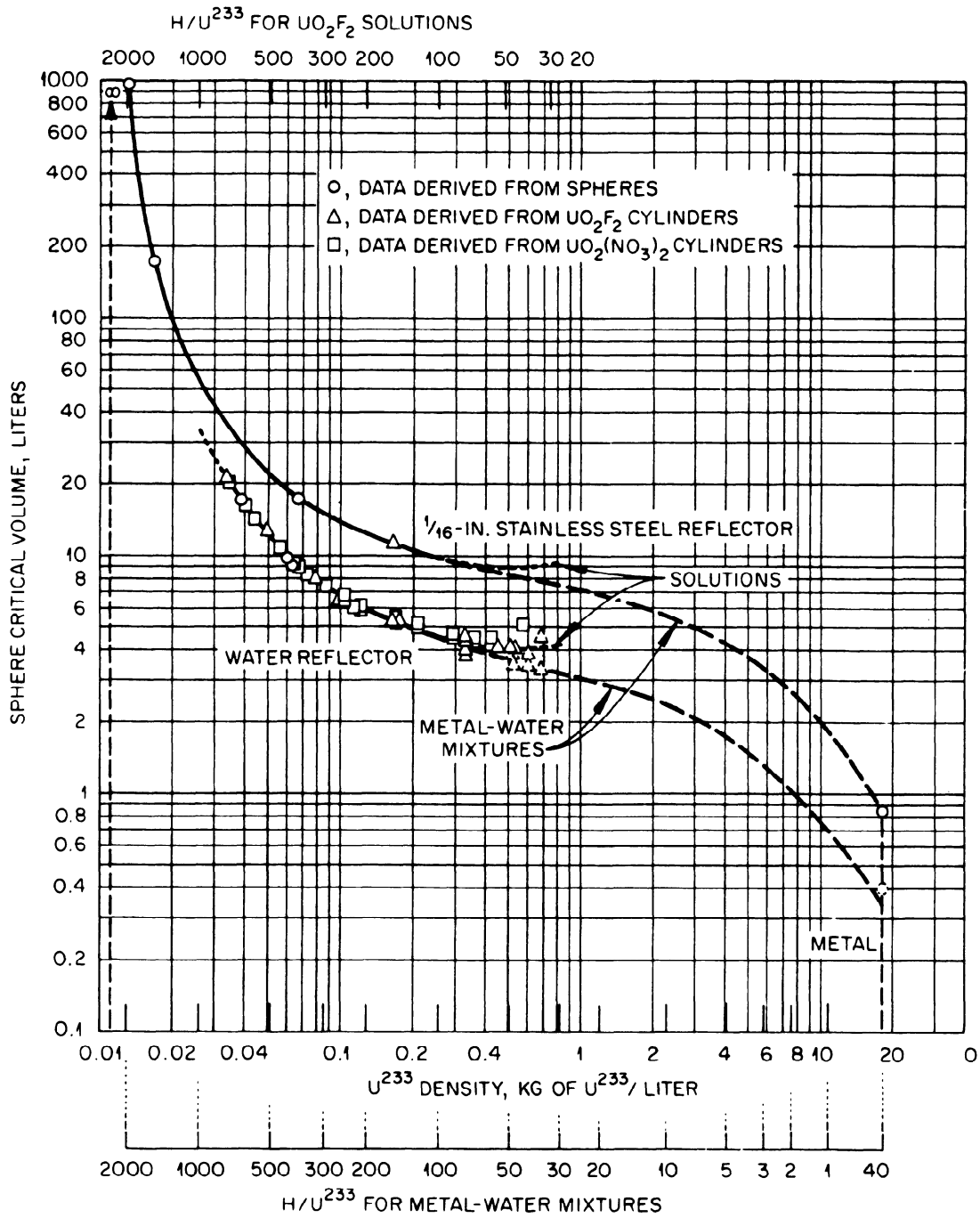


Fig. 35 – Critical volumes of homogeneous water-moderated U²³³ spheres. Dashed symbols represent less certain values.

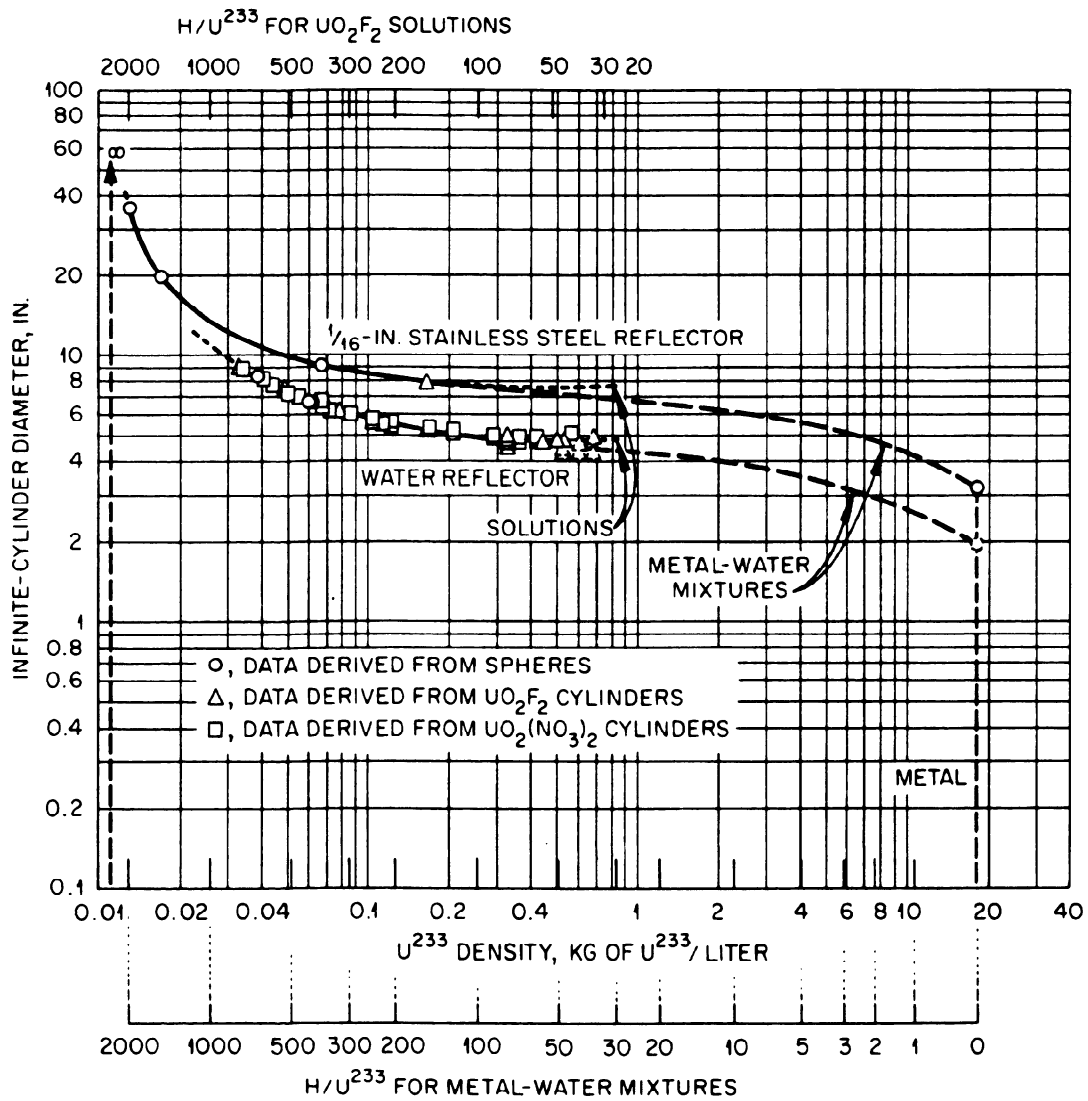


Fig. 36 – Estimated critical diameters of infinite cylinders of homogeneous water-moderated U^{233} . Dashed symbols represent less certain values.

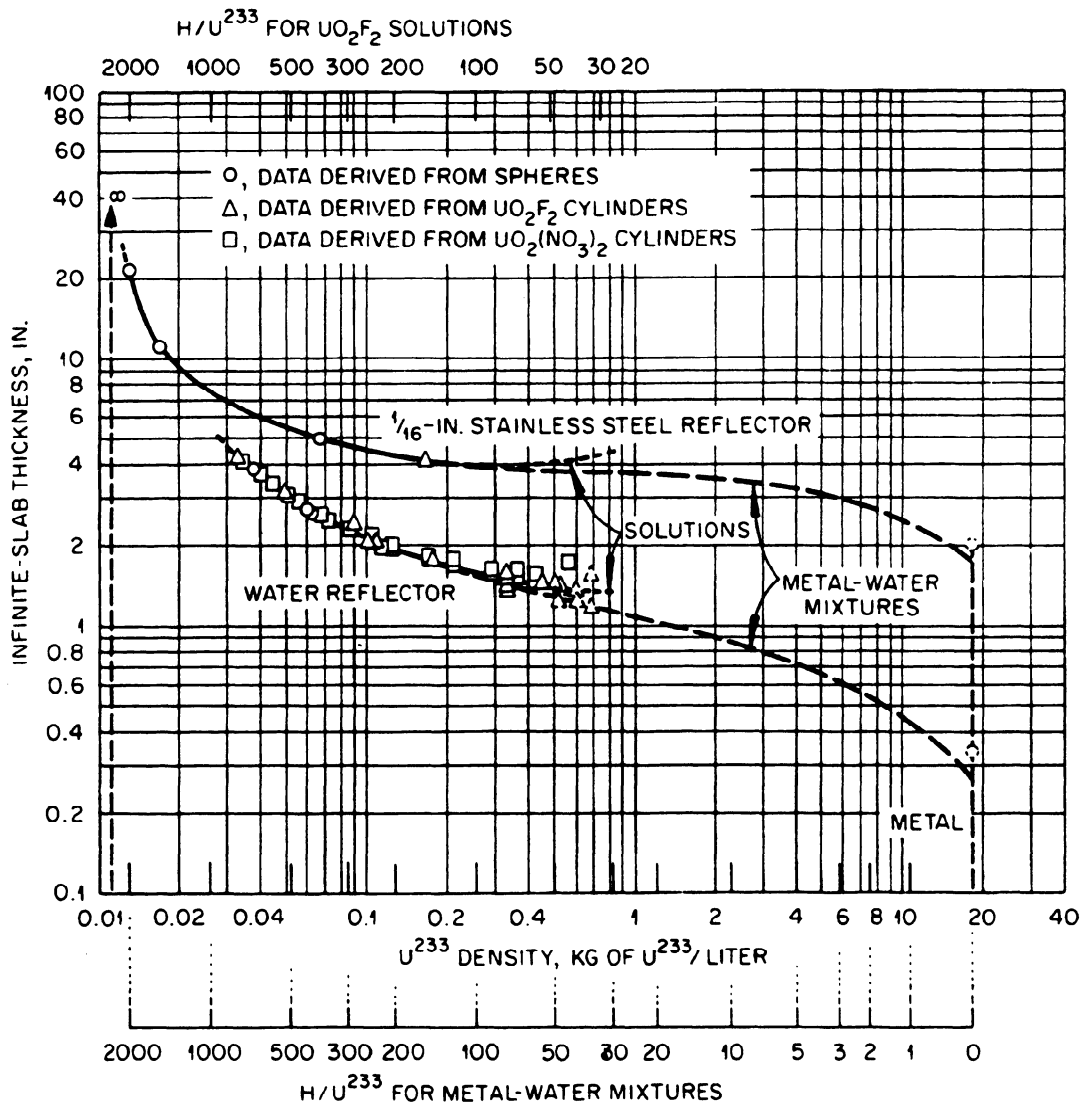


Fig. 37 – Estimated critical thicknesses of infinite slabs of homogeneous water-moderated U^{233} . Dashed symbols represent less certain values.

Poisoned Solutions

U^{235} SOLUTIONS

Soluble Poisons

At ORNL changes in the concentration of uranium required to compensate for boric acid additions to certain critical $UO_2(NO_3)_2$ solutions have been established.⁷² All assemblies were essentially unreflected 27-in.-diameter spheres. For U(93.2), initially critical at $H/U^{235} = 1380$, the system remained critical when ~ 1.55 atoms of U^{235} were added per atom of boron at concentrations up to 0.23 g of boron/liter. Data also exist on poisoning by nitrate, phosphate, and bismuth.²⁶

Additional information about homogeneously distributed poison is yielded by PCTR experiment namely, the concentration required in a fissile mixture to reduce k_∞ to unity. Figure 38 gives results for the effect of boron in homogeneous $U(3.04)O_3$ -polyethylene mixtures at various H/U^{235} atomic ratios⁷³ and also includes a point for a $U(2.00)F_4$ -paraffin mixture.⁴⁴ Figure 39 gives the boron concentration required to reduce to unity the k_∞ of optimally moderated homogeneous mixtures of hydrogen and uranium as a function of the U^{235} content. The values at 3.04 and 1.03 wt. % are established experimentally; the value at 2.00 wt. % is estimated from the one measurement shown in Fig. 38. The extrapolation beyond 3 wt. % is tentative. This extrapolation was obtained by DSN calculations with the use of Hansen-Roach²⁹ cross sections; it is applicable only when $H/U^{235} \geq 20$.

Heterogeneous Poisons

Recent experiments at ORNL⁷⁴ have resulted in definitive measures of the effectiveness of borosilicate-glass Raschig rings as fixed poisons in $U(93)O_2(NO_3)_2$ aqueous solutions. All measurements were made in cylindrical geometry, with the amount of boron in the rings, the cylinder diameter, the ring dimensions, and the solution concentration as variables. The natural boron content of the rings varied from 0.5 to 5.7 wt. %, and the volume of the vessel occupied by the glass ranged from 20.9 to 30 percent. Solutions containing between 415 and 63.3 g of uranium/liter were used. Results from exponential experiments, with the use of a critical layer of solution above the column of solution-ring mixture as a neutron source, provided estimates of the material buckling of the subcritical mixture as a function of solution concentration, boron content of the glass, and the glass volume present. These experiments indicated that $k_\infty \leq 1$ if the rings are uniformly distributed under the following conditions:

Minimum volume fraction occupied by glass, %	Minimum boron content of glass, wt. %	Maximum concentration of solution, g of U/liter
24	3.3	415
22	4.0	415
24	0.5	72

At Rocky Flats two series of subcritical measurements were conducted on hydrogen-moderated $U(93)O_2(NO_3)_2$ systems that contained randomly packed borosilicate-glass Raschig rings. In one case the glass displaced 17.8 vol. % of aqueous solution in a 42-in.-diameter tank that was surrounded by a close-fitting square concrete vault with open top.⁷⁵ The rings, 1.5 in. long and 1.5 in. in outside diameter with 0.078-in.-thick walls, contained 0.090 g of boron/cm³. Results indicate that such a tank if filled to infinite height would not be critical with a solution containing ~325 g of U^{235} /liter. Transformation of dimensions gives 27.5 in. as the radius of the corresponding critical sphere and 26.5 in. as the thickness of an infinite critical slab.

In the other case, rings (1.75 in. long and 1.5 in. in outside diameter with 0.125-in.-thick walls and containing 0.135 g of boron/cm³) were imbedded in slabs of $U(93)O_2(NO_3)_2 \cdot 4H_2O$, which were then built into concrete-reflected parallelepipeds.⁷⁶ In some instances moderation was increased by the insertion of thin Plexiglas sheets. Typically, a 22.67-in.-square core was judged to be critical at a height of 17.7 in. with $H/U = 8.01$, $\rho(U) = 1.058$ g/cm³, and 18.8 vol. % glass. The critical height was judged to be 15.7 in. with $H/U = 11.14$, $\rho(U) = 0.934$ g/cm³, and 16.4 vol. % glass. The corresponding estimated sphere radii were 12.2 in. for the first composition and 11.7 in. for the second. Infinite-cylinder diameters were about 15.7 and 15.2 in., and infinite-slab thicknesses were about 8.2 and 7.8 in.

Figures 40 to 42 are generalizations of the above Raschig-ring data, giving the percentage of dimensional increase per volume percent of glass (at ~15 to 19 vol. % glass) for spheres, infinitely long cylinders, and infinite slabs. These data apply to unreflected cores; thus they are expected to be slight underestimates for water-reflected systems.

Additional ORNL critical data^{6,77-79} and Rocky Flats⁷⁶ subcritical extrapolations on the effects of heterogeneous poisons on solutions containing U^{235} at high enrichments are listed in Table 5. Shape conversions of the Rocky Flats results for parallel boron-steel plates in $U(93)O_2(NO_3)_2$ solutions lead to the following conclusions for essentially unreflected systems. At $H/U^{235} = 82$ and with 1.0-in. plate separation, the steel increases the critical sphere radius by the factor 2.4. At $H/U^{235} = 100$ the corresponding factor is 4.0 with 1.0-in. plate separation and 1.9 with 1.5-in. separation. At $H/U^{235} = 225$, the factor is 3.7 with 1.5-in. separation and 1.4 with 2 $\frac{1}{8}$ -in. separation. The increases of infinite-cylinder diameters are only slightly greater than the above ratios. Corresponding increases of infinite-slab thicknesses are 13 to 24% larger than the sphere-radius factors ($\geq 20\%$ when the sphere factor is ≥ 2).

Data originating at Hanford,⁸⁰ BNL,^{50,81} and Westinghouse Electric Corp. Atomic Power Department⁸² from subcritical lattices of uranium rods of low enrichment poisoned with boron homogeneously distributed in the moderator water are given in Table 6. The amount of boron required for $B^2 = 0$ was determined by interpolation of bucklings measured with different amounts of poison in the moderator.

PLUTONIUM SOLUTIONS

Hanford measurements of the influence of excess nitrate on the critical mass of water-reflected plutonium solutions¹³ are represented in Fig. 43. There is less extensive information from the same source on poisoning by lithium, bismuth, and phosphate.

U^{233} SOLUTIONS

The only experimental data on poisons in U^{233} solutions are from ORNL. It was observed that critical masses of $U^{233}O_2(NO_3)_2$ solutions (with little excess nitrate) are about 1% greater than those of $U^{233}O_2F_2$ solutions¹⁴ over the range $60 < H/U^{233} < 760$.

The following effect of boron on the critical concentration of U(97.7 wt. % U^{233}) in an unreflected 27.24-in.-diameter sphere was measured in another experiment.^{7,2} The critical concentration in the absence of boron was 16.74 g of U^{233} /liter ($H/U^{233} = 1533$); increasing the concentration to 19.37 g of U^{233} /liter ($H/U^{233} = 1324$) required the addition of H_3BO_3 in an amount corresponding to 0.0912 g of boron/liter to retain criticality. This addition represents approximately 0.75 atom of boron per added atom of U^{233} . The required additions to retain criticality appear to be linear over the above U^{233} concentration range.

Table 5 – CRITICAL DIMENSIONS OF U²³⁵ SOLUTIONS WITH HETEROGENEOUS POISONS

Container	Solution	Reflector	Poison	Critical height, in.
15-in.-diameter stainless-steel cylinder ⁷⁸	U(93)O ₂ F ₂ H/U ²³⁵ = 73.0	Water	136 steel rods $\frac{7}{8}$ in. in diameter (49.2 vol.% of core)	37.5
30- by 60-in. aluminum tank ⁷⁷	U(93)O ₂ F ₂ H/U ²³⁵ = 78.7	Water (half reflected)	10 parallel $\frac{1}{4}$ -in.-thick Boral plates containing about 0.3 g of boron/cm ² , clad with $\frac{1}{16}$ -in. stainless steel, and spaced 2.3 in. apart	6.9
10-in.-diameter aluminum cylinder in $\frac{1}{4}$ -in.-thick copper ⁶⁶	U(93)O ₂ F ₂ H/U ²³⁵ = 52.6	Water on sides and bottom	Close-packed copper tubing 1.66 in. in outside diameter and having an 0.22-in. effective wall thickness occupying 33.7 vol.%	60.0
20-in.-diameter stainless-steel cylinder ⁷⁹	U(87)O ₂ (NO ₃) ₂ H/U ²³⁵ = 81.4	Water on sides and bottom	Pyrex tubing or rings ≤ 2 in. in inside diameter and containing ~ 0.28 g of boron/cm ³	
			7.8 vol.% glass	9.75
			9.45 vol.% glass	11.6
			11.5 vol.% glass	13.6
			13.3 vol.% glass	19.6
			13.95 vol.% glass	30.1
				Subcritical at 36 in.
20-in.-diameter stainless-steel cylinder ⁷⁹	U(87)O ₂ (NO ₃) ₂ H/U ²³⁵ = 141	Water on sides and bottom	Pyrex tubing 2 in. in inside diameter and containing ~ 0.28 g of boron/cm ³ 7.8 vol.% glass	12.5
20-in.-diameter stainless-steel cylinder ⁷⁹	U(87)O ₂ (NO ₃) ₂ H/U ²³⁵ = 276	Water on sides and bottom	Pyrex tubing 2 in. in inside diameter and containing ~ 0.28 g of boron/cm ³ 7.8 vol.% glass	Subcritical at 36 in.
35 $\frac{1}{8}$ -in.-diameter steel tank with a $\frac{3}{16}$ -in. wall ⁷⁶	U(93)O ₂ (NO ₃) ₂ H/U ²³⁵ = 82 H/U ²³⁵ = 100 H/U ²³⁵ = 100 H/U ²³⁵ = 225 H/U ²³⁵ = 225	Concrete on base only	Parallel 0.138-in.-thick stainless-steel plates containing 0.080 to 0.094 g of boron/cm ³ and extending $\frac{7}{8}$ in. from bottom and 1 $\frac{1}{2}$ -in. from wall	
			1.0-in.-plate separation	17.3
			1.0-in. plate separation	∞
			1.5-in. plate separation	12.4
			1.5-in. plate separation	∞
2.1-in. plate separation	12.2			

Table 6 – HEXAGONAL LATTICES OF SLIGHTLY ENRICHED URANIUM IN
BORON-POISONED MODERATOR WATER

Center-to-center spacing, in.	Water-to-uranium volume ratio	B^2 unpoisoned, 10^{-6} cm^{-2}	Boron concentration required for $B^2 = 0$	
			g/liter	B/U^{235}
U(1.007) metal (0.925 in. in diameter and 44 in. long clad with 0.997-in.-OD aluminum ⁸⁰)				
1.40	1.37	2794	0.71	0.109
1.50	1.74	3294	0.60	0.117
1.60	2.15	3341	0.57	0.137
U(1.143) metal (0.60 in. in diameter and 48 in. long clad with 0.028 in.-thick aluminum ⁸¹)				
0.854	1	2133	0.82	0.080
0.944	1.5	4023	1.01	0.147
1.027	2	4822	0.83	0.162
1.175	3	4712	0.49	0.145
1.307	4	3603	0.28	0.111
U(1.299) metal (0.60 in. in diameter and 48 in. long clad with 0.028-in.-thick aluminum ⁸¹)				
0.854	1	3211	1.22	0.106
0.944	1.5	5187	1.39	0.180
1.027	2	6108	1.12	0.193
1.175	3	6099	0.69	0.174
1.307	4	5038	0.43	0.148
U(1.143) metal (0.387 in. in diameter and 48 in. long clad with 0.028-in.-thick aluminum ⁵⁰)				
0.568	1	1203	~0.66	~0.066
0.625	1.5	3121	0.91	0.134
0.677	2	4226	0.86	0.171
0.770	3	4618	0.60	0.176
0.854	4	4014	0.39	0.154
U(2.7)O ₂ [0.3 in. in diameter clad with 0.306-in.-ID 0.016-in.-thick stainless steel; $\rho(\text{UO}_2) = 10.18 \text{ g/cm}^3$] (Ref. 82)				
0.404	2.2	4075	1.6	0.149
0.435	2.9	5323	1.5	0.182
0.470	3.9	6326	1.3	0.212

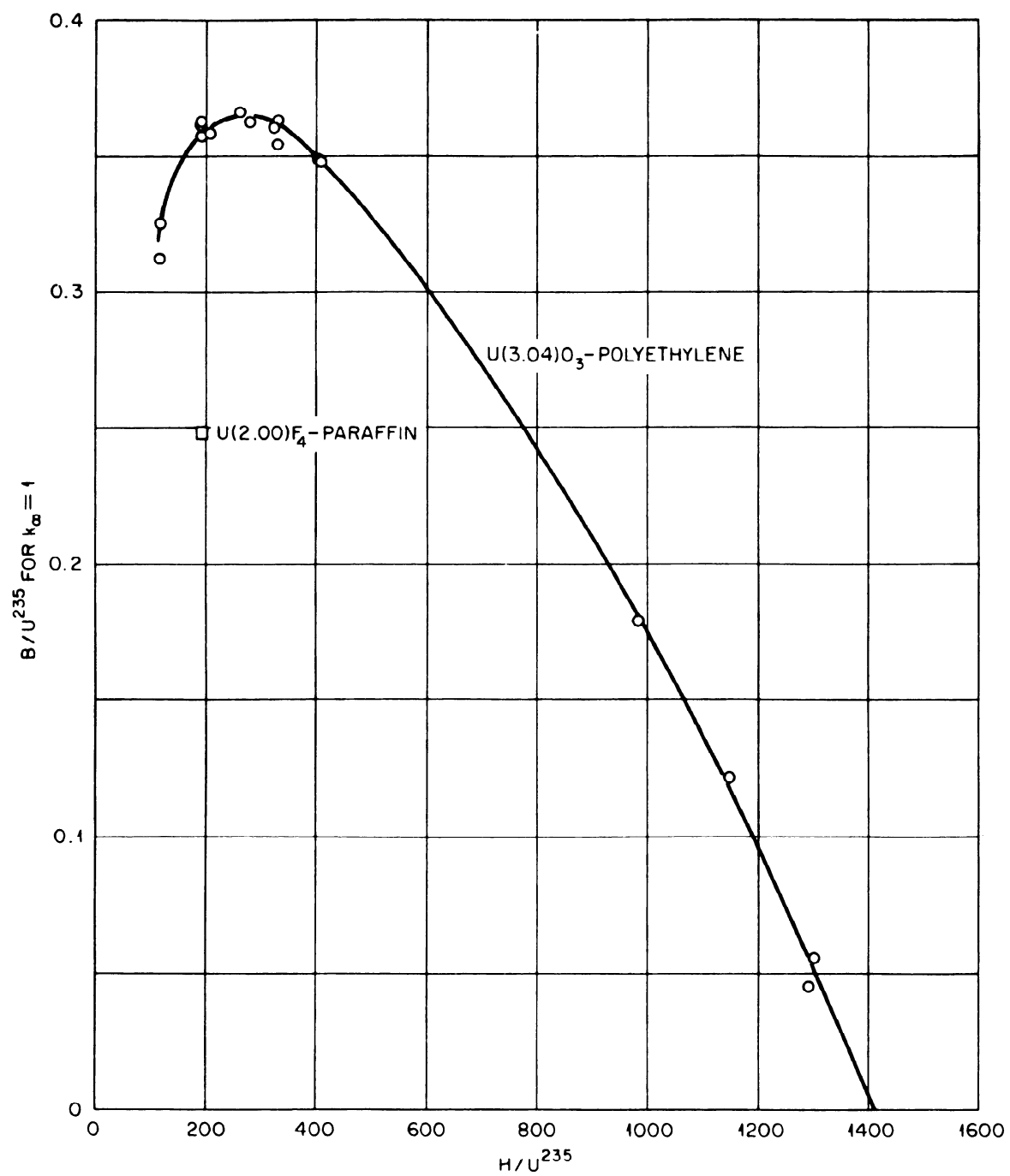


Fig. 38 – Boron concentrations required for $k_{\infty} = 1$. Homogeneous hydrogen-moderated enriched uranium.

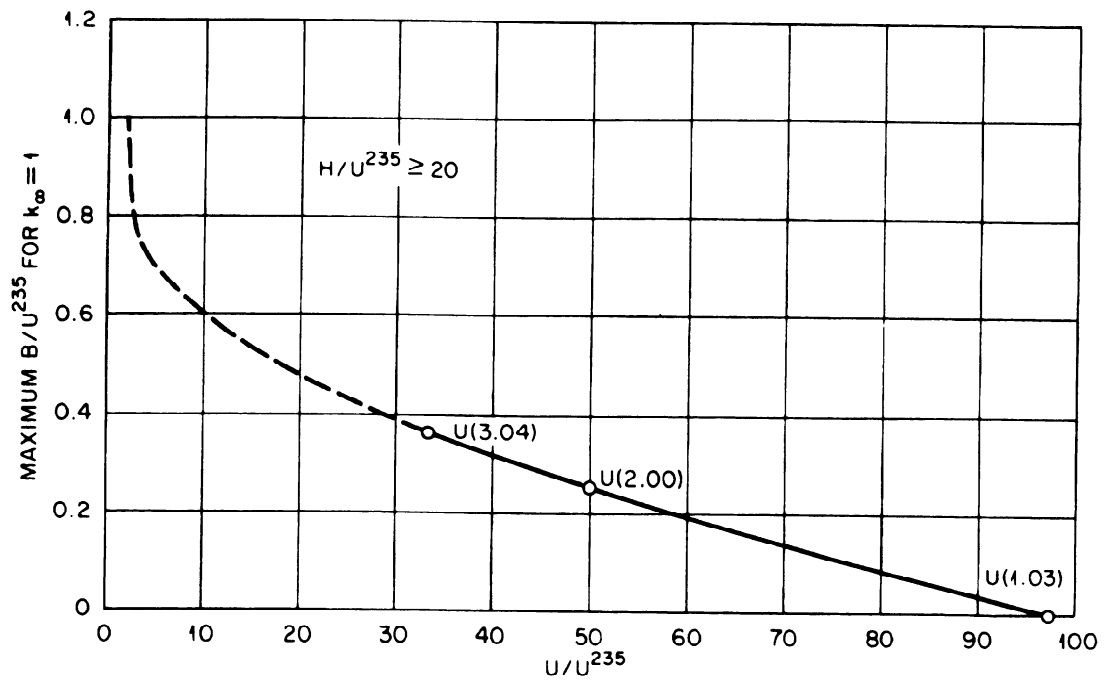


Fig. 39 – Maximum boron concentrations required for $k_{\infty} = 1$. Homogeneous hydrogen-moderated enriched uranium. $H/U^{235} \geq 20$.

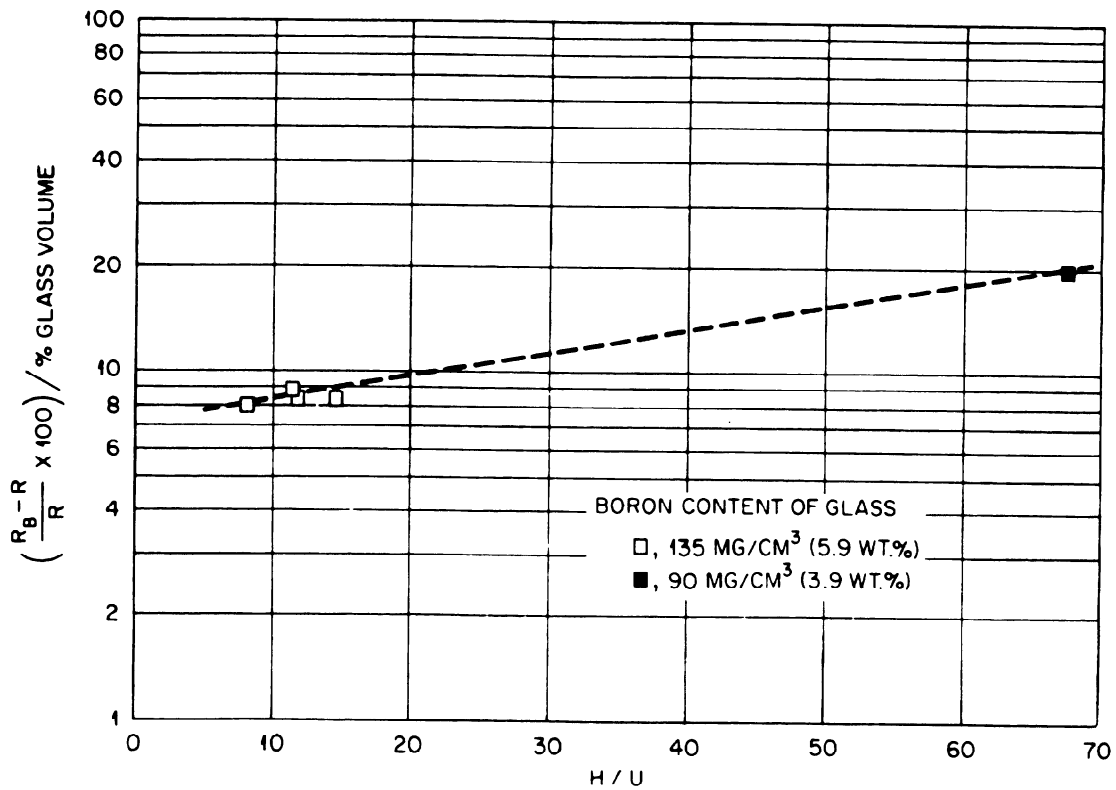


Fig. 40 – Increase of critical sphere radius per percent volume occupied by borosilicate glass in homogeneous hydrogen-moderated U(93).

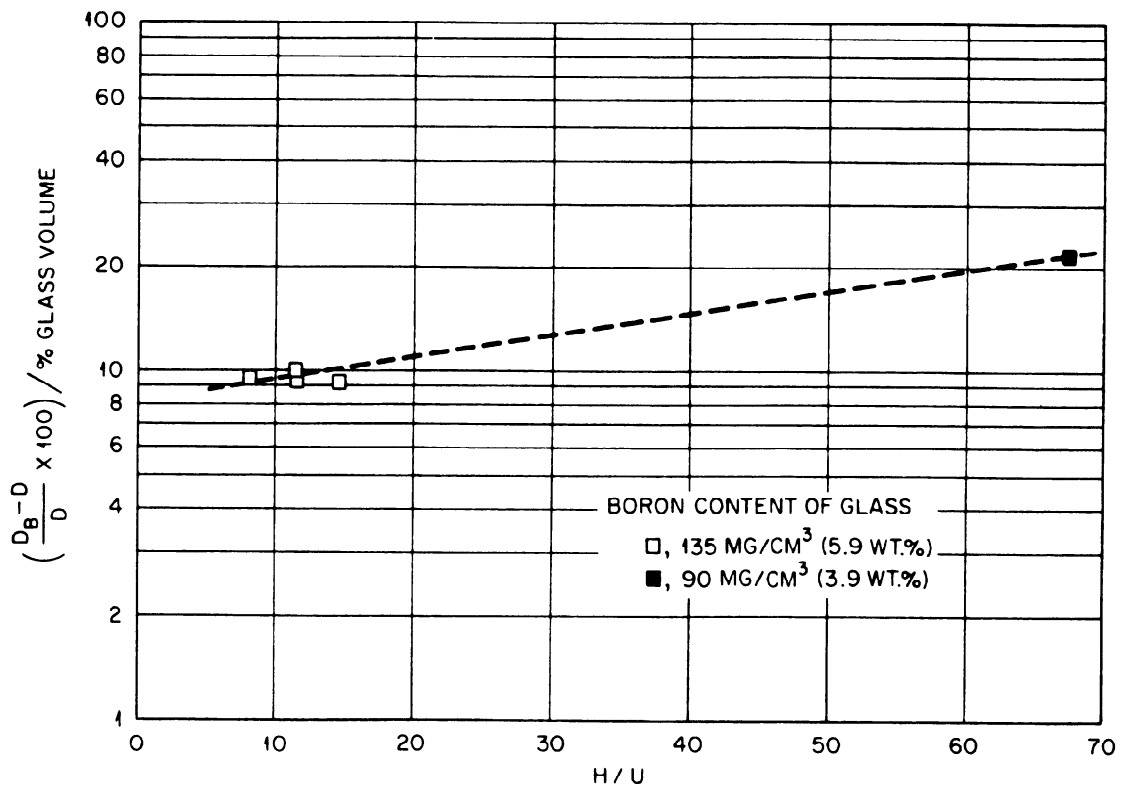


Fig. 41 – Increase of critical infinite-cylinder diameter per percent volume occupied by borosilicate glass in homogeneous hydrogen-moderated U(93).

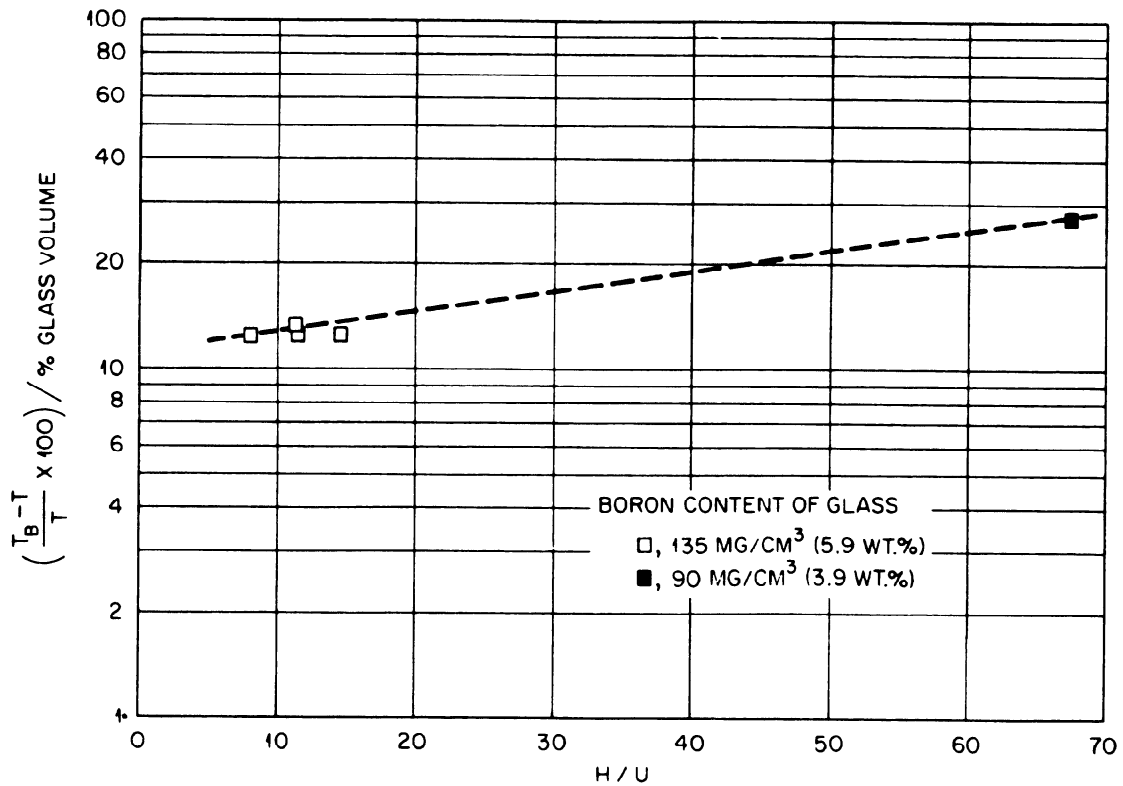


Fig. 42 – Increase of critical infinite-slab thickness per percent volume occupied by borosilicate glass in homogeneous hydrogen-moderated U(93).

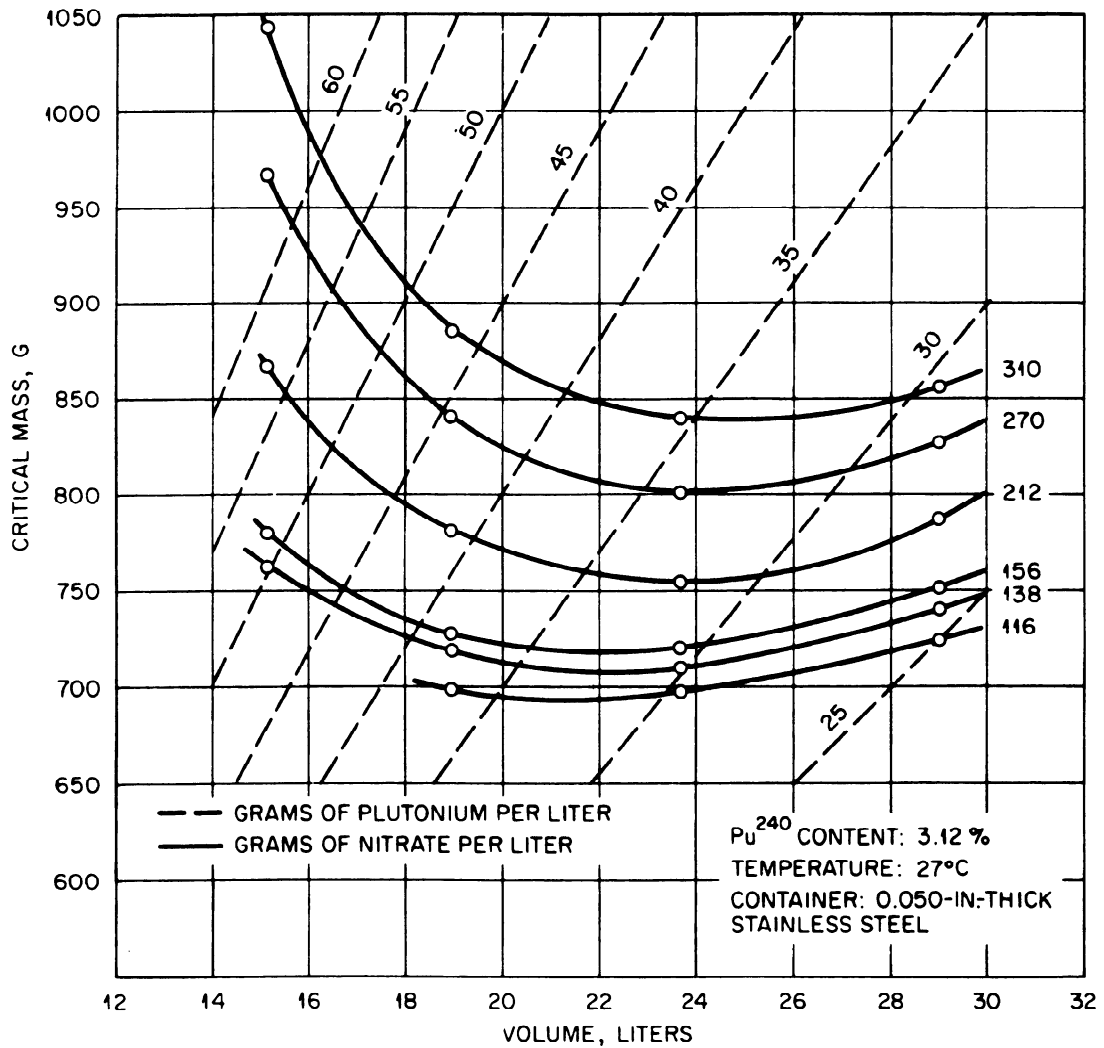


Fig. 43 – Spherical critical masses and volumes of water-reflected aqueous $Pu(NO_3)_4$ solutions with several nitrate concentrations.

Fissile Units with Various Reflectors

REFLECTORS ABOUT URANIUM AND PLUTONIUM METAL

Critical masses of unmoderated U(93.5) metal spheres are given for various reflectors as a function of reflector thickness in Fig. 44 and Table 7. These data originated at LASL^{11,33,83,84} and Livermore.^{36,37}

The critical dimensions of U(45) metal surrounded by various nonhydrogenous reflectors²³ are given in Table 8. Table 9 gives data derived at Aldermaston²³ from U(37.67) metal cubes surrounded with various reflectors. In some cases there was a 0.015-in.-thick cadmium sheet between the core and the reflector.

The less-complete information about reflected spheres of U²³³ metal, δ -phase Pu²³⁹, and α -phase Pu²³⁹ can be represented, as in Fig. 45, in terms of the critical mass of U(93.5) metal enclosed in a reflector of the same composition and thickness. These correlations (based on data from LASL,^{33,65,71,83} Livermore,^{21,67} and supported by Rocky Flats²²), which show no systematic distinction between nonmoderating and moderating reflectors, provide a means of estimating critical masses of the other metals from the more abundant data for enriched uranium.

REFLECTORS ABOUT HYDROGEN-MODERATED UNITS

The reflector savings effected by several materials on assemblies of U(1.42)F₄-paraffin mixtures were measured at Dounreay.¹⁰⁶ The results lead to Fig. 46. Figure 47 shows the ratio of the critical volumes of water-reflected spheres to those of unreflected spheres from the appropriate curves of Figs. 44 and 46 and, from ORNL, reflector savings measurements on U(93)O₂F₂ solutions.⁸

Additional information about reflected and moderated systems includes the Aldermaston subcritical observations²³ that 8-in.-thick Perspex is more effective about U(30.14)O₂-paraffin at H/U²³⁵ = 8.14 and 16.3 than is polyethylene of the same thickness. This comparison is shown in Table 2. According to ORNL critical results²⁶ for U(93)O₂(NO₃)₂ solution at H/U²³⁵ = 62 to 490, there is little influence on critical size if the inner 2½ in. of a thick water reflector is replaced by stainless steel. An infinite reflector of steel is more effective than one of water.

Table 7 – CRITICAL MASSES OF SPHERICAL U(93.5) METAL WITH VARIOUS REFLECTORS

Reflector material	Density, g/cm ³	Critical mass,* kg of U ²³⁵ at a density of 17.6 g/cm ³				
		Reflector thickness, in.				
		0.5 [†]	1 [†]	2	4	Infinite
Beryllium, type QMV	1.84	35.1	28.1	20.8	14.1	
BeO	2.69			21.3	15.5	~8.9
WC	14.7			21.3	16.5	~16.0
Uranium	19.0	35.6	29.3	23.5	18.4	16.1
Tungsten alloy (~92 wt.% tungsten)	17.4	35.8	29.7	24.1	19.4	
Molybdenum (99.8 wt.%)	10.53	36.9	31.0			
Paraffin						21.8
Polyethylene	0.921	38.9	30.8			
H ₂ O				~24.0	22.9	22.8
D ₂ O				~27.0	21.0	~13.6
Cobalt	8.72	36.7	31.2			
Copper	8.88	37.1	31.3	25.4	20.7	
Nickel	8.88	36.4	31.2	25.7	~21.5	19.6
Al ₂ O ₃	2.76	38.9	34.1			
Graphite, type CS-312	1.67	39.3	34.4	29.5	24.2	~16.7
Iron	7.87	38.5	34.1	29.3	25.3	23.2
Zinc	7.04			29.8	25.0	
Thorium	11.48			33.3		
Lead	11.3				29.5	
Aluminum, type 1100	2.70	41.7	38.1	~35.5	~32	<30.0
Titanium (96.5 wt.%)	4.50	41.9	38.6			
Magnesium	1.77	42.5	39.9			

*Some of these masses have been adjusted to the indicated reflector thicknesses by interpolation of the measurements.

[†] These masses were obtained by transformation of results from cylinders.

Table 8 — CRITICAL PARAMETERS OF U(45) METAL WITH VARIOUS REFLECTORS

Reflector		Effective radius, in.	Critical height, in.
Material	Thickness, in.		
None		5.79	7.26 ± 0.03
		8.46	5.47 ± 0.04
Natural uranium ($\rho = 18.83 \text{ g/cm}^3$)	6*	5.79	3.27 ± 0.02
		8.46	2.36 ± 0.02
		11.05	2.01 ± 0.02
Graphite ($\rho = 1.8 \text{ g/cm}^3$)	6*	5.79	3.00 ± 0.02
		8.46	2.14 ± 0.02
		11.05	1.78 ± 0.02
Borated graphite (~5 wt.% boron)	8 [†]	5.79	3.76 ± 0.03
		8.46	2.80 ± 0.03
		11.05	2.40 ± 0.02
Steel ($\rho = \sim 8.0 \text{ g/cm}^3$)	6 [‡]	5.79	4.12 ± 0.03
		8.46	2.98 ± 0.02
		11.05	2.57 ± 0.02
Aluminum ($\rho = \sim 2.75 \text{ g/cm}^3$)	6 [§]	5.79	4.40 ± 0.02
		8.46	3.21 ± 0.02
		11.05	2.72 ± 0.02

*Critical heights corrected for the effect of additional 0.130-in.- and 0.067-in.-thick steel reflector on top and bottom, respectively.

[†]6-in.-thick graphite reflector on sides; additional 0.38-in.-thick aluminum on top and 0.067-in.-thick steel on bottom.

[‡]6.56-in.-thick top reflector.

[§]6.38-in.-thick top reflector.

Table 9 - CRITICAL DIMENSIONS OF U(37.67) METAL CUBES WITH VARIOUS REFLECTORS
(U²³⁵ density = 6.71 g/cm³)

Material	Reflector		Critical dimensions*		
	Density, g/cm ³	Thickness, in.	Length, in.	Volume, liters	Mass, kg of U ²³⁵
Air			10.40 ± 0.08	18.4 ± 0.4	123.6 ± 3.0
Polythene	0.919	1	8.87 ± 0.06	11.4 ± 0.3	76.7 ± 2.0
Polythene	0.919	2	7.90 ± 0.06	8.08 ± 0.2	54.2 ± 1.4
Polythene	0.919	4	7.36 ± 0.06	6.53 ± 0.2	43.8 ± 1.4
Polythene	0.919	6	7.26 ± 0.06	6.27 ± 0.2	42.1 ± 1.8
Polythene	0.919	8	7.223 ± 0.007	6.17 ± 0.02	41.4 ± 0.2
Polythene plus cadmium [†]	0.919	8	9.05 ± 0.06	12.15 ± 0.2	81.5 ± 2.0
Wood	0.73	8	7.94 ± 0.06	8.20 ± 0.2	55.0 ± 1.4
Wood plus cadmium [†]		8	9.00 ± 0.06	11.9 ± 0.2	80.1 ± 2.0
Concrete	2.37	8	7.28 ± 0.06	6.36 ± 0.2	42.6 ± 1.0
Concrete plus cadmium [†]		8	8.07 ± 0.06	8.61 ± 0.2	57.8 ± 1.4
Water		8	7.47 ± 0.06	6.83 ± 0.2	45.8 ± 1.0

*Transformed from measurements with rectangular parallelepipeds.

[†] 0.015-in.-thick cadmium between core and reflector.

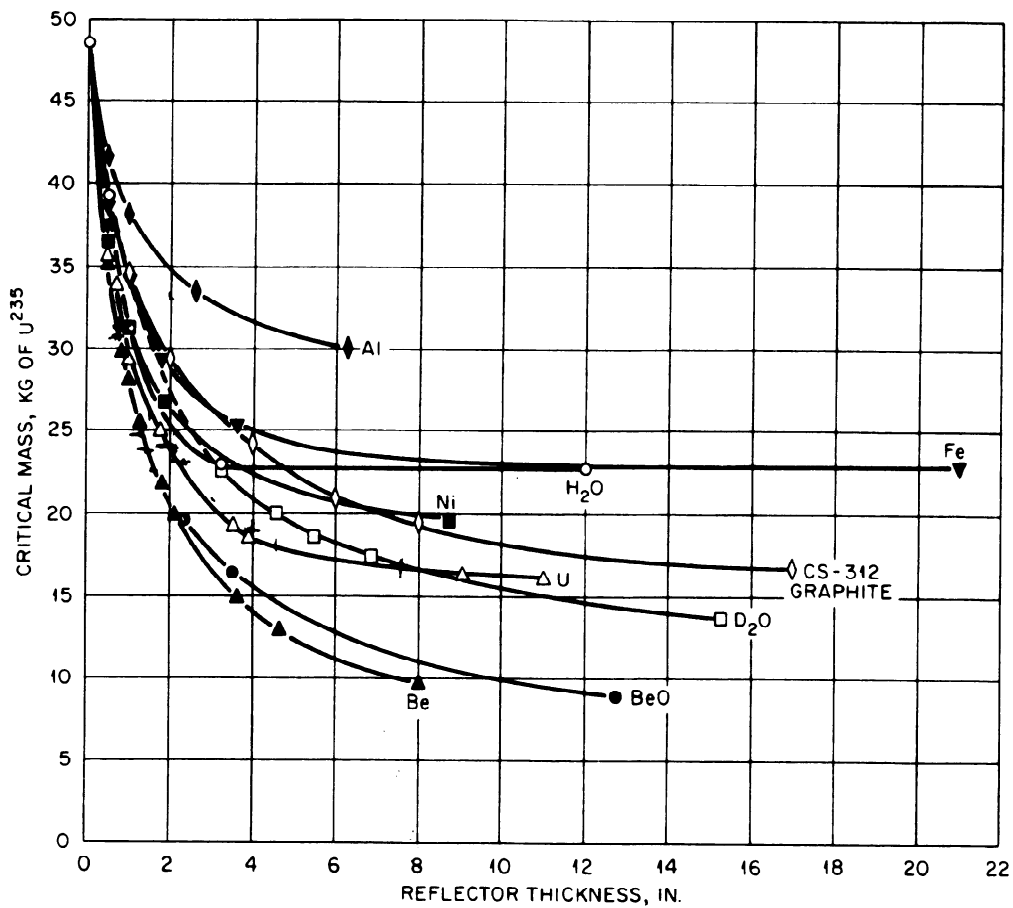


Fig. 44 – Critical masses of U(93.5) metal spheres in various reflectors. $\rho(U) = 18.8 \text{ g/cm}^3$.

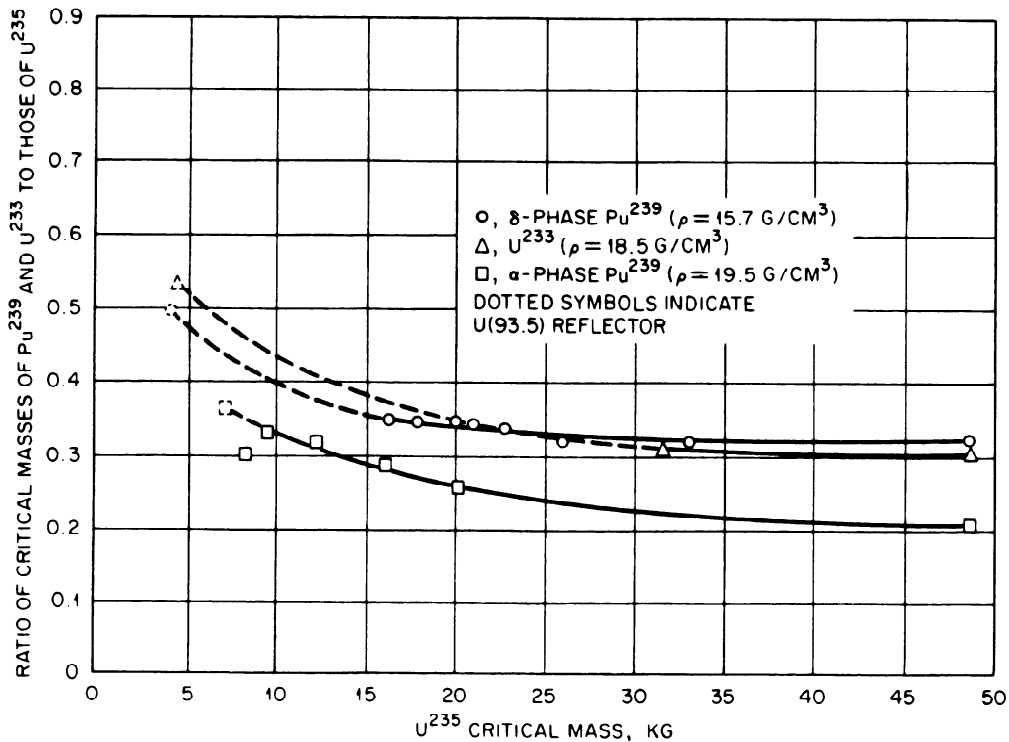


Fig. 45 – Critical masses of spheres of unmoderated Pu²³⁹ and U²³³ relative to those of U(93.5) spheres with the same reflector.

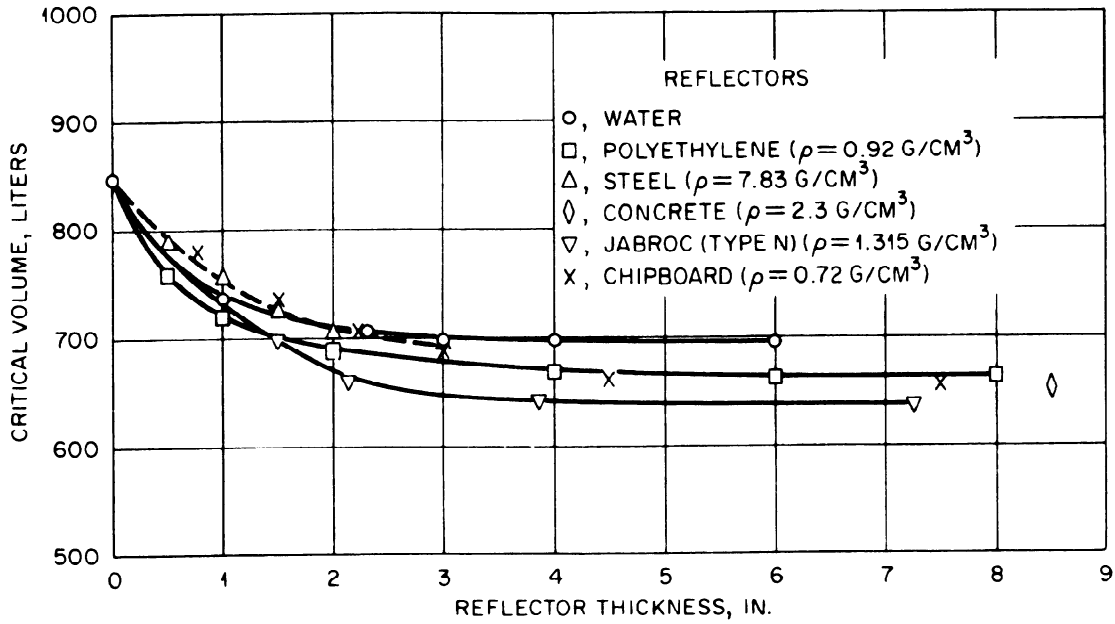


Fig. 46 - Sphere critical volume vs. reflector thickness for $U(1.42)F_4$ -paraffin at $H/U^{2.35} = 422$ and $\rho(U) = 2.5 \text{ g/cm}^3$. (Jabroc is a wood product containing about 45% carbon, 6% hydrogen, and 37% oxygen.)

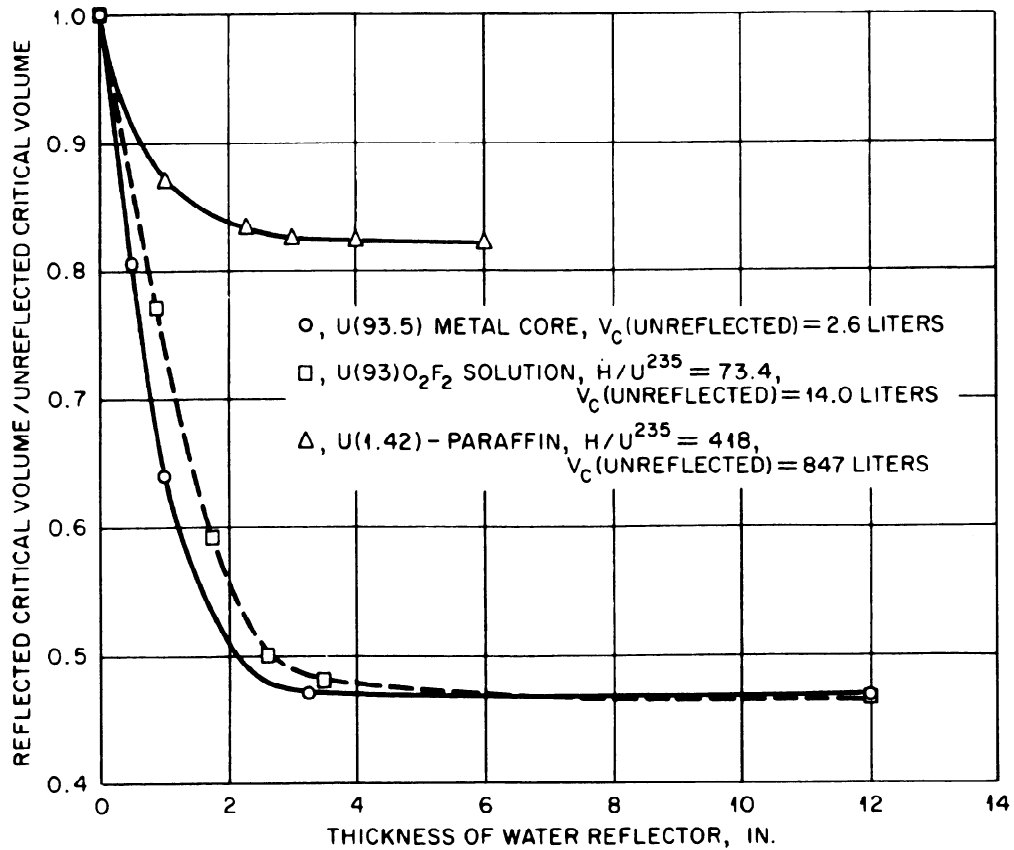


Fig. 47 - Ratio of water-reflected-sphere to unreflected-sphere critical volumes vs. reflector thickness.

Systems with Nonhydrogenous Diluents

“Dilution exponents” for various elements in U(94) metal, both unreflected and reflected by ~9-in.-thick natural uranium, and unreflected δ -phase plutonium spheres have been deduced at LASL from reactivity coefficients.⁸⁵ In terms of the dilution exponent, $n(x)$, the critical mass of fissile material when diluted homogeneously with the volume fraction, F , of the material, x , is

$$m_c = m_{c_0} (1 - F)^{-n} \quad F \ll 1$$

where m_{c_0} is the critical mass of the undiluted system. Although there is no assurance from reactivity coefficient data that dilution exponents hold over an extended dilution range, Figs. 48 to 52 and Fig. 17 show that reasonably constant-power relations hold up to 50 to 70 vol. % of aluminum, iron, copper, zinc, tungsten, thorium, or U²³⁸ when mixed with U(93) or plutonium metal.^{38,86} For the efficient moderators, H₂O, D₂O, beryllium, BeO, and graphite, constant dilution exponents apply to smaller ranges (Livermore,⁸⁷⁻⁹⁰ ORNL,⁹¹ LASL⁹²). Critical masses of spheres of U(93.5) diluted by these materials appear in Fig. 53.

Additional experiments with alternating disks of δ -phase plutonium and graphite have been performed at Rocky Flats.^{93,94} Although the data are applicable in instances where the conditions of the experiment are matched, the gross heterogeneity of the assemblies and their restricted geometry preclude generalization of them.

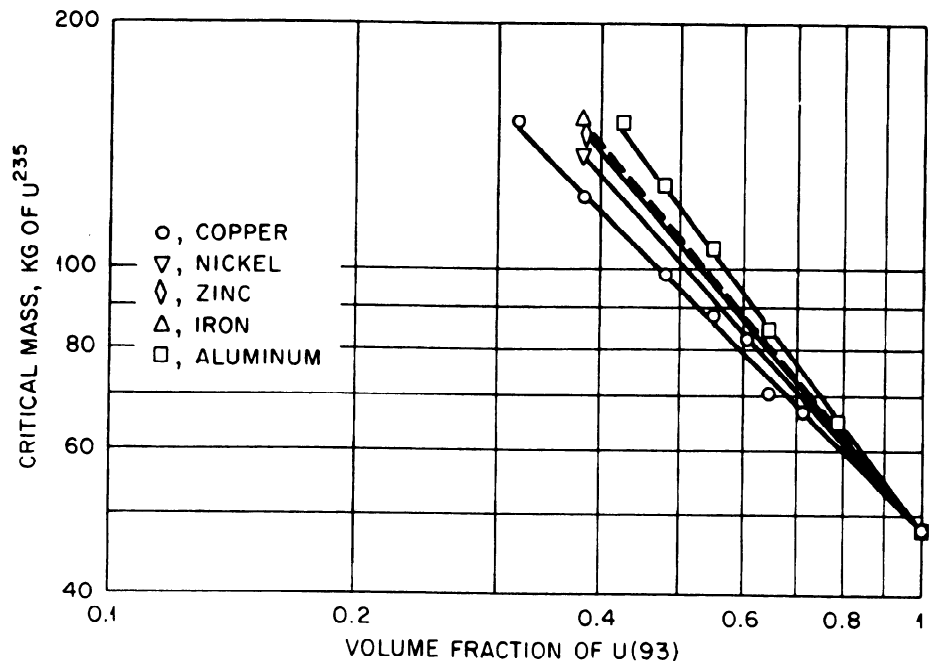


Fig. 48 – Critical masses of unreflected spheres of U(93) diluted with aluminum, iron, nickel, copper, or zinc.

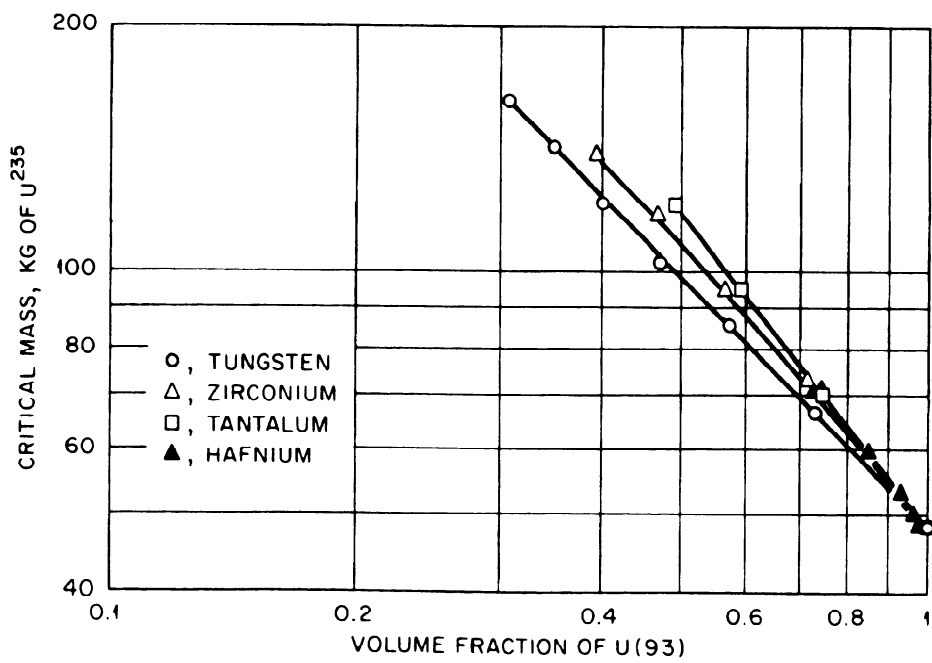


Fig. 49 – Critical masses of unreflected spheres of U(93) diluted with zirconium, tantalum, tungsten, or hafnium.

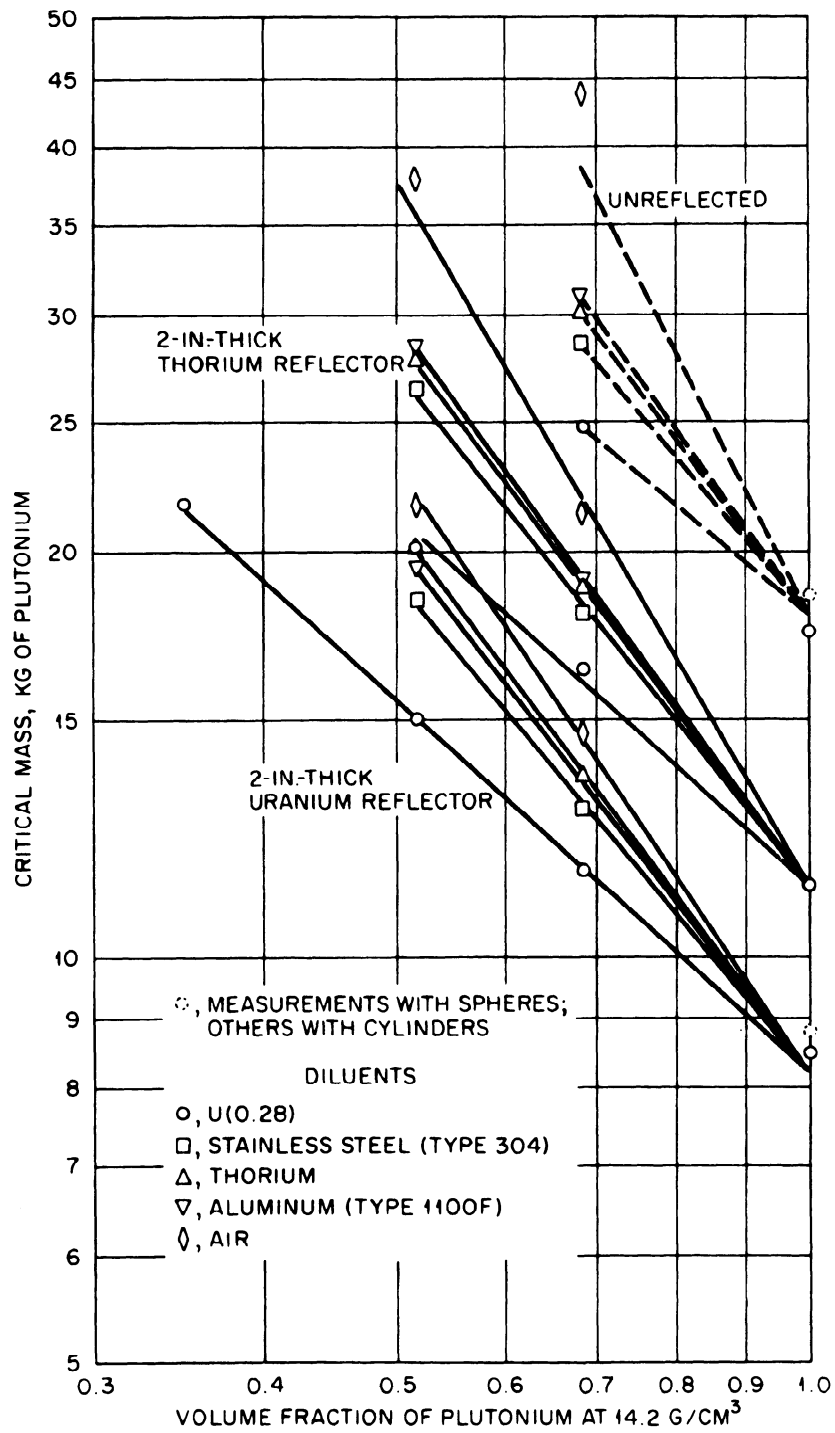


Fig. 50 – Critical masses of δ -phase plutonium spheres diluted with various metals.

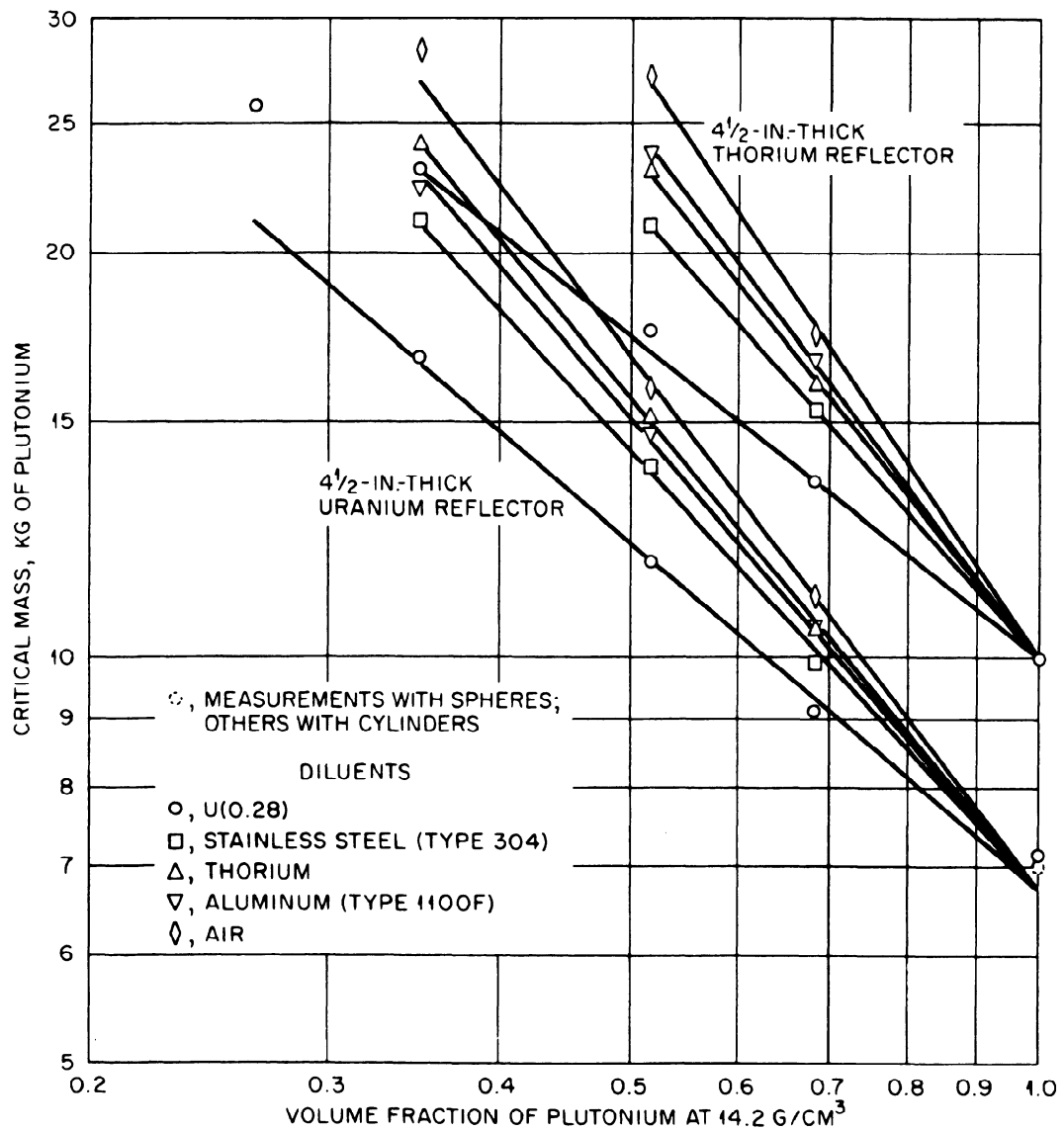


Fig. 51 - Critical masses of δ -phase plutonium spheres diluted with various metals.

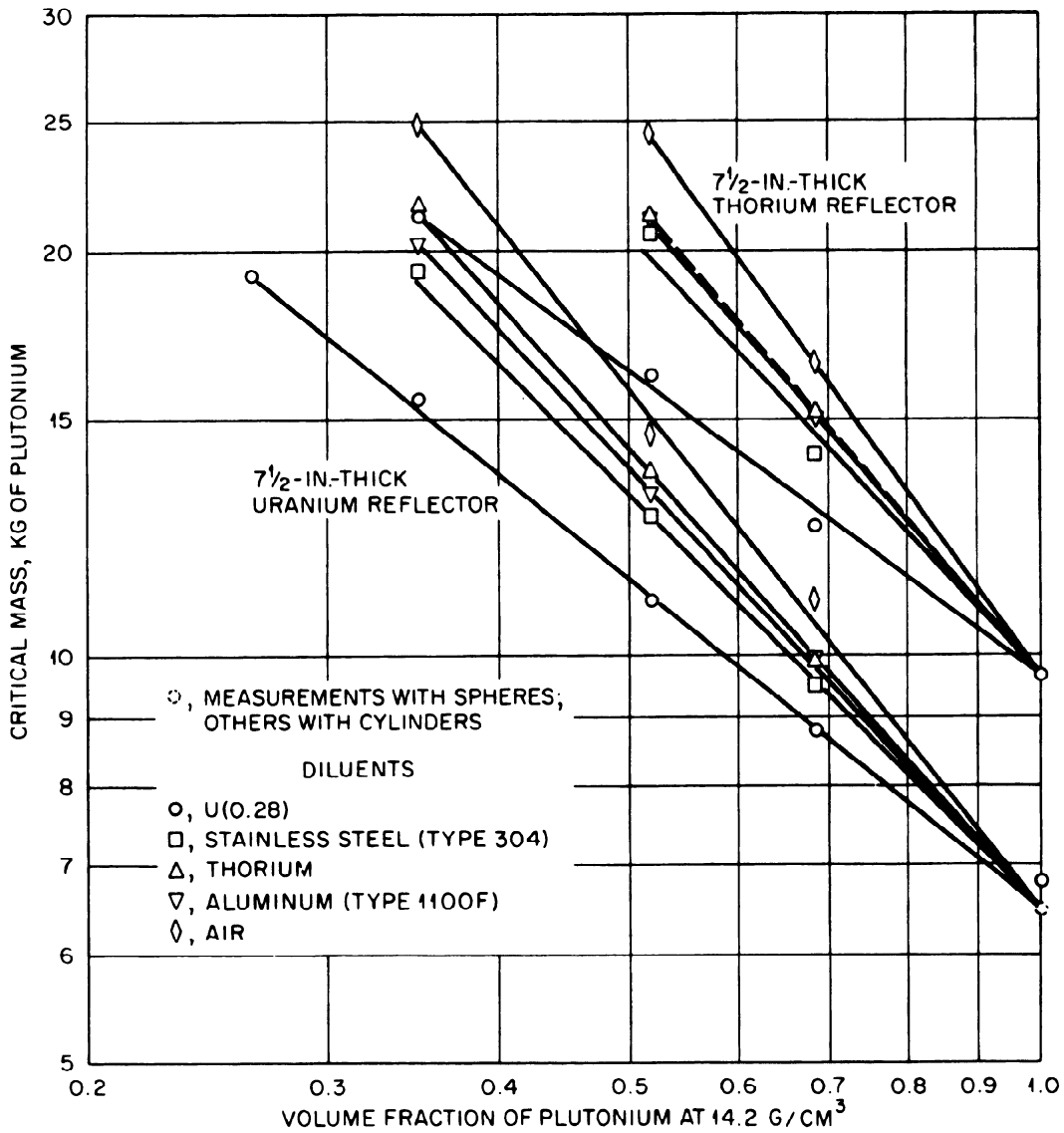


Fig. 52 – Critical masses of δ -phase plutonium spheres diluted with various metals.

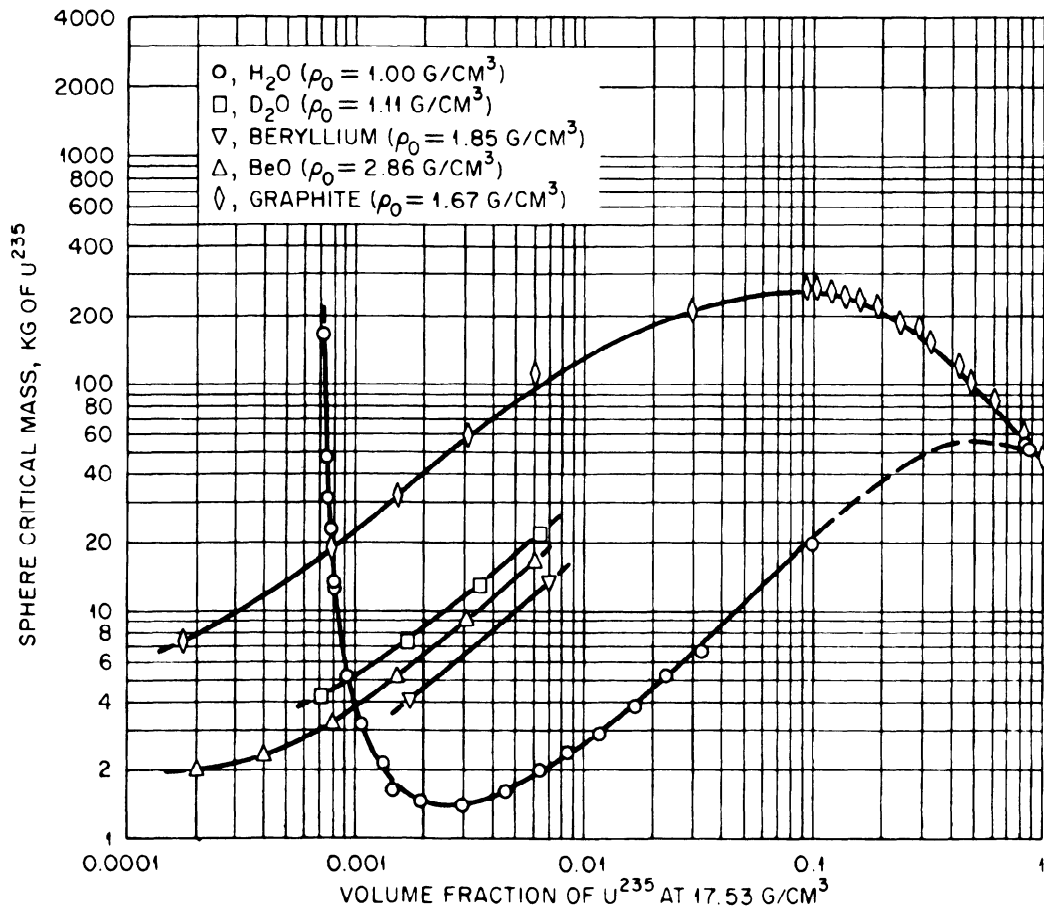


Fig. 53 – Critical masses of unreflected spheres of U(93) diluted with moderators.



Complex Shapes

ANNULI

The possibility of large-volume solution storage in annular cylinders prompted ORNL investigations that led to the critical data given in Figs. 54 and 55 for annuli of $U(93.2)O_2F_2$ aqueous solution with the inner cylinder lined with cadmium and filled with water.⁹⁵ From the results of tests with a solution having an $H/U^{235} = 50.4$ in annuli of a range of dimensions, it was concluded that the critical, infinitely high, externally reflected annulus would have a minimum thickness of between 2.5 and 3 in. (see Fig. 54). Decreasing the solution concentration to an $H/U^{235} = 309$ increased the minimum thickness of the infinitely high reflected annulus to between 3.5 and 4 in. (see Fig. 55). Information was also obtained⁸ for solutions at an H/U^{235} of between 72.4 and 74.6 with the inner cylinder containing either air or water, both with and without a cadmium liner. From these latter data Schuske and Bidinger⁹⁶ have developed empirical equations that permit the calculation of the critical dimensions of infinitely long annuli.

Limited results on critical $U(93)$ metal annuli with various reflectors from LASL^{2,98} and with no reflector from ORNL⁹⁹ are listed in Table 10.

PIPE INTERSECTIONS

Results of ORNL observations on several solution-filled "crosses" and Y-shaped pipe intersections⁸ appear in Table 11. Generalizations of these data have been made by Newlon¹⁰⁰ and by Schuske.^{101,102}

Table 10 - CRITICAL MASSES OF U(²³⁵) METAL ANNULI

Reflector	Core		Critical mass of core, kg of U	Critical height of core, in.
	Outside diameter, in.	Inside diameter, in.		
1-in. natural uranium, complete	12.25	6	82.7 ± 0.3	3.01
3-in. natural uranium, inside filled	12.25	6	55.9 ± 0.3	2.03
3-in. polyethylene, inside filled	12.25	6	60.6 ± 0.3	2.20
2-in. CS-312 graphite, inside filled	12.25	6	78.5 ± 0.3	2.86
2-in. CS-312 graphite, without top reflector (wall 5 in. above core base)	12.25	6	97 ± 2	3.54
1-in. natural uranium in 2-in. polyethylene, complete	12.25	6	54.5 ± 0.3	1.98
1-in. natural uranium, no internal reflector	12.25	6	60.8 ± 0.3	2.21
Thick water, inside filled	5.00	3.85	~160*	~66*
9 ¹ / ₂ -in. graphite, inside not filled	5.00	3.85	Probably subcritical at infinite height	
Thick water, inside filled	6.14	5	~110*	~36*
9-in. graphite, inside not filled	6.14	5	Probably subcritical at infinite height	
Thick water, inside filled	6.14	3.85	31.8 ± 0.5	5.75 ± 0.1
9-in. graphite, inside not filled	6.14	3.85	35.1 ± 0.4	6.36 ± 0.06
No reflector outside or inside the annulus	15	9	205.6 ± 2.8	5.93 ± 0.08
	15	7	179.0 ± 2.5	4.23 ± 0.06
	13	7	165.6 ± 2.9	5.74 ± 0.10

*Uncertain extrapolation since only 43 kg of metal was available for the experiment.

Table 11 - CRITICAL PARAMETERS OF U(\sim 93) SOLUTIONS IN CYLINDRICAL
30° "Y" AND 90° "CROSS" GEOMETRIES

Diameter of cylinder, in.	Geometry	Solution concentration		Critical height,* in.
		H/U ²³⁵	kg of U ²³⁵ /liter	
Effectively infinite water reflector except at top				
4	Cross	44.3	0.538	†
5	Cross	44.3	0.538	5.75
5	Cross	73.4	0.337	7.8
5	Y	73.4	0.337	15.6
No reflector				
5	Y	73.4	0.337	†
5	Cross	73.4	0.337	†
7.5	Cross	44.3	0.538	†
7.5	Cross	72.4	0.342	†

*Height above intersection of the axes.

† Extrapolation of the measurements from a height at least 14 in. above the intersection of the centerlines indicates that the solution in this geometry will not be critical at any height.

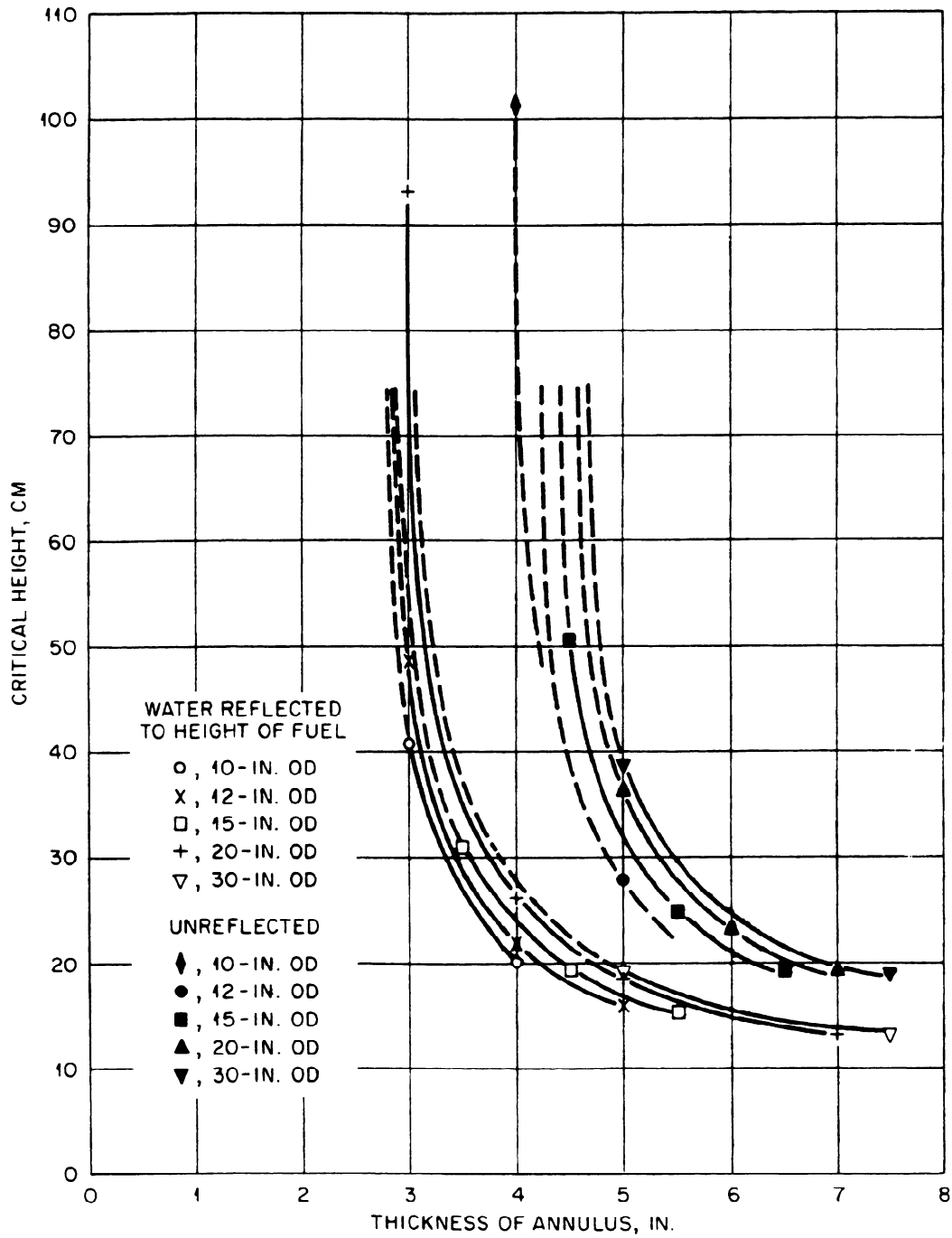


Fig. 54 - Critical heights of cylindrical annuli containing aqueous solutions of $U(93)O_2F_2$ as a function of the thicknesses of the annuli. H/U^{235} atomic ratio = 50.4.

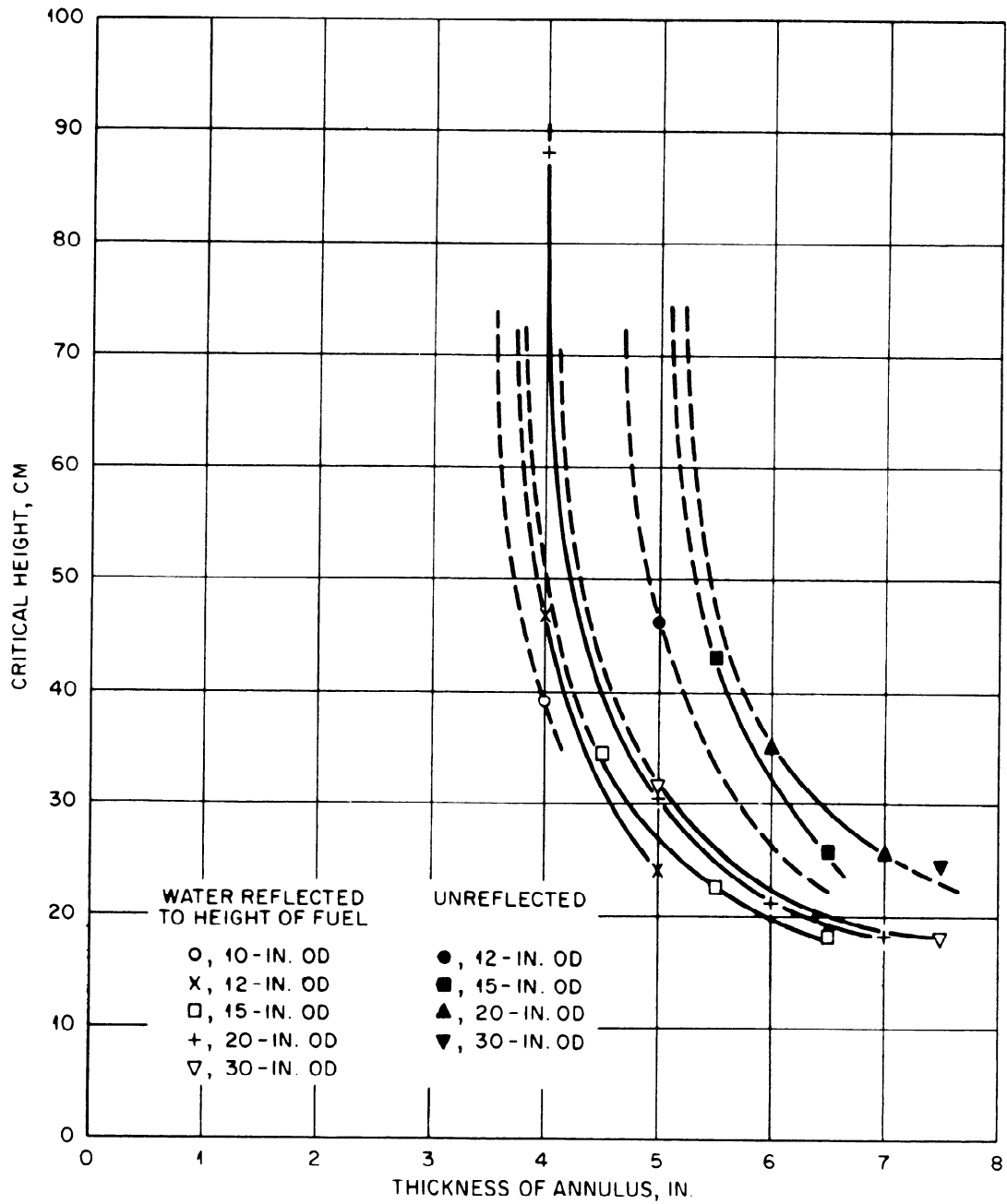



Fig. 55 - Critical heights of cylindrical annuli containing aqueous solutions of $U(93)O_2F_2$ as a function of the thicknesses of the annuli. H/U^{235} atomic ratio = 309.



PART II

MULTIPLE-UNIT

ARRAYS

Although the information presented thus far in this compilation describes the properties of critical single units of the fissile materials, this section concerns critical arrays of units which are individually subcritical under the conditions of the experiment. Unlike the correlations in Part I which, in many cases, are the results of generalization of data by reduction to common geometrical shapes, Part II reports, primarily, data obtained directly from experiment. The available information principally describes arrays of uranium at several enrichments and, for purposes of presentation, is divided into two sections. The first section includes aqueous solutions and other homogeneously hydrogen-moderated materials, and the second describes arrays consisting of metal units. The arrays are designated as being linear, planar, or spatial, depending upon whether the centers of the units establish a line, a plane, or a three-dimensional lattice. Within planar arrays the units were arranged with their centers at the corners of quadrilaterals or at the apexes of triangles, forming square and triangular patterns, respectively; in spatial arrays the units were arranged at the corners of rectangular parallelepipeds. Planar arrays are characterized by their boundaries as being hexagonal, square, rectangular, etc. In general, arrays of units in triangular patterns were hexagonal in outline; those in square patterns were square in outline. Spatial arrays are cubes or parallelepipeds, depending upon their boundaries.

Homogeneously Moderated Units

PLANAR ARRAYS

Cylindrical Units of U(~ 90)

Cylindrical volumes of aqueous solutions of uranyl fluoride and uranyl nitrate highly enriched in U^{235} and of various concentrations have been arranged in critical or near-critical arrays, the common height of the solution within the components of an array being the controlling parameter. Results were obtained with as many as 19 cylinders of solution contained, in most cases, within aluminum vessels arranged in both linear and planar arrays. In some cases water separated the cylinders and surrounded the arrays, thereby serving as both moderator and reflector. It was not practical, however, to provide reflector above the surface of the solution. The aqueous solutions constituting the units of the arrays were, of course, moderated. Cadmium surrounded the cylinders in some experiments. Not all arrays tested were critical, owing to limitations imposed either by the height of the containing vessels or by the available inventory of fissile material.

In addition to these regular arrays, some specialized experiments were performed. In one, for example, two of three cylinders of solution remained fixed, and the third was revolved about the center of one of them, the critical height being measured as a function of the position of the third cylinder. In another, each of two cylinders was solution of a different concentration. The effect of partial and complete reflection by water was determined for another pair of solution cylinders.

The important parameters describing the individual experiments of this series are given in Table 12. The results appear in Figs. 56 to 73, to which reference is made in the table.

Two general conclusions may be drawn from these data: (1) the optimum solution concentration for arrays is approximately that for the minimum critical volume of single unit (Fig. 57), and (2) the presence of cadmium on the lateral surface of units reduces the spacing required for critical moderated arrays (Figs. 59 and 63) but is less effective in unmoderated arrays (Figs. 60 and 64).

The effect on the critical spacing of the insertion of slabs of concrete, water, and Plexiglas between rows of cylindrical units of aqueous uranyl nitrate solution is shown in Fig. 74 (Ref. 115). In some cases this addition may be considered as hydrogenous material placed between two critical arrays. The results serve as estimates of the effectiveness of various materials in isolating arrays. It is clear that partial isolation does take place; particularly, had the arrays been located from the slabs a distance equal to half the surface separation between units in an array, the coupling between arrays would have been further reduced. When the units are in contact with the slab, its effect is, in some cases, almost equivalent to half the array. The lower three diagrams show that concrete is a better reflector than water.

Several additional sets of experiments were performed at ORNL in which the volume and composition of each cylindrical unit of fissile solution were unchanged throughout a set.^{111,116}

The material comprising each unit was an aqueous solution of $U(92.6)O_2(NO_3)_2$. The chemical concentration of the solution was varied between sets of experiments. Although in each case the solution was contained in polyethylene cylinders, the capacity of the containers and the extent to which they were filled were also varied. The two types of containers available for these experiments are described in Table 13 and in Fig. 77. Nonuniformity in the thickness of the walls of most of the containers adds to the uncertainty in the analysis of the results but does not detract from their application to safety problems associated with these widely used containers. In one set of experiments,¹¹⁵ reported in Table 13, the individual units were arranged in contact in both linear and planar arrays. In another set, the critical surface separation (equal in two directions) of the cylinders was determined for each of several heights of solution in its containers.¹¹¹ In most of these experiments the containers were arranged with their bases in a horizontal plane. For one test, however, containers were arranged coaxially in two tiers in the manner illustrated in Fig. 75, thereby producing an array of effectively elongated units. The units were arranged in a square pattern for all but one measurement. The results of this set of experiments are given in Figs. 76 and 77. It is noted in Fig. 76 that a triangular pattern of units is more reactive than the same number of units in a square pattern.

The effect of a methacrylate plastic, Plexiglas, as a reflector and moderator of these arrays of solution cylinders is reported in Table 14 (Ref. 111). [Information on Plexiglas as a moderator of arrays of $U(93)$ units of metal is given in the subsection "Spatial Arrays."]

In another set of experiments an array of units was constructed inside a tank to which water could be added to provide a neutron reflector and moderator.¹¹¹ An array of 36 units, constructed within the tank at the spacing observed in a critical array of the same size located outside the tank, was found to be supercritical owing to the reflection of neutrons by the tank wall. Nevertheless, this pattern was basic to a study of water moderator and reflector. The results are given in Table 15 and include those from a test of the effect of a spray from a fire-hose nozzle. The criticality index in these experiments was the height of solution in five centrally located units.

A critical 6 by 6 unit array of cylindrical units of solution, arranged in a square pattern, served as a relative reactivity measuring device in a set of experiments in which various materials were substituted for one of the units.¹¹⁵ The change in solution height in five of the units, serving as a control, necessary to maintain criticality was observed. The results are presented in Table 16 together with a schema of the array locating the controlling units and the position where the substitutions were made.

The relative effect on reactivity due to displacement of units within an array was observed¹¹⁵ in two experiments with unreflected and unmoderated arrays. In the first, a row of units adjacent to the peripheral row of a 5 by 5 unit array was rotated through 90 deg, causing the axes of these cylinders to be horizontal instead of vertical. Since the boundary of the array was a cube, the rotation of these five units did not alter the shape or dimensions of the array. In the second case, an entire half of a 6 by 6 unit array was displaced from the other half, in a direction parallel to the midplane, a distance equal to one-half the center spacing of the units, preserving the original spacing within each half. In both cases the reactivity was decreased. The data are given in Table 17.

Two additional planar arrays with 5-liter units of aqueous $U(92.6)O_2(NO_3)_2$ solution were reported from ORNL.¹¹⁷ Each unit was a right-circular cylinder 19.04 cm in diameter and 17.77 cm high contained in a 0.64-cm-thick methacrylate plastic (Plexiglas, $\rho = 1.18$ g/cm³) vessel. The U^{235} concentration of the solution was 384 g/liter, and the U^{235} content of a unit was 1.92 kg. In one array, 19 units arranged with the centers in a triangular pattern were critical, unreflected, at a surface separation of 1.35 cm. In the other, four units in a square pattern with a surface separation of 3.94 cm were critical when reflected by 15.2-cm-thick paraffin.

Source neutron multiplication measurements¹¹⁸ with a planar array of as many as 25 cylinders of aqueous $U(90)O_2(NO_3)_2$ solution, arranged in a square pattern having a 61-cm center spacing and bottom reflected by a concrete floor, indicated that an unlimited number of units would be subcritical. The U^{235} concentration of the solution was 300 g/liter, corresponding

to an H/U^{235} of 75. The containers were double-walled steel, spray-coated with cadmium, 14.7 cm in inside diameter, and 108.0 cm long. Each held 4.1 kg of U^{235} .

Slab Units of $U(\sim 90)$

Slabs of aqueous $U(93.2)O_2F_2$ solution were arranged with their bases in the same plane and their surfaces of maximum area either parallel or perpendicular in order to study interaction between large areas of fissile material.¹⁰⁸ The aluminum vessels containing the solution were 120.6 cm wide and either 7.6 or 15.1 cm thick; the solution height was adjusted for criticality under the various experimental conditions. Data from several combinations and arrangements of these slabs are shown in Figs. 78 to 81; the environment varied from no moderator or reflector to complete submersion of the array to the height of the solution in the containers. Figure 82 shows the data resulting from the arrangement of these solution slabs in "T" and "L" geometries.

In experiments exploring the effect of hydrogenous material separating units of fissile material,¹¹⁹ a pair of unreflected, parallel slabs of aqueous $U(93.2)O_2F_2$ solution, about 15 cm thick, were separated 30.5 cm. Plexiglas of increasing thickness was first centered between the slabs and then placed on the inside facing surfaces. Figure 83 shows the common solution height of the two slabs as a function of the total thickness of moderator between the slabs. The two curves must meet, of course, when the space between the slabs is completely filled with the moderator.

Combinations of Slab and Cylindrical Units of $U(\sim 90)$

Data¹¹⁴ describing critical planar arrays of a slab and a cylinder of aqueous $U(93.2)O_2F_2$ solutions, unreflected and unmoderated, are given in Table 18. The U^{235} concentration of the cylinders remained constant throughout the experiments, while that of the slab was varied. Results of interaction between a slab and a cylinder of solution of the same concentration,¹⁰⁸ unreflected and partially reflected, are shown in Fig. 84.

Units of $U(<90)$

The critical dimensions of an unreflected array of two slabs dissimilar both in U^{235} enrichment and in chemical composition were determined at ORNL as a function of their separation.¹²⁰ One of the slabs was an aqueous solution of $U(93.2)O_2F_2$ having a U^{235} concentration of about 480 g/liter, corresponding to an H/U^{235} of 50.1. The slab was vertical, 120.6 cm wide, 7.6 cm thick, and of variable height. The second slab, parallel to the first, was constructed of blocks of an homogenized mixture of CF_2 and $U(37)F_4$. The uranium density was 3.1 g/cm^3 , and absorbed moisture accounted for an H/U^{235} of 0.1. In some experiments Plexiglas was placed between adjacent layers of blocks and between adjacent rows in one direction, producing an H/U^{235} of 16. The critical height of the solution slab as a function of the surface separation of the two components is shown in Fig. 85 for various slabs of $U(37)CF_6$ blocks.

Dounreay¹⁰⁶ is the source of data on a pair of slabs of an aqueous solution of $U(30.45)O_2F_2$. Each slab was 6.09 cm thick and 120 cm wide. They were filled to the common height necessary for criticality when the space between the two was occupied by various materials. Figure 86 shows the data for a solution at a uranium concentration of 599 g/liter (U^{235} concentration of 182 g/liter) corresponding to an H/U^{235} of 130, and Fig. 87 gives more limited results for a uranium solution concentration of 375 g/liter corresponding to an H/U^{235} of 214.

Neutron interaction between two identical parallelepipeds of homogeneous $U(30.14)O_2$ - paraffin (CH_2) reflected by 20.3-cm-thick polyethylene on all sides except the facing ones was

investigated at Aldermaston.²³ The mixture had a U^{235} density of 0.331 g/cm^3 and an H/U^{235} of 81.8. The parallelepipeds were 20.3 by 20.3 cm in cross section and of variable, but common, height. The critical separation was determined as a function of the separation of the two components and of the material between them. The experimental arrangement and data are shown in Fig. 88.

Measurements have also been made at Dounreay¹²¹ on two identical rectangular parallelepipeds of $U(1.42)F_4$ -paraffin, containing 2.5 g of uranium/ cm^3 , with an H/U^{235} of 420. The parallelepipeds were reflected by 20.3-cm-thick polyethylene on all sides except the facing ones. The critical separation was measured as a function of the total uranium in both components under two different conditions: with the facing areas constant and the lengths of the two components of the array varied, and with the base areas constant and the heights varied equally. The critical separation and dimensions are given in Table 19. Figure 89 shows the influence of separation on the critical mass under the two conditions. It can be seen that with a separation of 10 cm, the mass in each component is 80% of the critical mass of a completely reflected single array.

Also shown in Fig. 89 are results¹²¹ obtained with a similar $U(1.42)F_4$ -paraffin mixture containing 2.22 g of uranium/ cm^3 with an H/U^{235} of 562. These parallelepipeds were constructed with a constant facing area and with a 20.3-cm-thick polyethylene reflector on all surfaces except the facing areas. This moderation is approximately that yielding minimum critical mass.

Data on the neutron interaction of two homogeneously moderated units, one a slab of $U(30.45)O_2F_2$ solution 6.1 cm thick and the other a parallelepiped of $U(1.42)F_4$ -paraffin, separated by various materials, originated at Dounreay.¹⁰⁶ Table 20 and Fig. 90 show the experimental arrangement and the results.

Plutonium Solutions

An experiment to determine the safety of spaced cylinders of plutonium solution was performed at Rocky Flats.¹²² The $Pu(NO_3)_4$ solution, containing 5N excess HNO_3 , at a concentration of 400 g of plutonium/liter was held in stainless-steel pipe 14.1 cm in outside diameter and 12.8 cm in inside diameter. Five 19-liter vertical cylinders were arranged in a linear array with a 61-cm center spacing and located about 5 cm above a thick concrete slab. Benelex (pressed wood chips impregnated with a hydrocarbon plastic), with a density of 1.44 g/cm^3 and 10.2 cm thick, was located 15.3 cm from the sides of the array and 150 cm from the top of the solution. The array was apparently quite subcritical.

The neutron interaction between two identical annuli of plutonium nitrate solution in stainless-steel containers has been studied at the Station de Criticalite'¹²³ in France. The internal and external diameters of the containers were 30.0 and 50.0 cm, respectively. Three different solution concentrations were reported, with various conditions of reflection. Figures 91 to 93 show the results. No corrections were made for the effect of the Pu^{240} , of the NO_3^- , of the containers, or of temperature.

SPATIAL ARRAYS

The critical dimensions of spatial arrays of as many as 125 five-liter units of aqueous $U(92.6)O_2(NO_3)_2$ solution were reported from ORNL.¹¹⁷ Each unit was a right-circular cylinder 19.04 cm in diameter and 17.77 cm high contained in a 0.64-cm-thick methacrylate plastic (Plexiglas, $\rho = 1.18 \text{ g/cm}^3$) vessel. The U^{235} concentration of the solution was 384 g/liter, and the U^{235} content of a unit was 1.92 kg.

Unreflected arrays of 8, 27, 64, and 125 units with centers located at the corners of rectangular parallelepipeds were constructed with equal numbers of units in the three dimensions.

Arrays of 8 and 27 units were assembled with paraffin reflectors ($\rho = 0.93 \text{ g/cm}^3$) as thick as 15.2 cm and with Plexiglas reflectors 1.3 cm thick. In these arrays the reflector was located at the lattice cell boundary of the peripheral units, where a lattice cell is one occupied by a single unit. The data are given in Table 21. That similar units of lower concentration are less reactive is shown by the results in Table 22.

Table 12 – EXPERIMENTS WITH ARRAYS OF VARIABLE-HEIGHT CYLINDERS OF U (~ 90) SOLUTION

Cylinder diameter		Number of cylinders in array	Solution concentration, $H/U^{2.35}$	Array moderator and reflector	Figure No.	Reference No.
in.	cm					
5.0	12.7	7	50	None	56	108
5.0	12.7	2,6,7	50	Water	56	108,109
6.0	15.2	2	30–330	Water	57	109
6.0	15.2	2–6	44	Water	58	8
6.0	15.2	3,7	44	Water	59	8
6.0	15.2	7	44,309	None	60	8,110
6.0	15.2	7,9,16,19	59	None	61	111
8.0	20.3	2–5	44	None	62	8
8.0	20.3	2–5	44	Water	62	8
8.0	20.3	2,3,7	30–53	Water	63	8,109
8.0	20.3	3,7	44,309	None	64	8,110,112
9.5	24.1	2–7	297	None	65	110
10.0	25.4	2	50–325	None	66	109
10.0	25.4	2	30–329	Water	66	109
15.0	38.1	2	169,329	None	67	109
15.0	38.1	2	53–329	Water	67	109
20.0	50.8	2	169,329	None	68	109
11.8	30.0	2	260–520	None	69	113
6.0	15.2	3	44	Water	70	8
8.0	20.3	3	44	None	71	8
8.0	20.3	3	44	Water	71	8
10.0	25.4	2	50–328*	None	72	108
10.0	25.4	2	59	Partial water	73	114
10.0	25.4	2	59	Water	73	114

*These are arrays of dissimilar cylinders.

Table 13 - LINEAR AND PLANAR ARRAYS OF CYLINDERS OF $U(92.6)O_2(NO_3)_2$ SOLUTION

Description of Solution:		Description of Container:	
U^{235} concentration	384 g/liter	Composition	Polyethylene (CH ₂)
H/ U^{235}	59	Height (inside)	112.4 cm
Specific gravity	1.55	Diameter (inside) (av.)	11.9 cm
		Wall thickness	0.51-1.14 cm
		Nominal capacity	13 liters

Number of units in array	Array and pattern*	Solution height, cm	Comments
18	Linear	112.4	Not critical with one side of array reflected by 15.2-cm-thick Plexiglas
19	Linear	112.4 [†]	Critical with one side of array reflected by 15.2-cm-thick Plexiglas
3	Planar, triangular	112.4	Not critical
4	Planar, square	112.4	Not critical
4	Planar, triangular	87.8	Critical
5	Planar, triangular	39.9	Critical
5	Planar, cross with four cylinders in contact only with center one	112.4	Not critical

*Surfaces of containers in contact.

[†]The five central units were filled to only 105.9 cm.

Table 14 – PLANAR ARRAYS OF CYLINDERS OF AQUEOUS $U(92.6)O_2(NO_3)_2$ SOLUTION
REFLECTED AND MODERATED BY PLEXIGLAS

Description of Units:		Description of Arrays:	
Height	112.4 cm	4 × 4 units in square pattern	
See Table 13 for additional description		Density of Plexiglas	1.18 g/cm ³
Thickness and location of moderator	Array reflector* thickness, cm	Surface separation of units, cm	Number of units in unreflected and unmoderated critical array at the same spacing (from Fig. 76)
1.3-cm shell closely surrounding each unit	0	12.0	26.5
1.3-cm layer centered between adjacent units	0.6	12.6	28.5
2.5-cm layer centered between adjacent units	0	12.2	27.3
2.5-cm layer centered between adjacent units	1.3	13.7	33.0
3.8-cm layer centered between adjacent units	1.9	13.4	31.8

*It is noted that the array reflector is constituted by the outer walls of the plastic matrices in which the peripheral units were centered.

**Table 15 – PLANAR ARRAYS OF CYLINDERS OF AQUEOUS $U(92.6)O_2(NO_3)_2$ SOLUTION
REFLECTED AND MODERATED BY WATER**

Description of Units:

Height 112.4 cm except as noted
See Table 13 for additional description

Description of Array:

Basic 6 × 6 units in square pattern
14.33 cm surface separation*

Description of Water Tank:

Diameter 2.74 m
Height 3.04 m

Number of units in array	Critical height of five central units, cm	Comments
36	73.7	No water reflector or moderator
30 [†]	88.1	No moderator; bottom reflector only
36	112.4	Fully submerged; subcritical
32.6 [†]	100.8	No water reflector or moderator
32 [†]	99.5	Water sprayed over array at a rate of 66.8 liters/min

*The basic array was critical at this separation when mounted outside the water tank.

†These were 5 × 6 unit arrays with necessary additional units in the sixth row.

Table 16 – CHANGES IN CRITICAL DIMENSIONS OF AN ARRAY UPON REPLACEMENT OF A UNIT BY VARIOUS MATERIALS

Description of Solution and Containers:

See Table 13

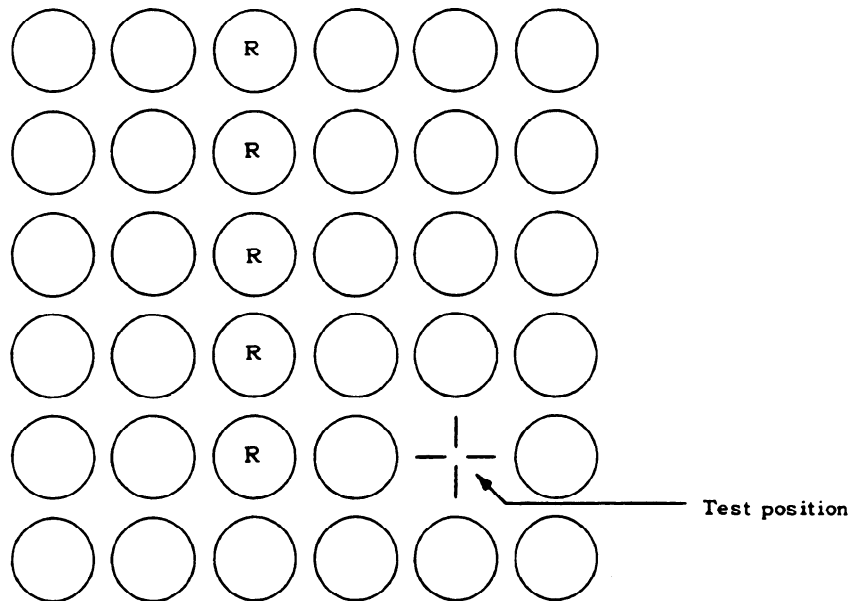
Description of Array:

6 × 6 units in square pattern

Surface separation 13.5 cm

Units designated "R" are the variable-height controls

All other units are 112.4 cm high



Description of material in test position	Critical solution height in five control units, cm
None	95.3
112.4-cm-high $U(92.6)O_2(NO_3)_2$ solution at a uranium concentration of 410 g/liter in a 13-liter capacity container*	74.8
112.4-cm-high $U(92.6)O_2(NO_3)_2$ solution at a uranium concentration of 410 g/liter in a 15-liter capacity container [†]	71.5
112.4-cm-high $U(37.1)O_2F_2$ solution at a uranium concentration of 516 g/liter in a 13-liter capacity container*	77.4
112.4-cm-high $U(37.1)O_2F_2$ solution at a uranium concentration of 516 g/liter in a 15-liter capacity container [†]	75.4
112.4-cm-high H_2O in a 13-liter capacity container*	91.4

Table 16 – (continued)

Description of material in test position	Critical solution height in five control units, cm
112.4-cm-high D ₂ O (99.7 at. % D) in a 13-liter capacity container*	87.2
10.2 × 10.1 × 111.8 cm column of U(2)F ₄ -paraffin (92 wt. % UF ₄ , ρ = 4.5 g/cm ³ , H/U ²³⁵ = 194)	87.4
10.2 × 10.2 × 111.8 cm column of U(3)F ₄ -paraffin (92 wt. % UF ₄ , ρ = 4.5 g/cm ³ , H/U ²³⁵ = 133)	86.6
* * * * *	

Samples were contained in two stainless-steel beakers 17.2 cm in diameter and 19.1 cm high. The beakers were mounted coaxially. The distance between centers of samples was 38.1 cm, and the center of gravity of the samples was at the midheight of the adjacent solution units.

Contents of beakers

(a) Empty	94.5
(b) Filled to depth of ~18 cm with U(93)F ₄ powder; ~14 kg of UF ₄ in each beaker	87.0
(c) Filled to a depth of ~6 cm with U(93)O ₂ ; ~6.5 kg of UO ₂ in each beaker	90.9
(d) Filled to a depth of ~11 cm with U(93) metal scraps; ~15 kg of U(93) in each beaker	88.1
(e) Filled to a depth of ~7 cm with U(93) metal "pellets"; 15.0 kg of U(93) in each beaker	88.6
(f) Filled to a depth of ~18 cm of U(92.6)O ₂ (NO ₃) ₂ solution (4 liters) at a concentration of 410 g of uranium/liter; 6.24 kg of solution in each beaker	83.0
* * * * *	

24.5 kg of U(93.4) metal centered with solution height in adjacent units

4 pieces, each 2.5 × 5.1 × 25.4 cm:




(a) Arranged in a column 2.5 × 5.1 × 101.6 cm high	89.5
(b)  × 25.4 cm high	87.6
(c)  × 25.4 cm high	87.9
(d)  × 25.4 cm high	87.8

Table 16 - (continued)

Description of material in test position	Critical solution height in five control units, cm
$10.2 \times 10.2 \times 111.8$ cm column of $U(0.21)F_4-CF_2$; Uranium density, 3.11 g/cm^3 , Total density, $4.78 \pm 0.07 \text{ g/cm}^3$	89.0
$10.8 \times 10.8 \times 111.8$ cm column of graphite; Average density, 1.54 g/cm^3	88.1
$10.8 \times 10.8 \times 111.8$ cm column of beryllium; Average density, 1.70 g/cm^3	86.3
Iron pipe: Inside diameter, 15.4 cm Length, 112.0 cm Wall thickness, 0.71 cm	91.8

*Described in Table 13.

†Described in Fig. 77.

Table 17 – EFFECT ON CRITICAL DIMENSIONS OF THE DISPLACEMENT OF UNITS
WITHIN PLANAR ARRAYS

Description of Units: See Table 13

Height = 112.4 cm except as noted

Description of arrays	Height of five central units, cm
A. 5 × 5 units with axes vertical; square pattern; surface separation, 11.6 cm	105.9
B. Same as A except the units in a row adjacent to a peripheral row had axes horizontal	112.4*
C. 6 × 6 units with axes vertical; square pattern; surface separation, 13.7 cm	83.3
D. Same as C except one-half of array was displaced laterally one-half the distance separating units	104.3

*The array was quite subcritical.

Table 18 – CRITICAL PLANAR ARRAYS OF A SLAB AND A CYLINDER
OF AQUEOUS U(93.2)O₂F₂ SOLUTION*

Surface separation of containers, cm	Cylinder [†]			Slab [‡]		
	g of U ²³⁵ /liter	H/U ²³⁵	Critical height, cm	g of U ²³⁵ /liter	H/U ²³⁵	Critical height, cm
5.0	79	325	53.3	79	325	28.3
15.2	79	325	67.6	79	325	34.3
30.5	79	325	85.1	79	325	39.1
5.0	79	325	49.5	101	254	28.7
15.2	79	325	67.6	101	254	33.5
30.5	79	325	83.8	101	254	39.4

*The lower surfaces of the solution slab and cylinder were coplanar, and the projection of the cylinder was centered horizontally on the face of the slab.

[†] Solution was contained in a 25.4-cm-diameter aluminum cylinder with 1.6-mm-thick walls.

[‡] Solution was contained in a 15.1-cm-thick aluminum slab with 3.2-mm-thick walls.

Table 19 – CRITICAL DIMENSIONS OF TWO PARALLELEPIPEDS OF HOMOGENEOUS
 $U(1.42)F_4$ –PARAFFIN REFLECTED BY 20.3 CM OF POLYETHYLENE
EXCEPT ON FACING AREAS

Separation, cm	Dimensions of each parallelepiped			Total volume, liters	Total mass, kg	
	Width, cm	Height, cm	Length, cm		Uranium	U^{235}
0.000	92.1	92.3	45.3	771	1920	27.3
0.533	92.1	92.3	46.2	785	1950	27.7
0.998	92.1	92.3	{ 46.2* 48.7*	808	2010	28.5
1.443	92.1	92.3	48.7	830	2070	29.4
2.121	92.1	92.3	51.3	873	2170	30.8
2.771	92.1	92.3	53.9	917	2280	32.4
3.899	92.1	92.3	{ 53.9* 61.5*	982	2450	34.8
4.943	92.1	92.3	61.5	1048	2610	37.1
6.541	92.1	92.3	66.7	1134	2820	40.1
8.438	92.1	92.3	71.8	1222	3040	43.2
10.363	92.1	92.3	75.7	1288	3210	45.6
0.000	46.1	90.6	92.3	771	1920	27.3
0.254	46.1	91.3	92.3	777	1930	27.4
0.508	46.1	92.5	92.3	787	1960	27.8
0.742	46.1	93.8	92.3	798	1990	28.3
0.935	46.1	95.1	92.3	809	2010	28.5
1.511	46.1	98.9	92.3	842	2100	29.8
2.731	46.1	109.2	92.3	929	2310	32.8
3.576	46.1	116.9	92.3	995	2480	35.2

*In these cases the lengths of the two components of the array were unequal.

Table 20 – CRITICAL DIMENSIONS OF PARALLELEPIPEDS OF U(1.42)F₄–PARAFFIN
AND U(30.45)O₂F₂ SOLUTION

Material separating components	Critical height, cm	Critical separation, cm	Mass, kg of U ²³⁵		Volume, liters	
			U(1.42)F ₄ –paraffin	U(30.45)O ₂ F ₂	U(1.42)F ₄ –paraffin	U(30.45)O ₂ F ₂
Air	63.7	0.25	145	10.68	478	46.2
	64.5	0.58	148	10.90	488	47.1
	67.1	1.12	154	11.32	509	49.0
	72.2	2.24	165.5	12.20	546	52.8
	77.4	3.34	177.8	13.08	586	56.5
	87.7	5.55	201.5	14.81	664	64.1
	98.0	7.85	225.0	16.55	742	71.6
	108.4	10.32	248.5	18.35	820	79.2
Polyethylene	61.6	0.64	141.2	10.40	466	95.0
	76.4	2.70	175.5	12.90	578	55.8
	108.4	4.44	248.5	18.35	820	79.2
Jabroc*	74.1	4.05	170.0	12.55	561	54.2
	81.1	4.92	186.5	13.74	614	59.9
	105.8	6.73	242.5	17.89	800	77.2
Concrete	70.8	2.70	162	11.91	535	51.6
	92.7	8.56	214	15.70	705	67.8
	106	11.43	244	17.90	805	77.4
Mild steel	74.2	0.69	170.5	12.55	562	54.2
	91.0	1.98	209	15.40	689	66.5
Stainless steel	75	0.58	172	12.70	567	54.8

*Jabroc is a wood product containing about 45% carbon, 6% hydrogen, and 37% oxygen; $\rho = 1.315$ g/cm³.

Table 21 – CRITICAL SPATIAL ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION
WITH REFLECTORS OF VARIOUS THICKNESSES

Units: Right-circular cylinder of 5 liters of aqueous $U(92.6)O_2(NO_3)_2$
solution 19.04 cm in diameter and 17.77 cm high contained in
0.64-cm-thick Plexiglas

U^{235} concentration: 384 g/liter; $H/U^{235} = 59$

Number of units in array	Reflector		Surface separation of units,* cm	Average uranium density in array, g/cm ³
	Material	Thickness, cm		
8	None	0	1.43	0.214
8	Paraffin	1.3	3.28	0.167
8	Paraffin	3.8	6.91	0.108
8	Paraffin	7.6	8.48	0.091
8	Paraffin	15.2	8.99	0.087
8	Plexiglas	1.3	3.00	0.173
27	None	0	6.48	0.113
27	Paraffin	1.3	9.02	0.086
27	Paraffin	3.8	13.69	0.055
27	Paraffin	15.2	16.53 [†]	0.043
27	Plexiglas	1.3	8.76	0.088
64	None	0	10.67	0.072
125	None	0	14.40	0.052

*The uncertainty in the separation is ± 0.13 cm.

[†]The separation was 16.91 cm when one face of the array was reflected by 15.2-cm-thick Plexiglas.

Table 22 – CRITICAL SPATIAL ARRAYS OF $U(92.6)O_2(NO_3)_2$ SOLUTION OF VARIOUS CONCENTRATIONS

Units: Right-circular cylinder of 5 liters of aqueous $U(92.6)O_2(NO_3)_2$ solution 19.04 cm in diameter and 17.77 cm high contained in 0.64-cm-thick Plexiglas

Number of units in array	H/U ²³⁵ atomic ratio	Uranium density in the solution, g/liter	Reflector		Surface separation of units,* cm	Average uranium density in array, g/cm ³
			Material	Thickness, cm		
8	92	279	Paraffin [†]	11.4	8.71	0.060
8	92	279	None		1.43	0.144
27	92	279	None		6.40	0.077
8	440	63.3	None		0 [‡]	0.040
27	440	63.3	None		2.41	0.029
27	§	§	None		6.41	0.107

*The uncertainty in the separation is ± 0.13 cm.

[†]The lower surface of the array was reflected by 15.2-cm-thick paraffin.

[‡]The array was subcritical, $k_{eff} \sim 0.6$.

[§]Five units in the center tier at an H/U²³⁵ of 92 and remaining 22 units at an H/U²³⁵ of 59.

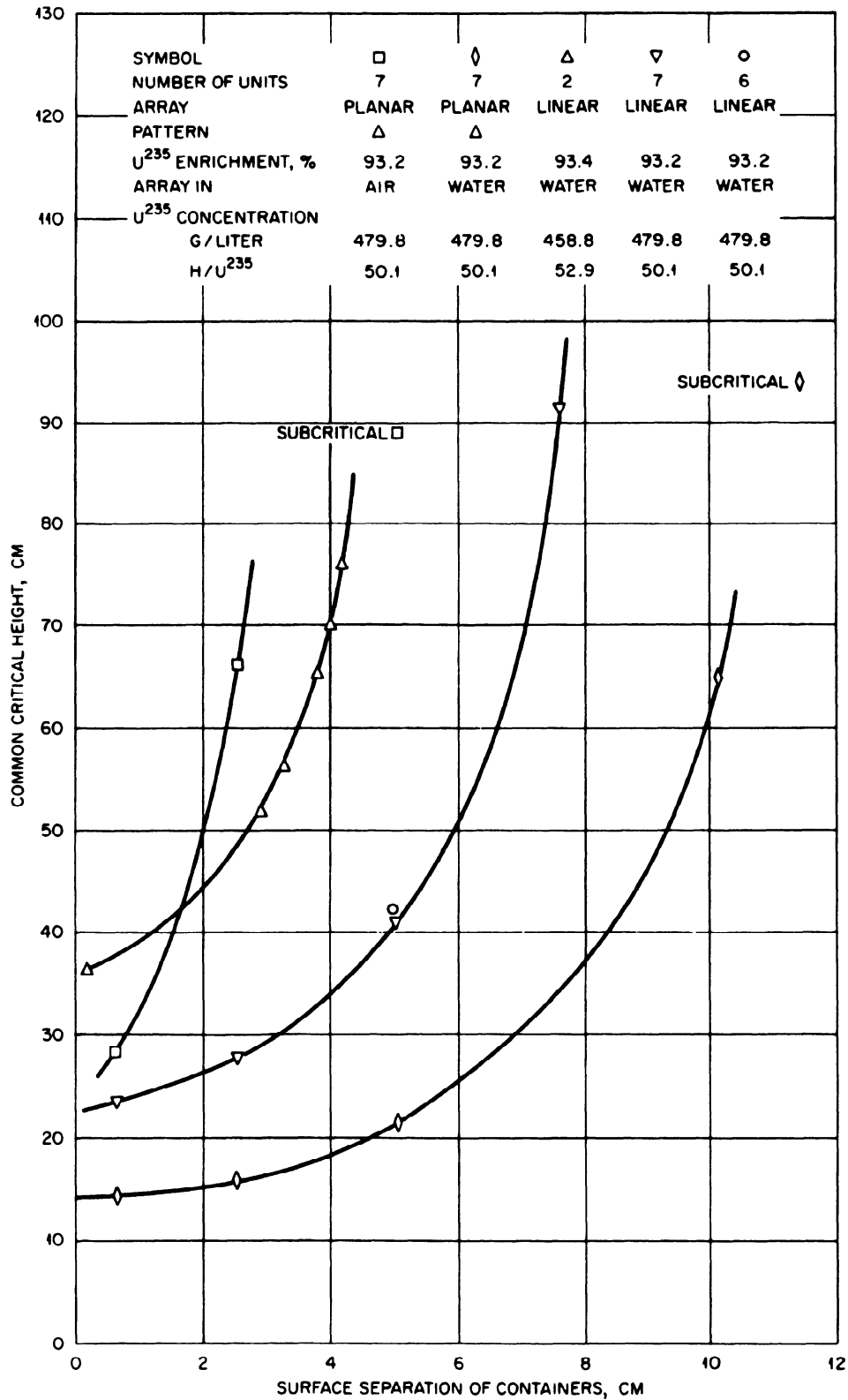


Fig. 56 – Planar arrays of 12.7-cm-dia cylinders of aqueous $U(93)O_2F_2$ solution (1.6-mm-thick aluminum containers).

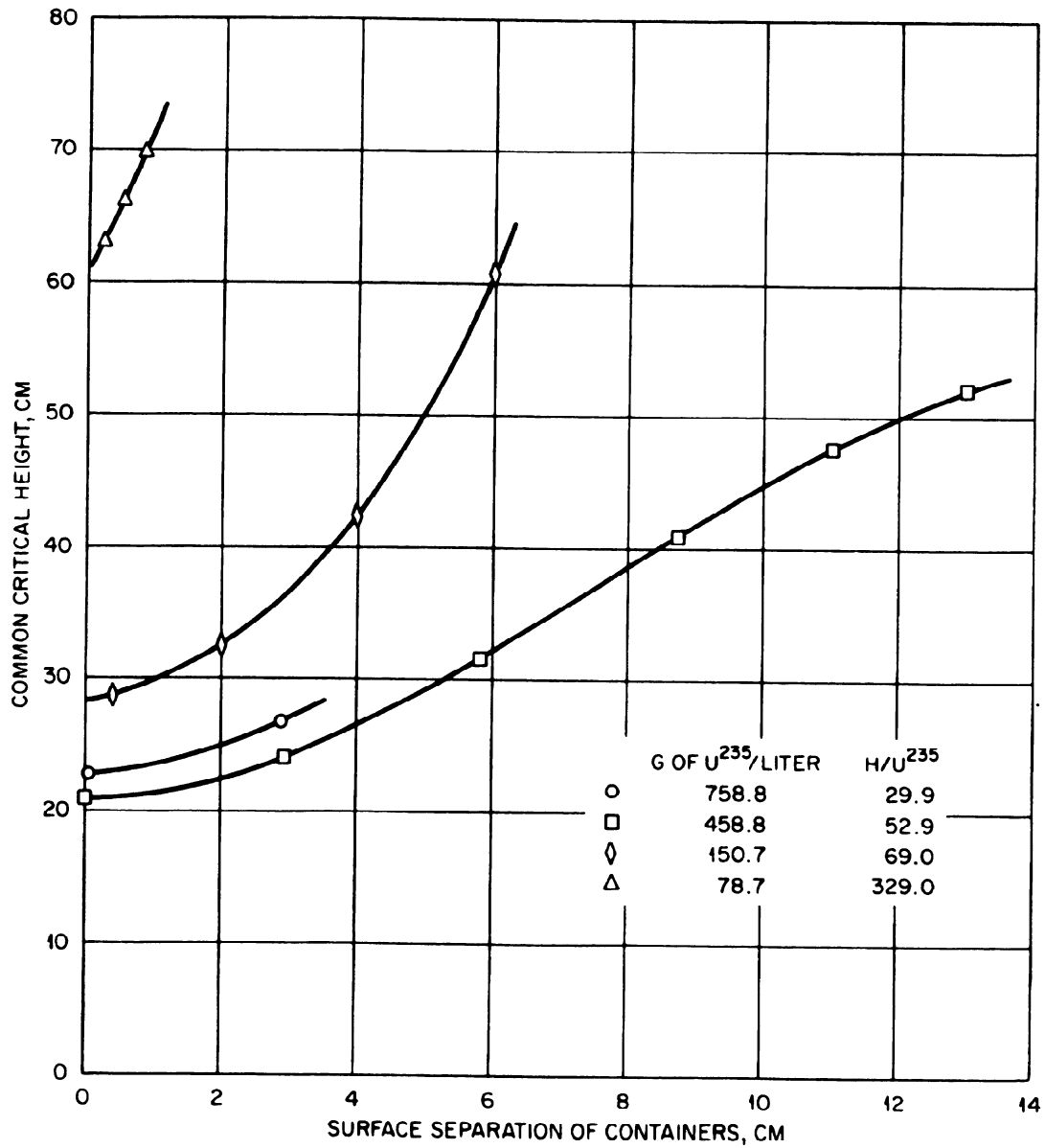


Fig. 57 - Water-moderated and -reflected two-unit planar arrays of 15.2-cm-dia cylinders of aqueous $U(93.4)O_2F_2$ solution (1.6-mm-thick aluminum containers).

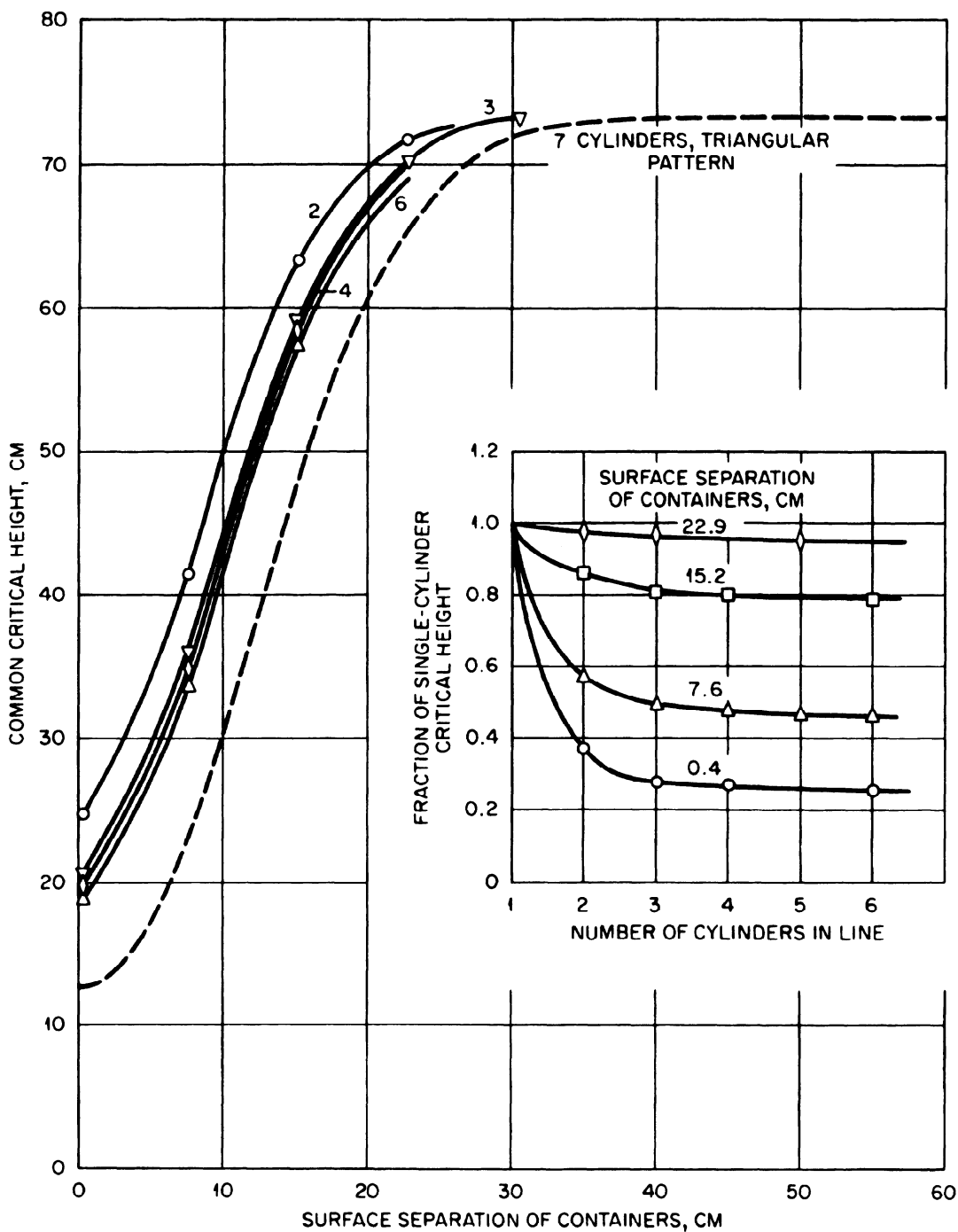


Fig. 58 – Water-moderated and -reflected linear arrays of 15.2-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 537.6 g/liter; $H/U^{235} = 44.3$. Containers: 1.6-mm-thick aluminum.

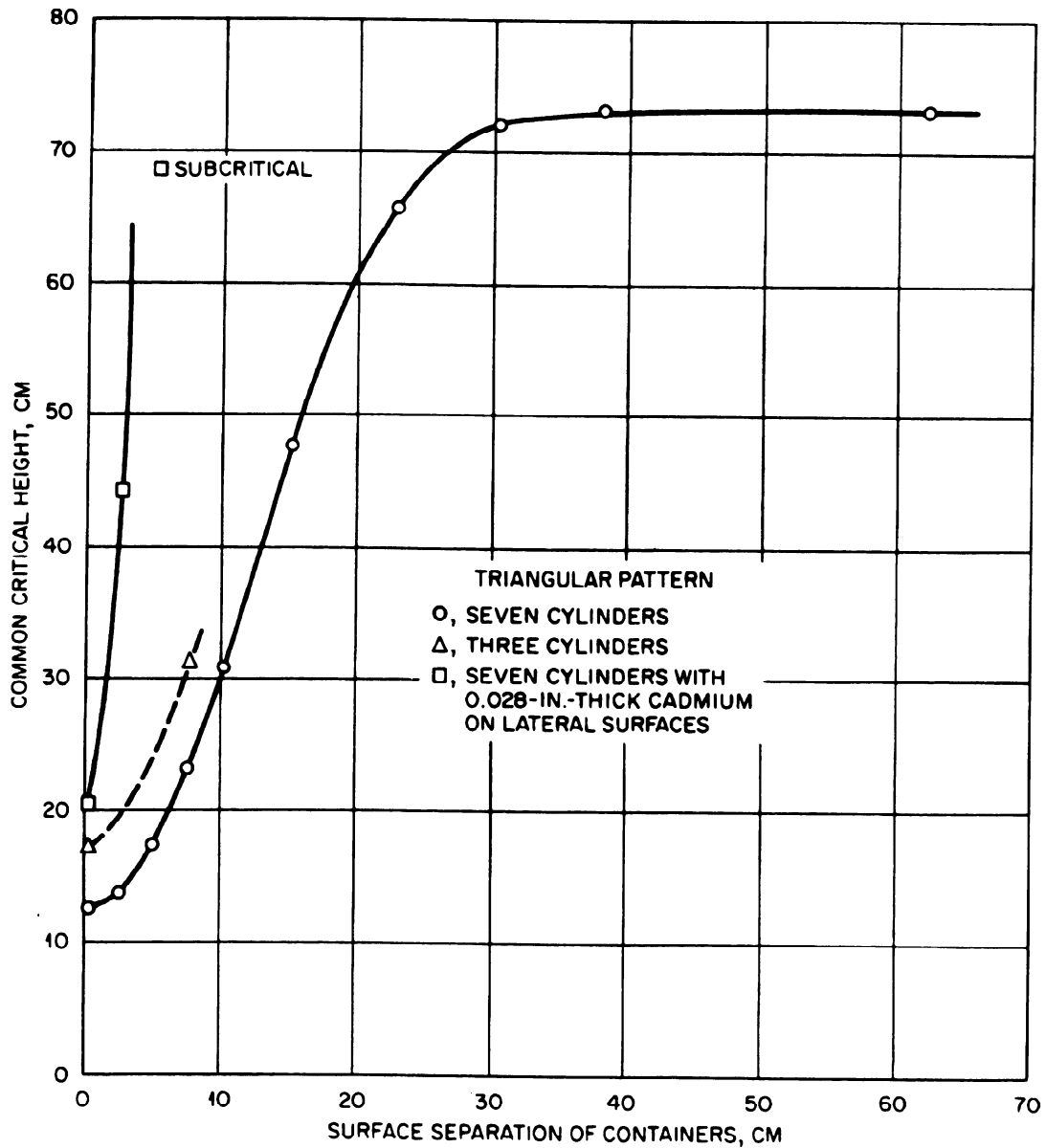


Fig. 59 – Water-moderated and -reflected planar arrays of 15.2-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution in triangular patterns. U^{235} concentration: 537.6 g/liter; $H/U^{235} = 44.3$. Containers: 1.6-mm-thick aluminum.

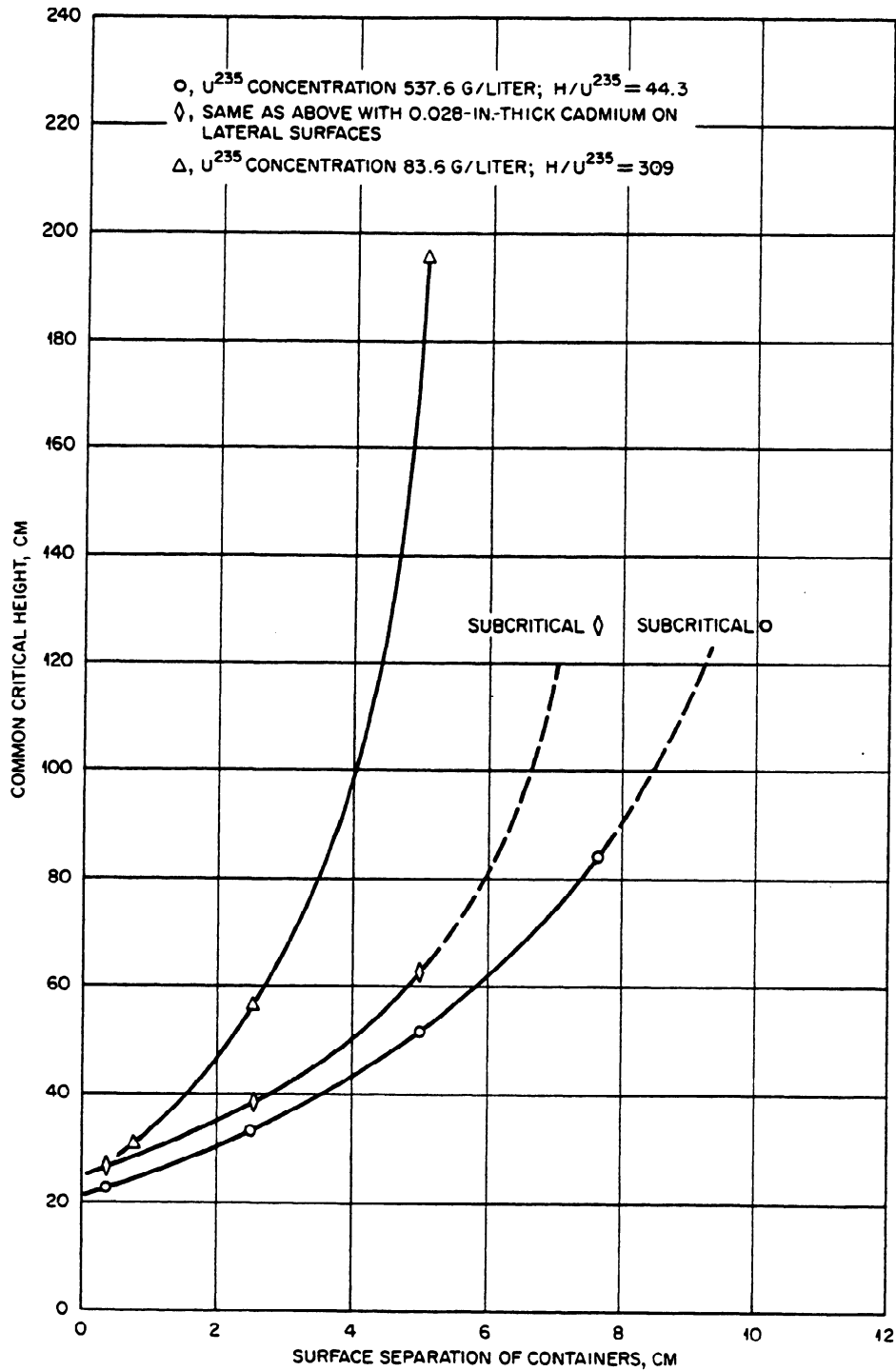


Fig. 60 - Unreflected and unmoderated seven-unit planar arrays of 15.2-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution in triangular patterns (1.6-mm-thick aluminum containers).

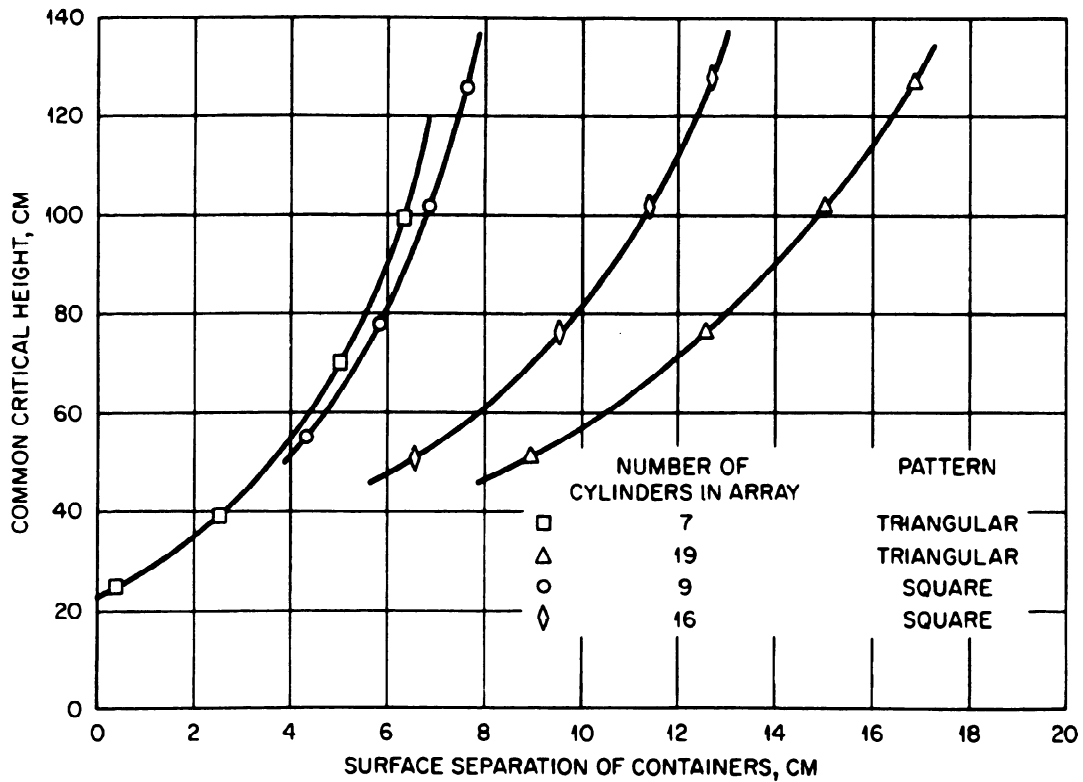


Fig. 61 - Unreflected and unmoderated planar arrays of 15.2-cm-dia cylinders of aqueous $U(92.6)O_2(NO_3)_2$ solution. U^{235} concentration: 384 g/liter; $H/U^{235} = 59$. Containers: 1.6-mm-thick aluminum.

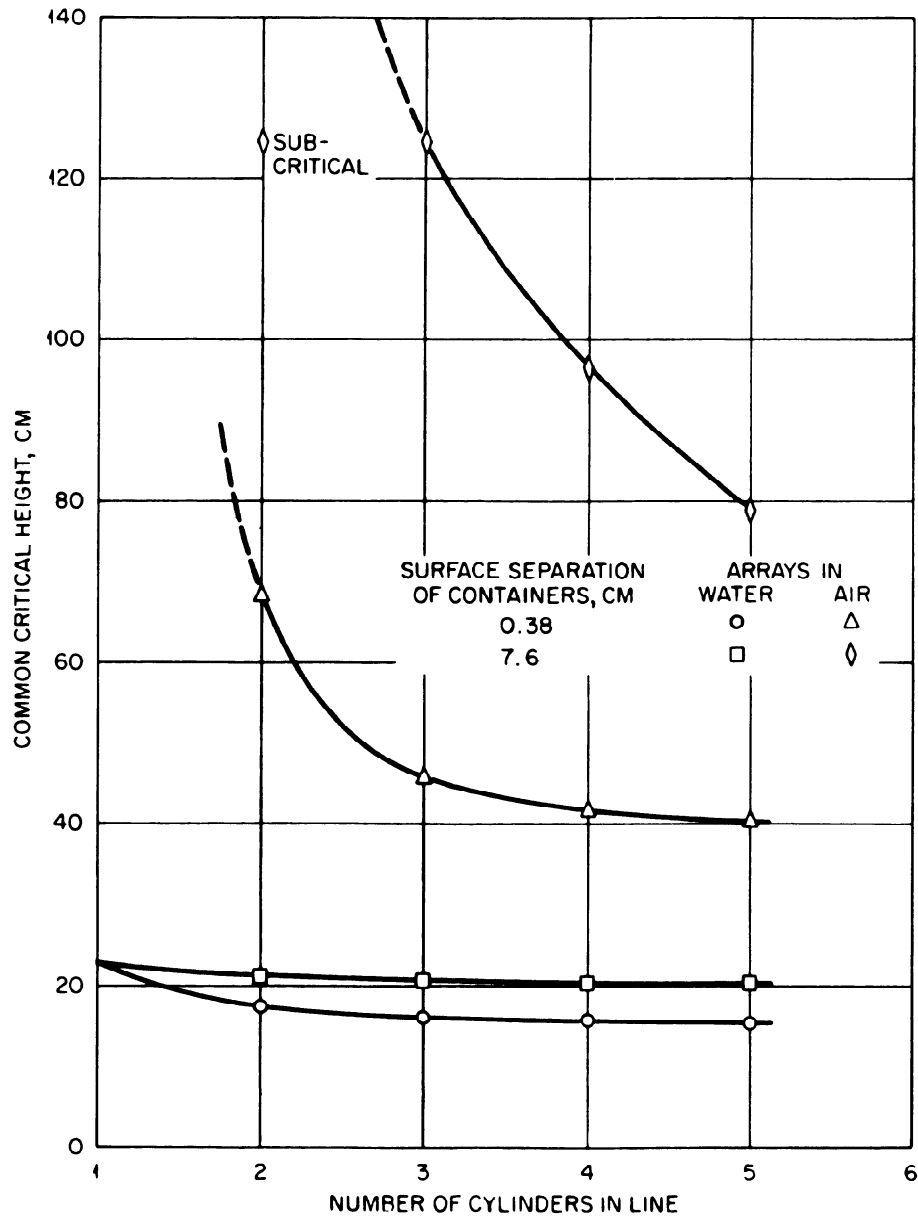


Fig. 62 - Linear arrays of 20.3-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 537.6 g/liter; $H/U^{235} = 44.3$. Containers: 1.6-mm-thick aluminum.

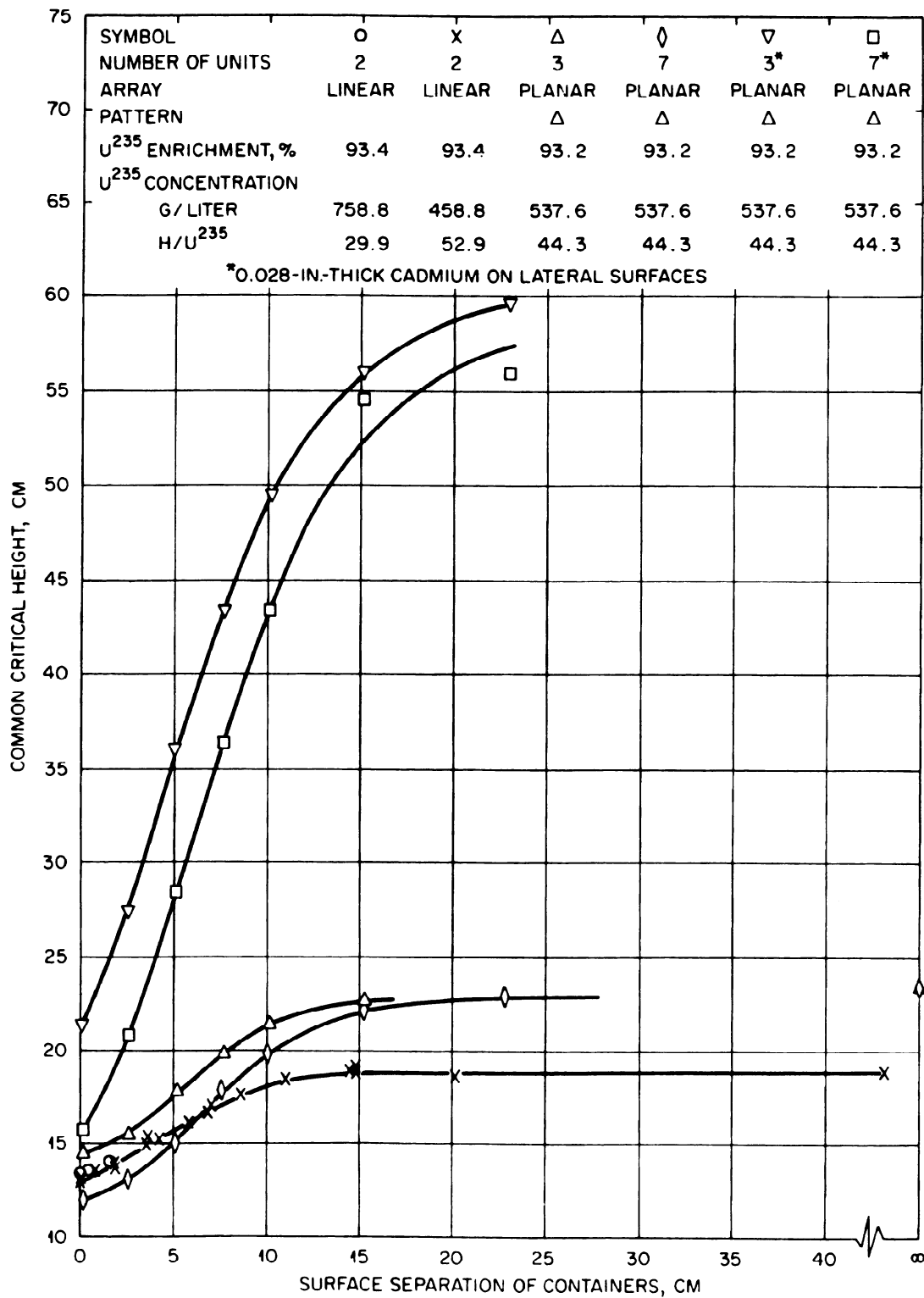


Fig. 63 – Water-reflected and -moderated linear and planar arrays of 20.3-cm-dia cylinders of aqueous $U(93)O_2F_2$ solution (1.6-mm-thick aluminum containers).

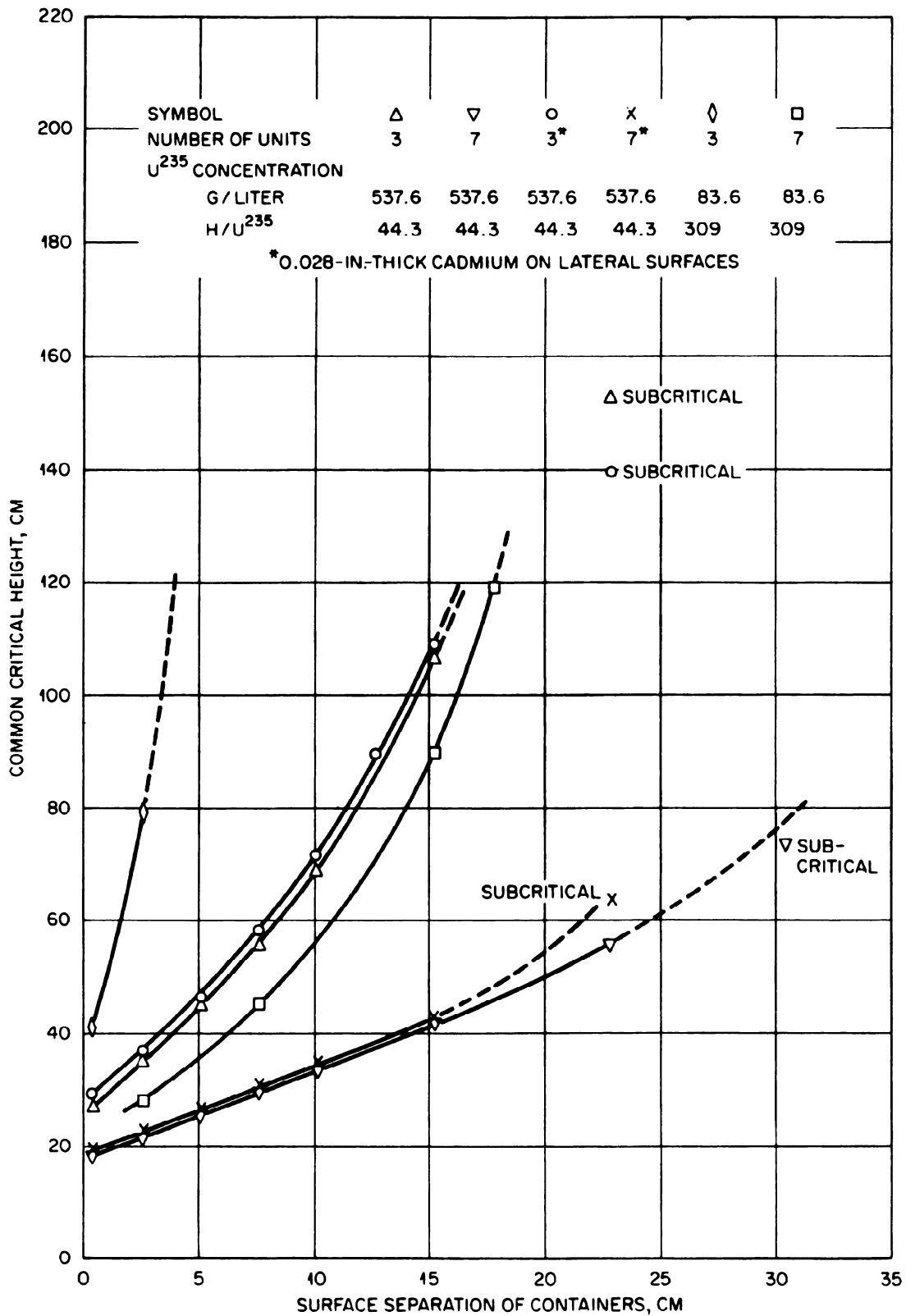


Fig. 64 – Unreflected and unmoderated planar arrays of 20.3-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution in triangular patterns (1.6-mm-thick aluminum containers).

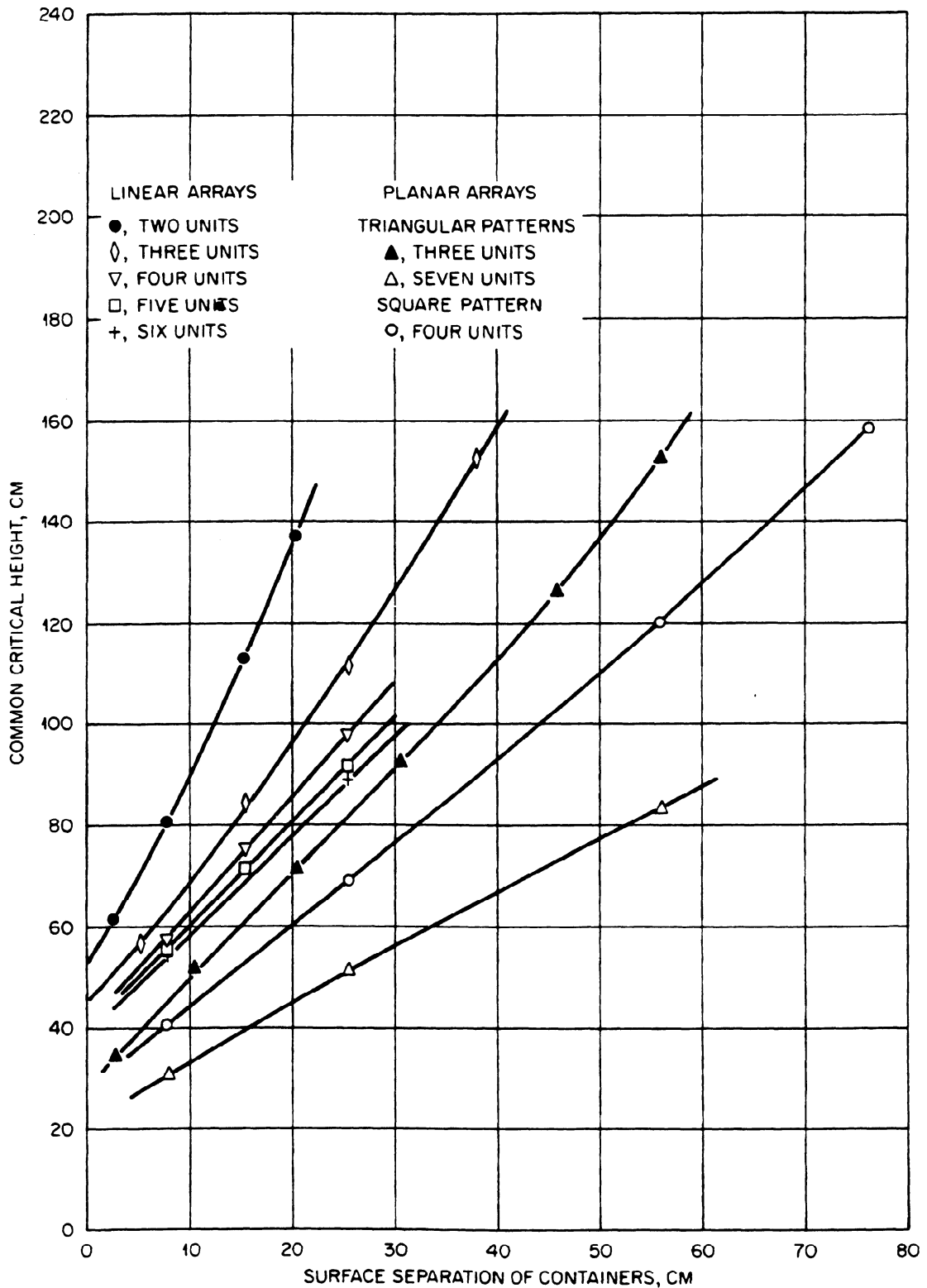


Fig. 65 – Unreflected and unmoderated linear and planar arrays of 24.1-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 86.8 g/liter; $H/U^{235} = 297$. Containers: 1.6-mm-thick aluminum.

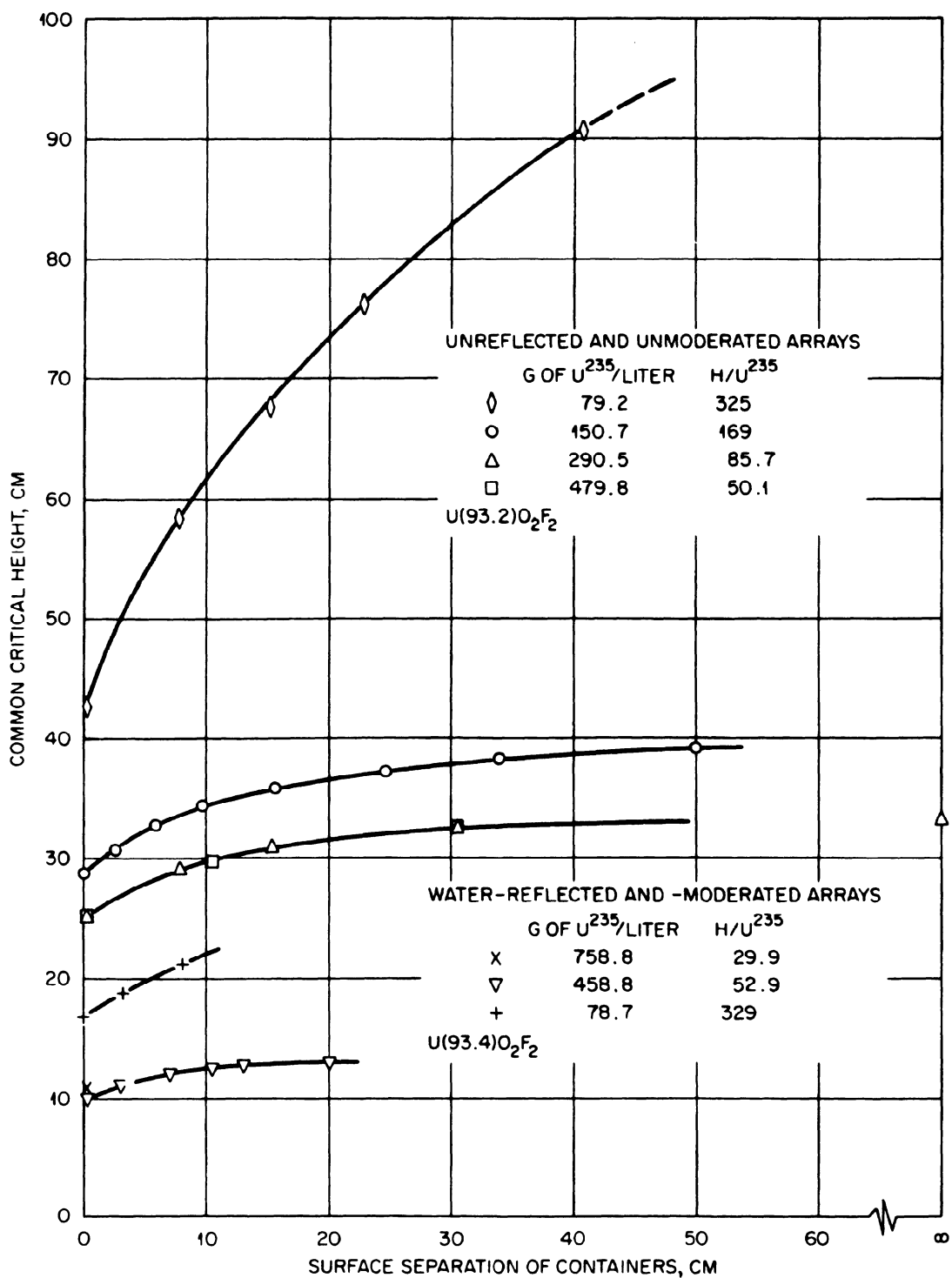


Fig. 66 – Two-unit planar arrays of 25.4-cm-dia cylinders of aqueous U(93.4)O₂F₂ solution (1.6-mm-thick aluminum containers).

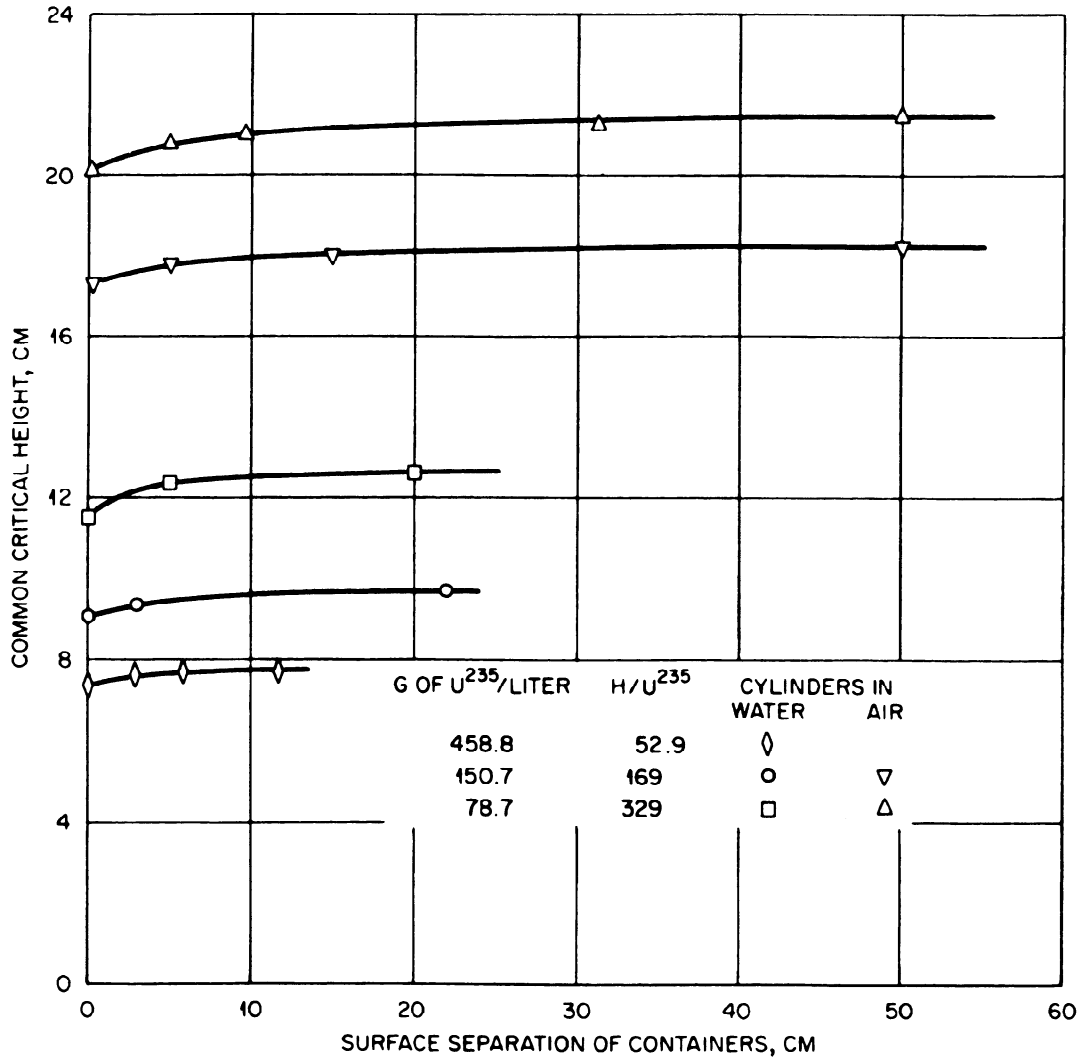


Fig. 67 - Two-unit planar arrays of 38.1-cm-dia cylinders of aqueous $U(93.4)O_2F_2$ solution (1.6-mm-thick aluminum containers).

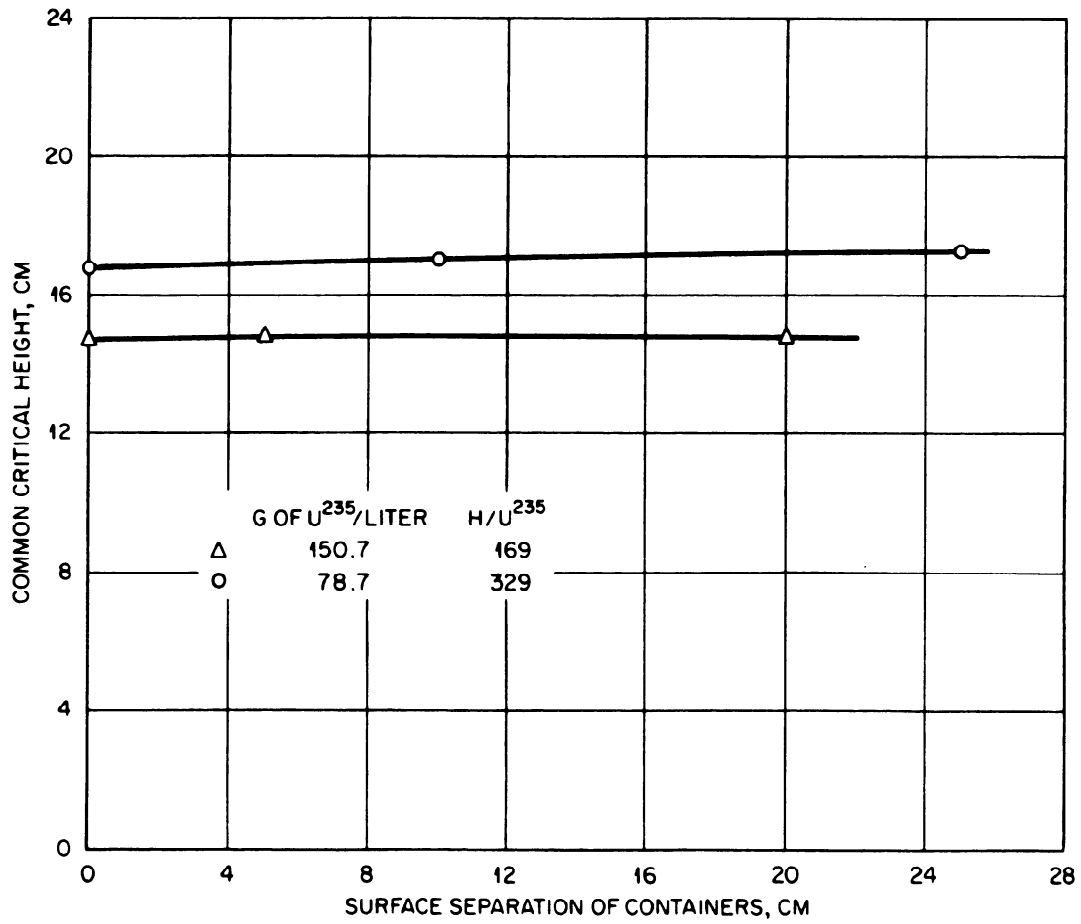


Fig. 68 – Unreflected and unmoderated two-unit planar arrays of 50.8-cm-dia cylinders of aqueous $U(93.4)O_2F_2$ solution (1.6-mm-thick stainless-steel containers).

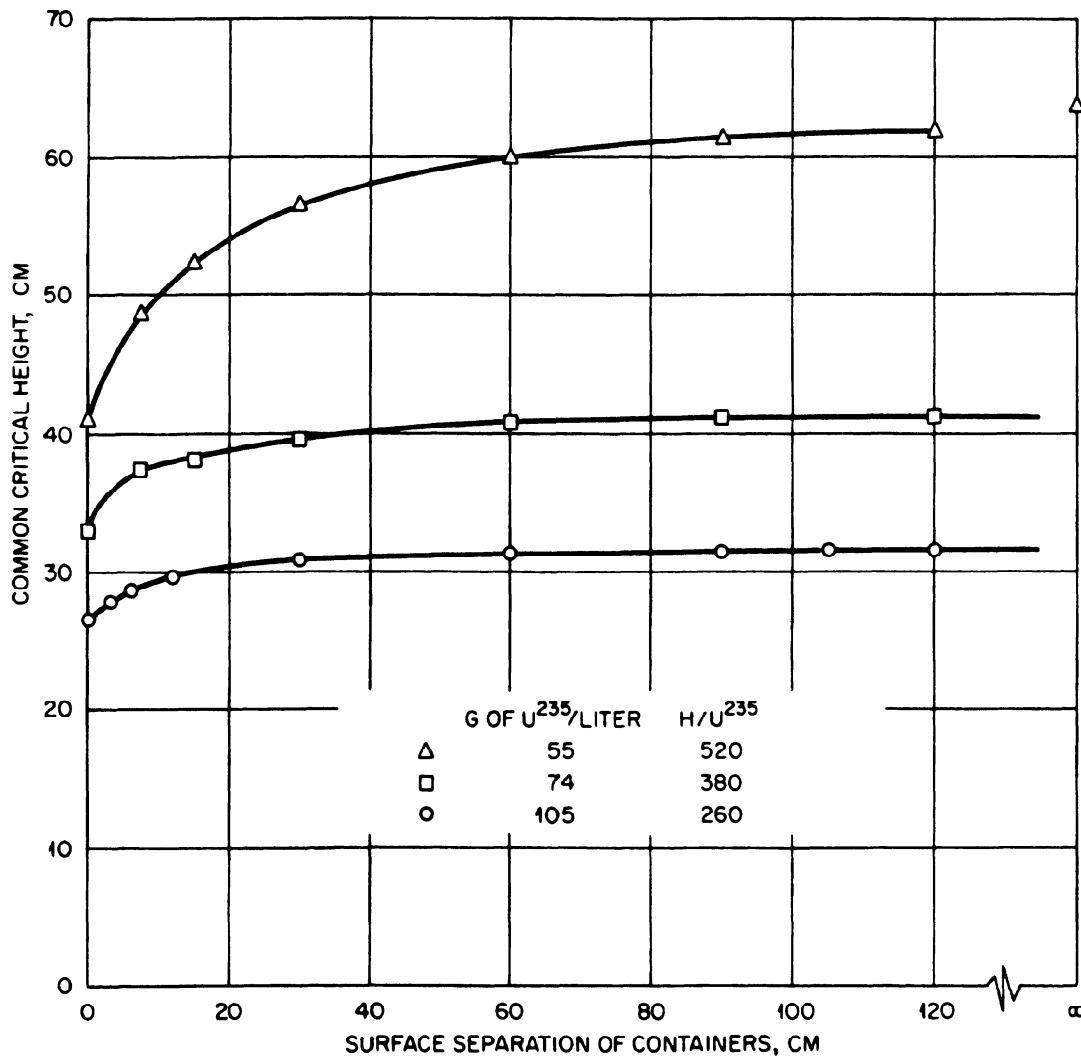


Fig. 69 - Unreflected and unmoderated two-unit planar arrays of 30.0-cm-dia cylinders of $U(90)O_2(NO_3)_2$ solution (1.5-mm-thick stainless-steel containers).

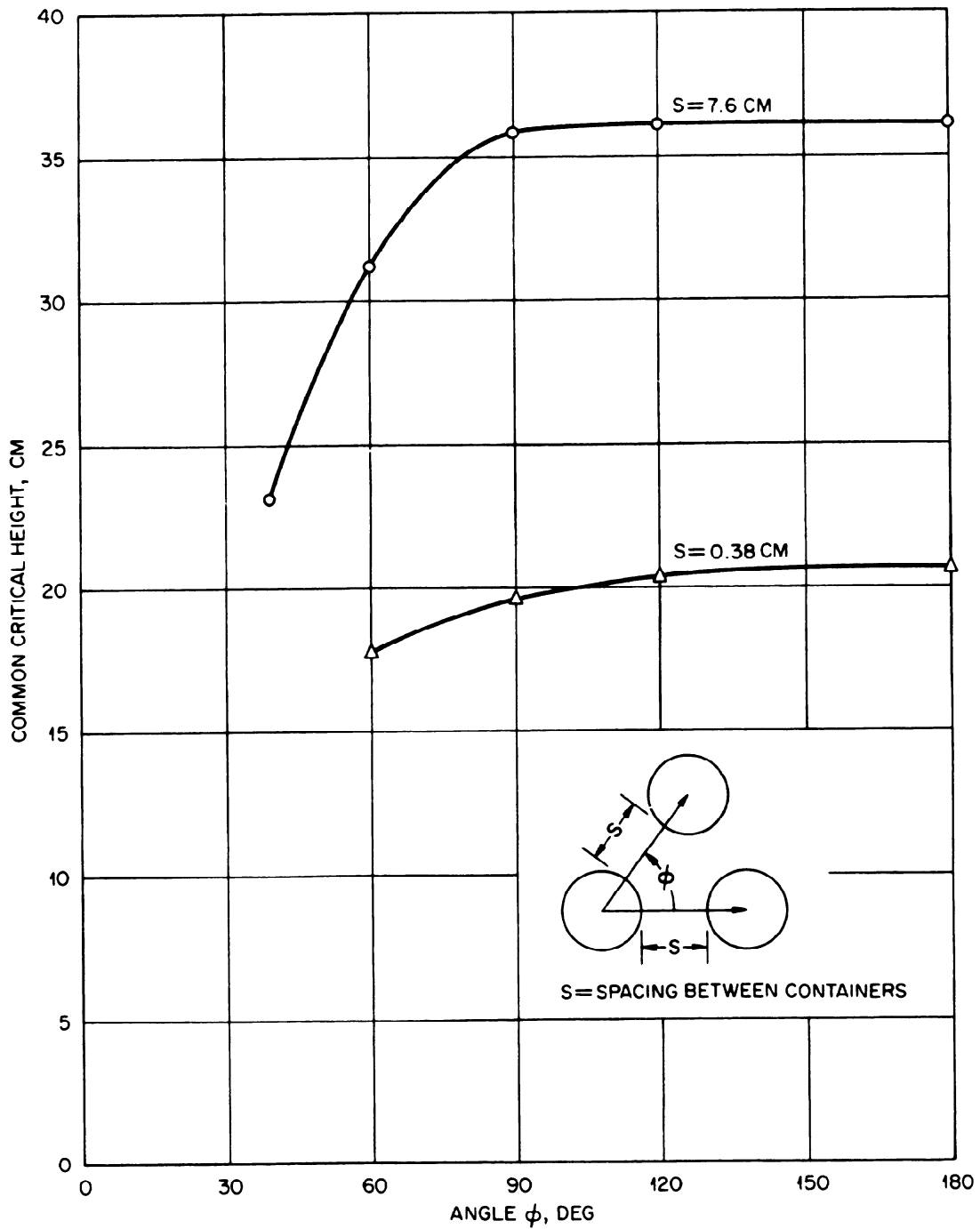


Fig. 70 – Water-moderated and -reflected three-unit planar arrays of 15.2-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution in isosceles triangular patterns. U^{235} concentration: 537.6 g/liter; $H/U^{235} = 44.3$. Containers: 1.6-mm-thick aluminum.

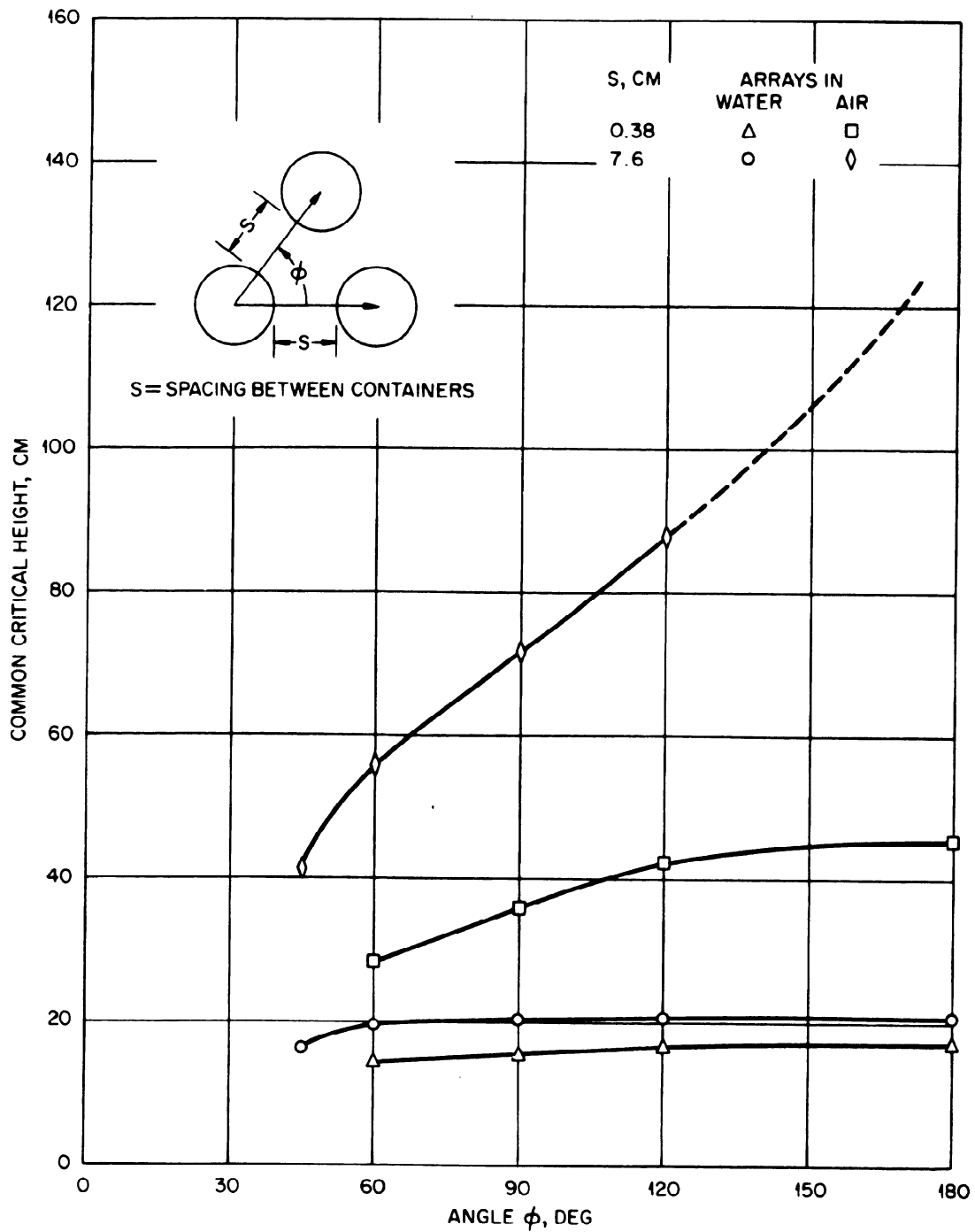


Fig. 71 - Three-unit planar arrays of 20.3-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution in isosceles triangular patterns. U^{235} concentration: 537.6 g/liter; $H/U^{235} = 44.3$. Containers: 1.6-mm-thick aluminum.

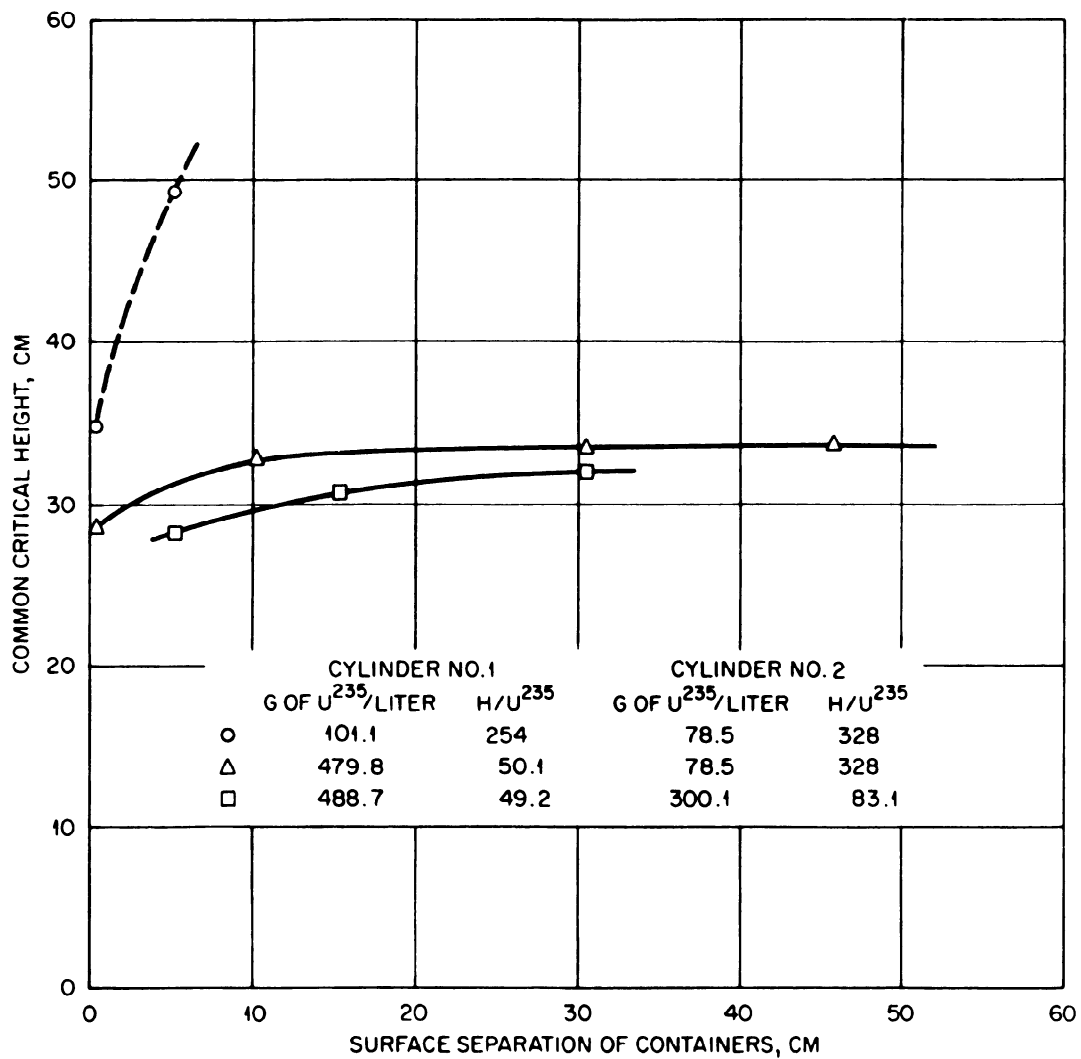


Fig. 72 – Unmoderated and unreflected two-unit planar arrays of 25.4-cm-dia cylinders of aqueous $U(93.2)O_2F_2$ solution of different concentration (1.6-mm-thick aluminum containers).

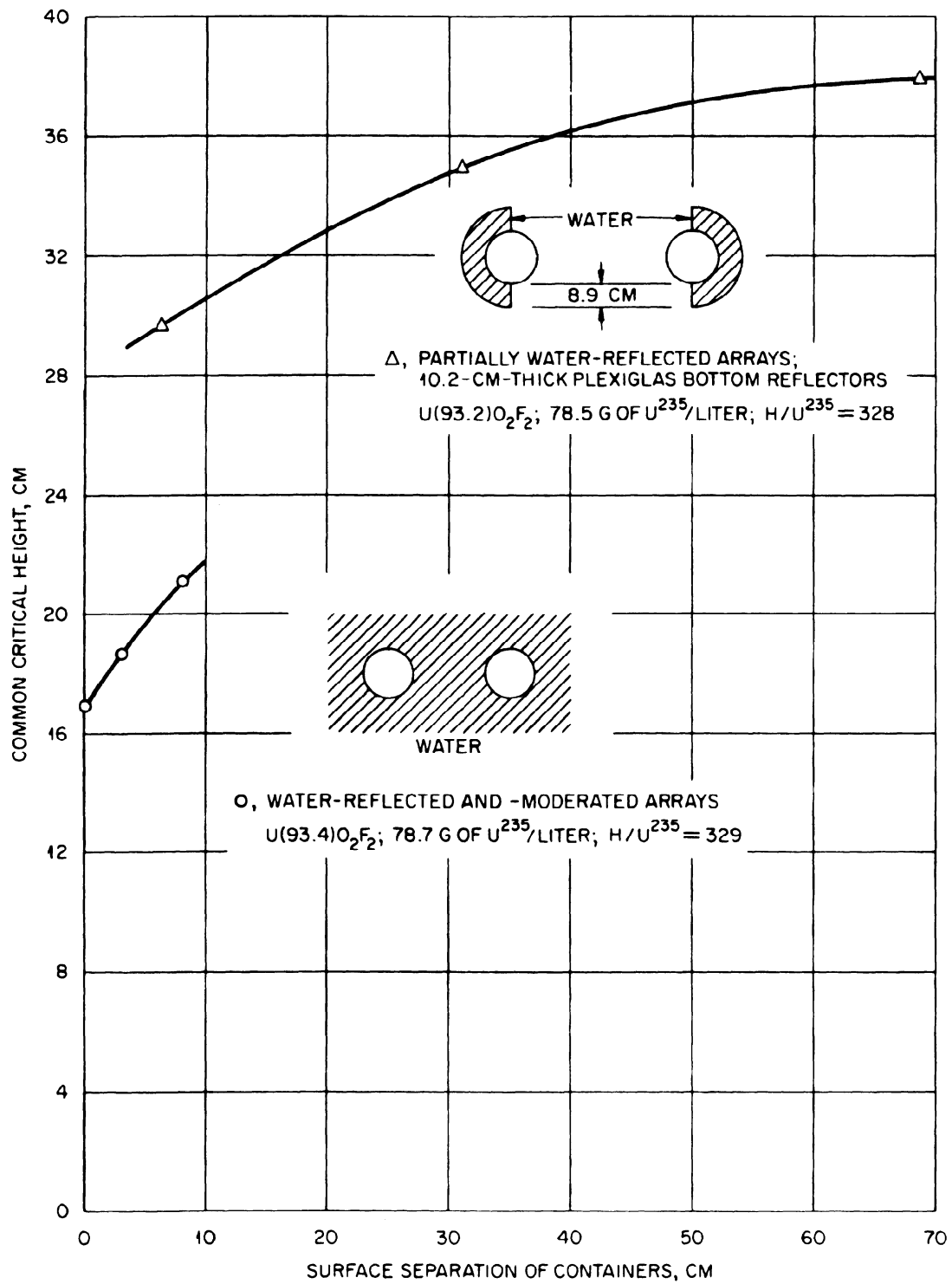


Fig. 73 - Two-unit planar arrays of 25.4-cm-dia cylinders of aqueous $U(93)O_2F_2$ solution partially and completely surrounded by water (1.6-mm-thick aluminum containers).

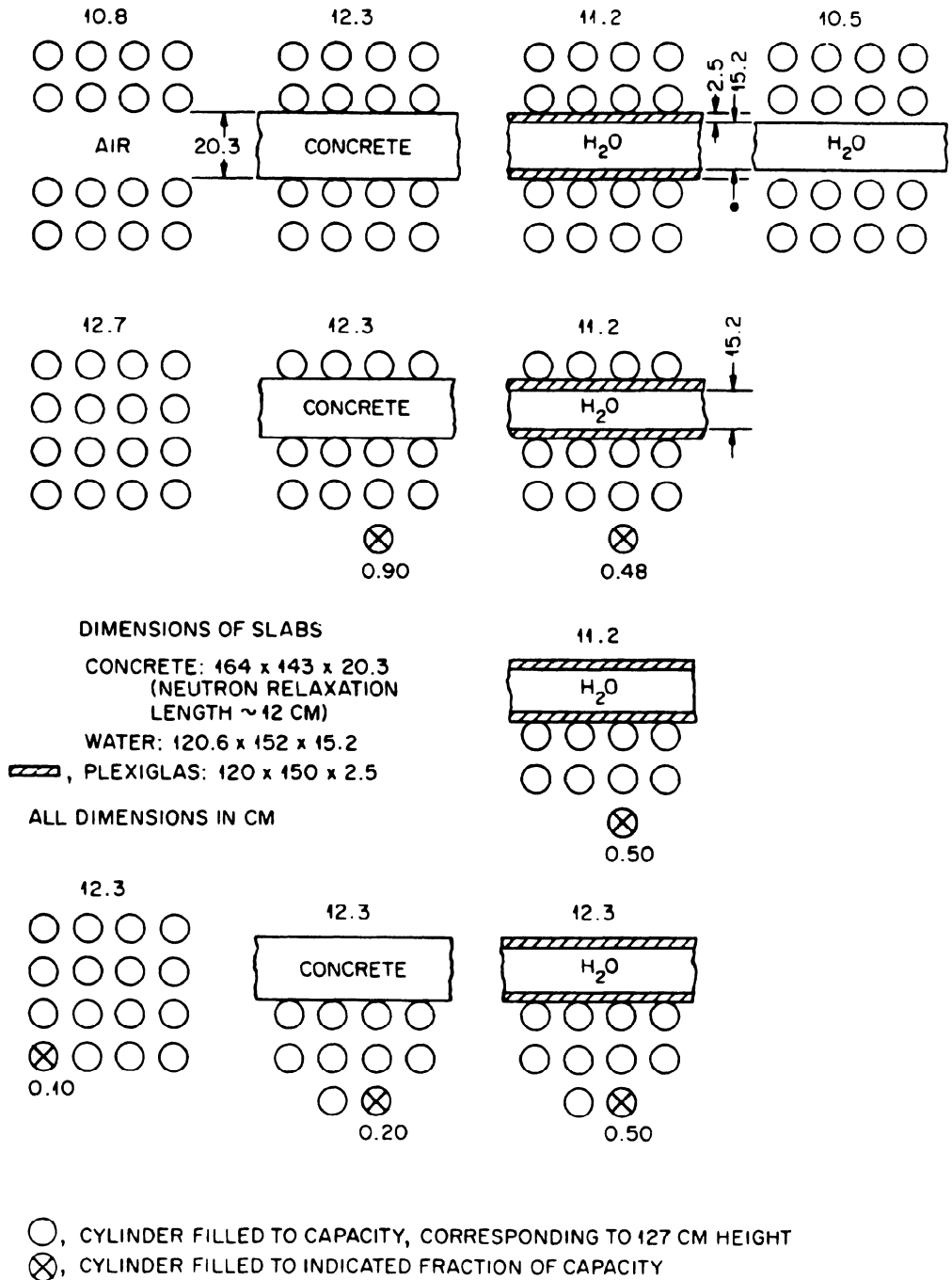


Fig. 74 – Planar arrays of 15.2-cm-dia cylinders of aqueous $U(92.6)O_2(NO_3)_2$ solution with concrete and hydrogenous reflector-moderators. U^{235} concentration: 384 g/liter; $H/U^{235} = 59$. Solution containers: 1.6-mm-thick aluminum. Water containers: 3.2-mm-thick aluminum. The surface separation of the containers (in centimeters) is noted above each configuration.

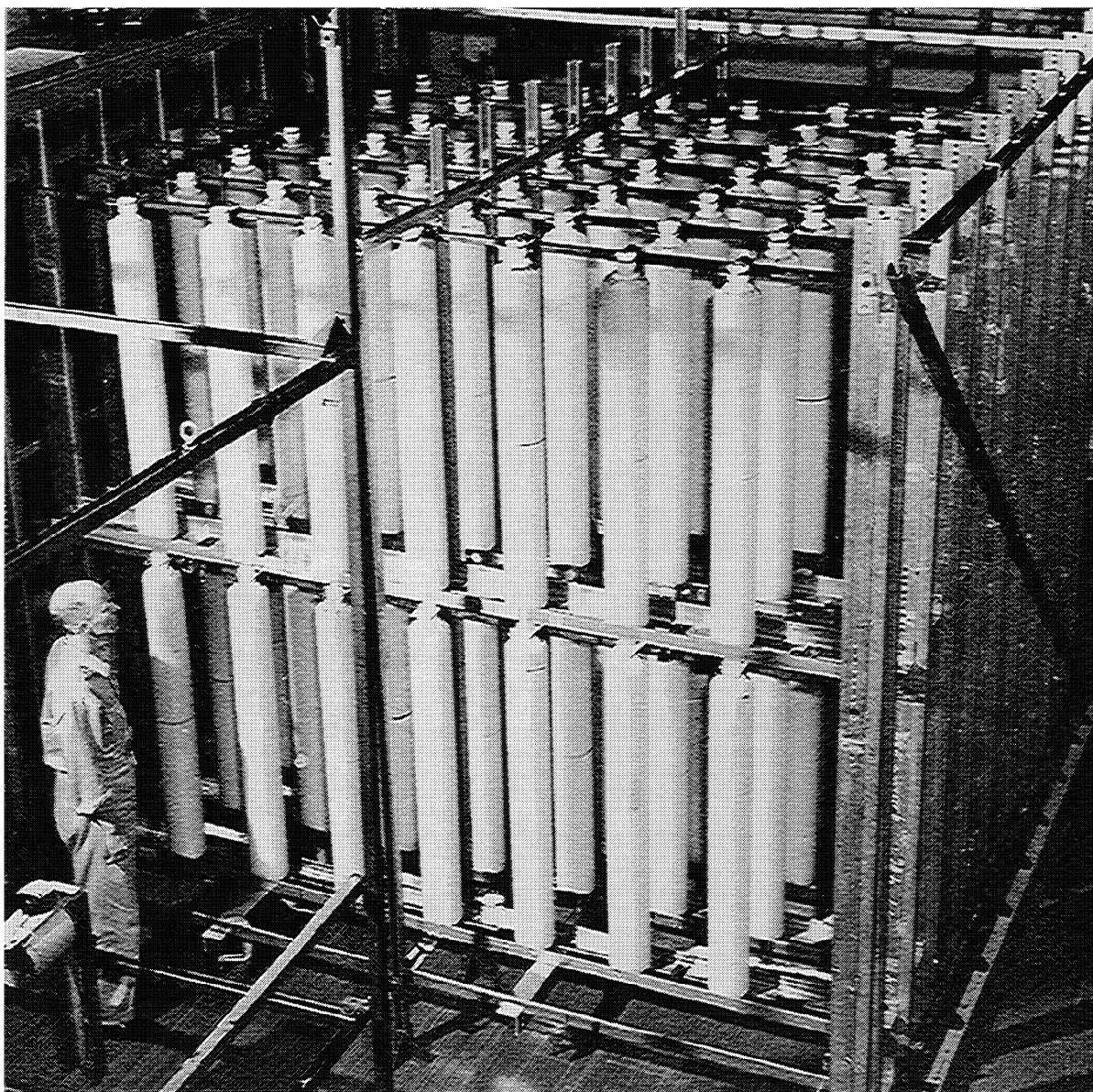


Fig. 75 – Double-tier array of cylinders of $U(92.6)O_2(NO_3)_2$ aqueous solution in 13.65-cm-dia polyethylene containers.

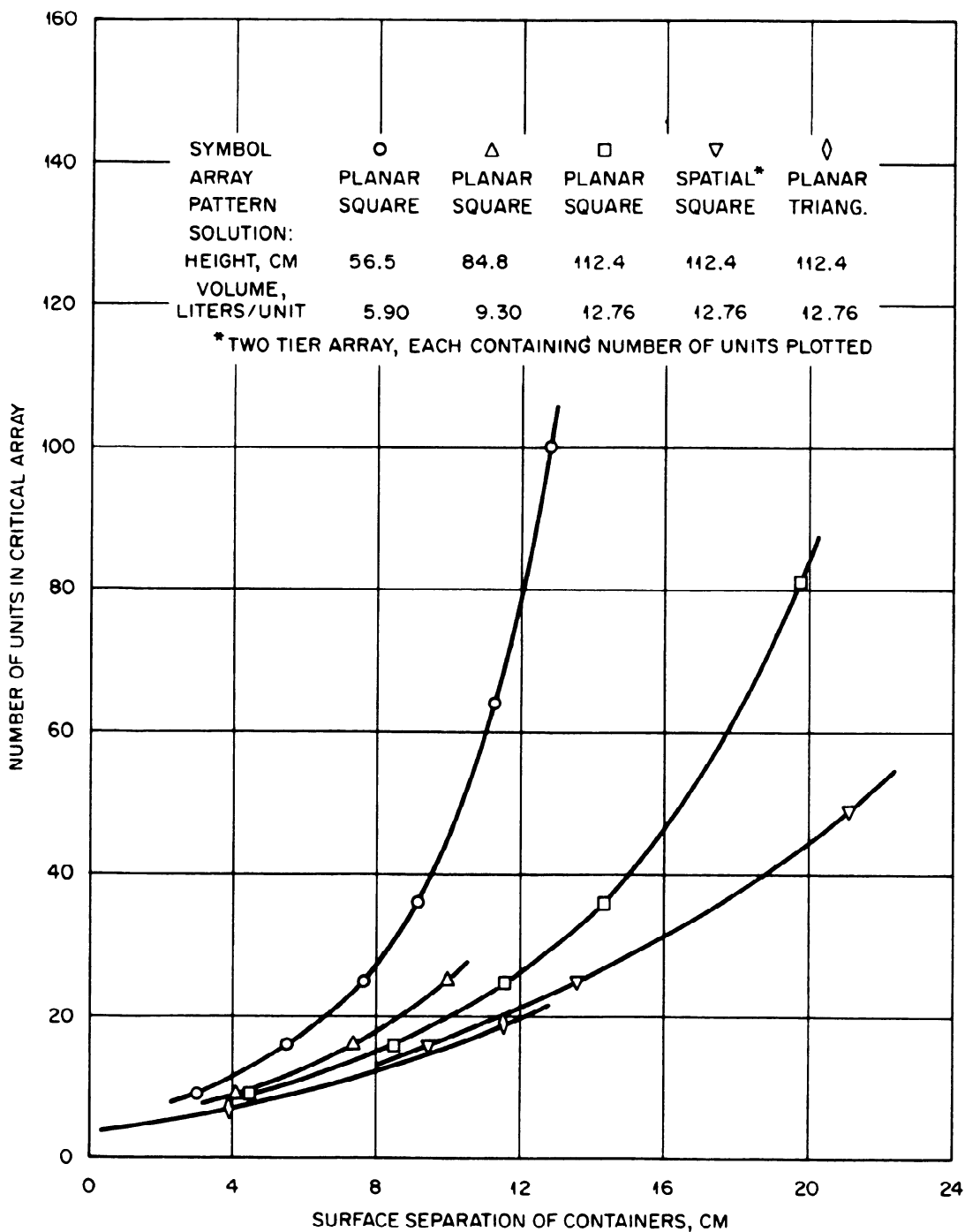


Fig. 76 – Unreflected and unmoderated arrays of cylinders of aqueous $U(92.6)O_2(NO_3)_2$ solution. U^{235} concentration: 384 g/liter; $H/U^{235} = 59$. The polyethylene containers are described in Table 13.

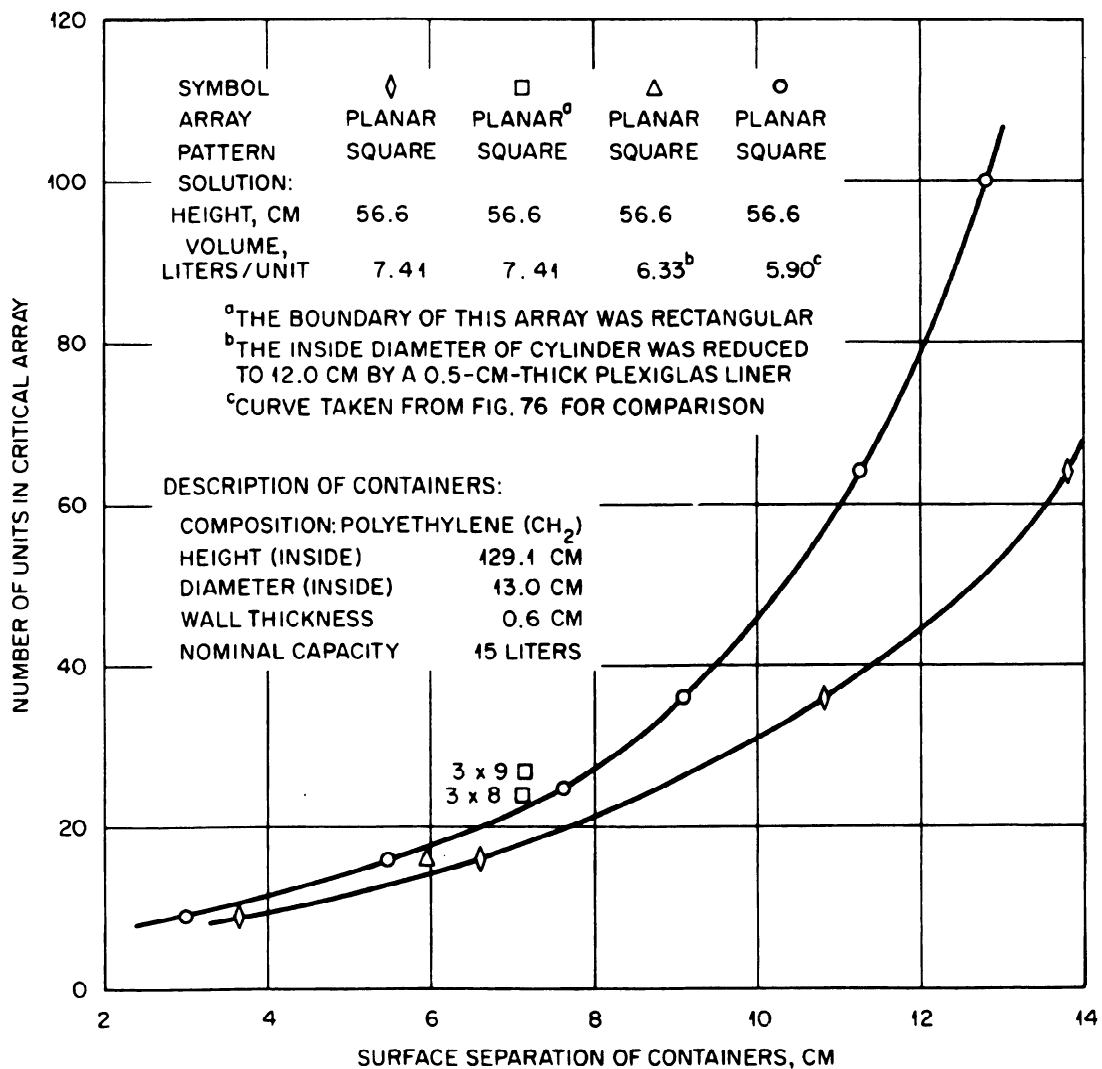


Fig. 77 – Unreflected and unmoderated planar arrays of cylinders of aqueous $U(92.6)O_2(NO_3)_2$ solution. U^{235} concentration: 384 g/liter; $H/U^{235} = 59$.

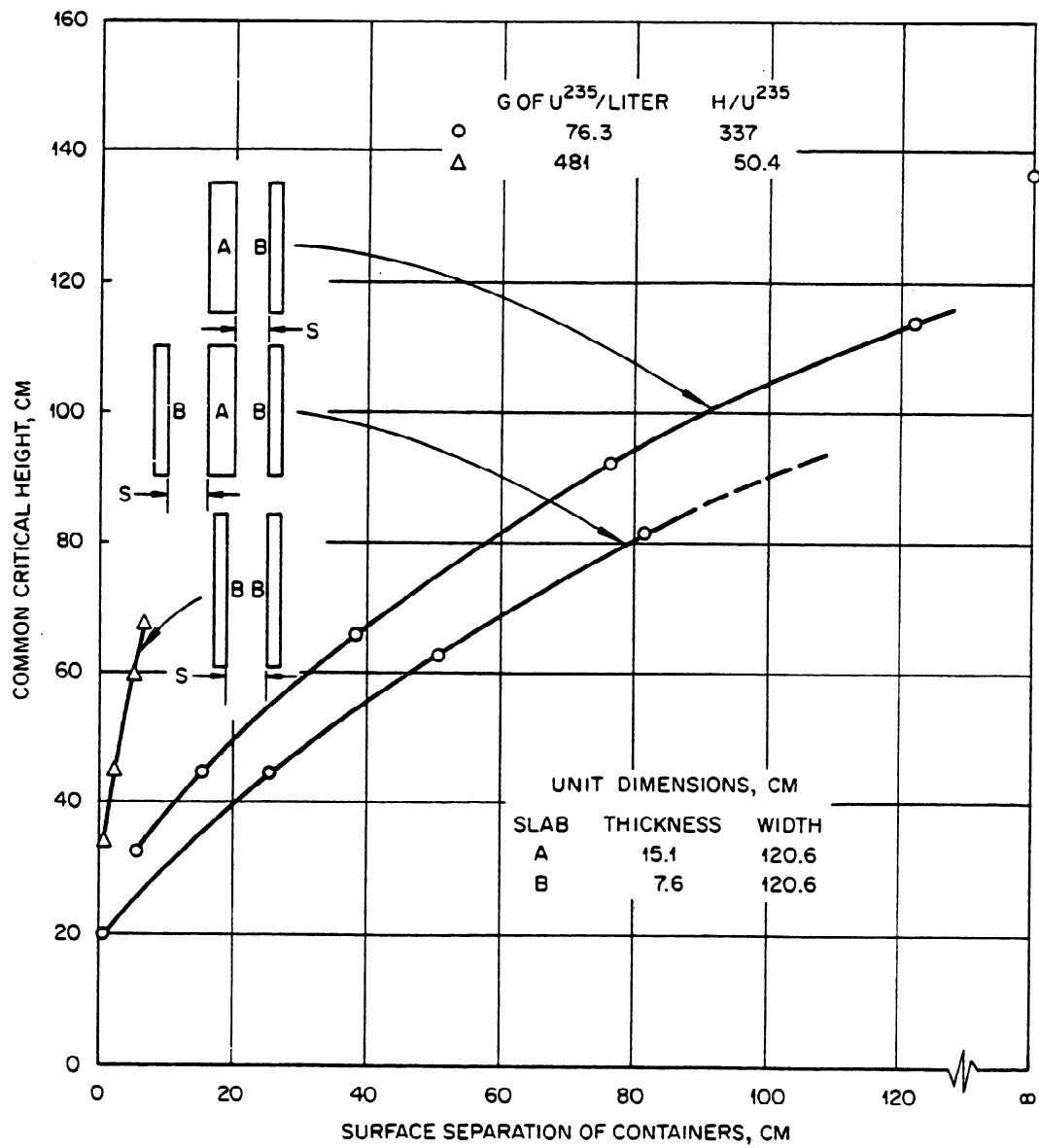


Fig. 78 – Unmoderated and unreflected planar arrays of slabs of aqueous $U(93.2)O_2F_2$ solution (3.2-mm-thick aluminum containers).

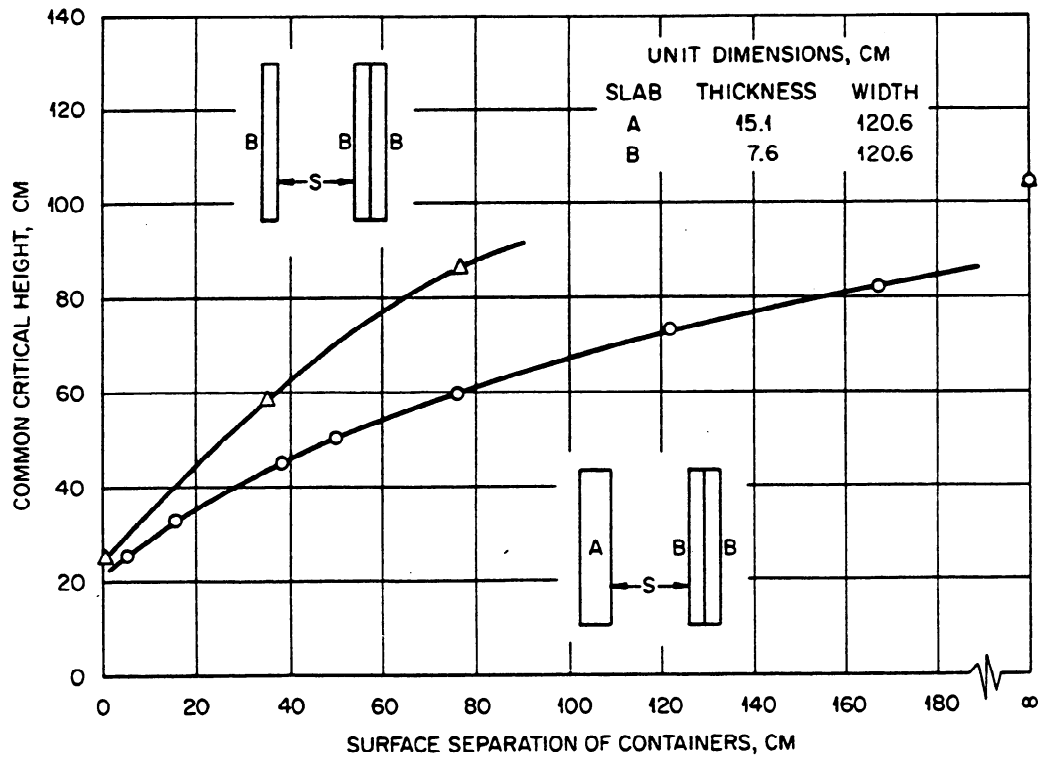


Fig. 79 - Unmoderated and unreflected planar arrays of slabs of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 76.3 g/liter; $H/U^{235} = 337$. Containers: 3.2-mm-thick aluminum.

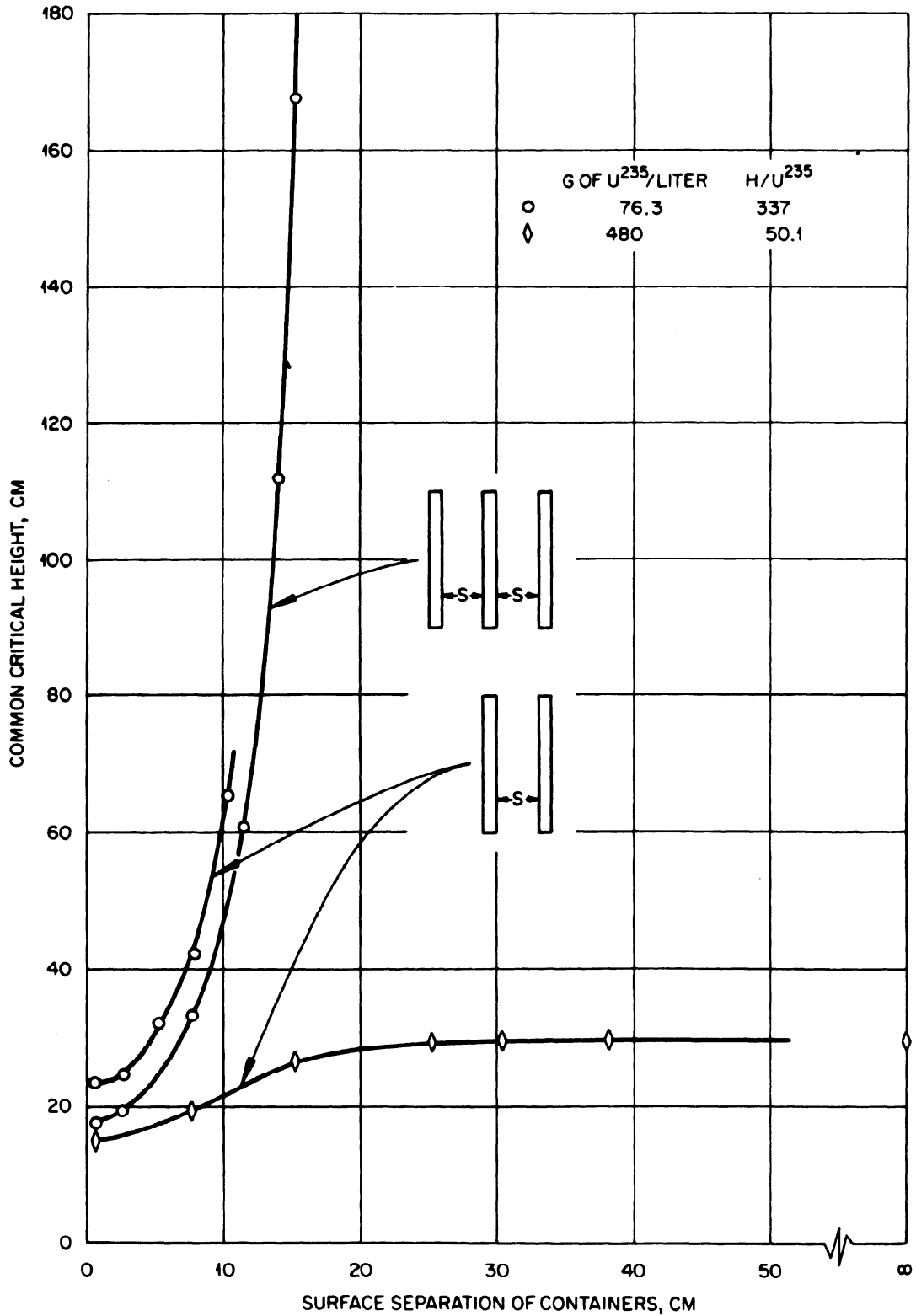


Fig. 80 – Water-moderated and -reflected planar arrays of slabs of aqueous $U(93.2)O_2F_2$ solution. Containers: 120.6-cm-wide and 7.6-cm-thick aluminum with 3.2-mm-thick walls.

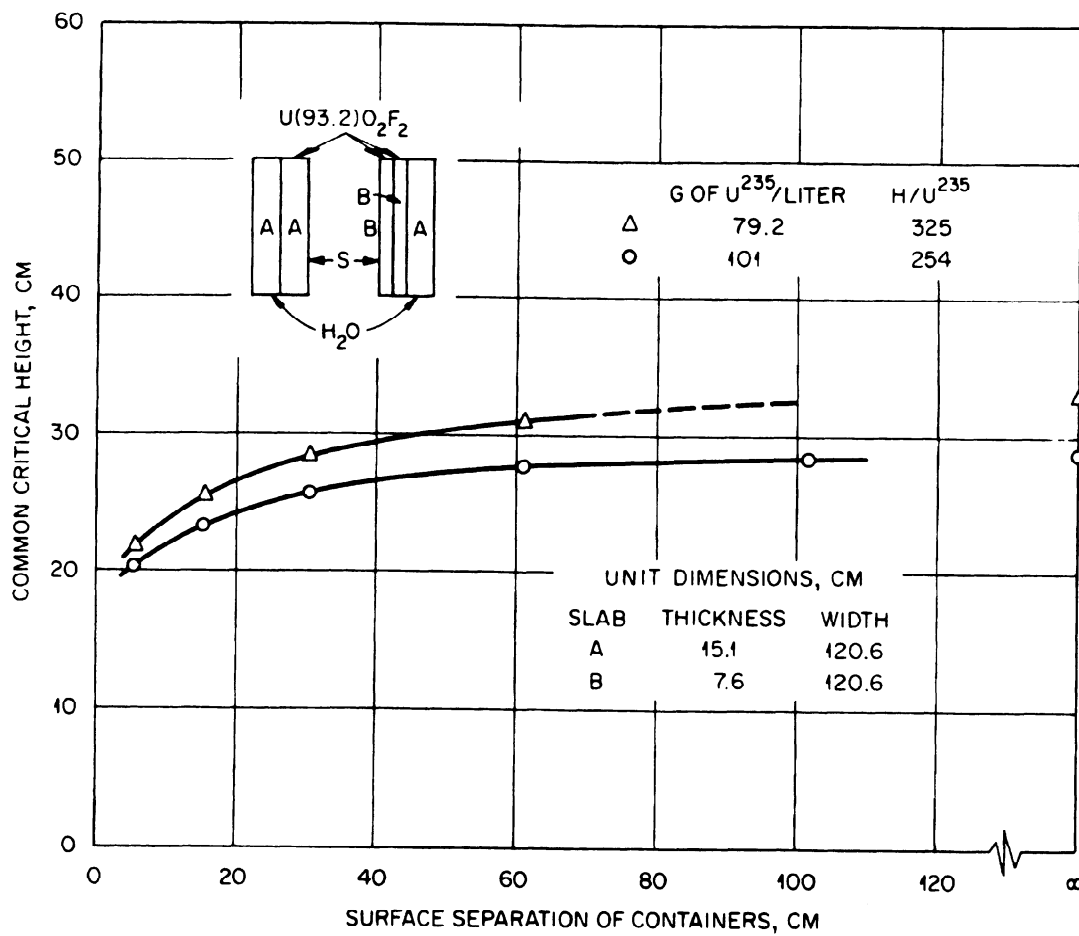


Fig. 81 - Partially reflected planar arrays of slabs of aqueous U(93.2)O₂F₂ solution (3.2-mm-thick aluminum containers).

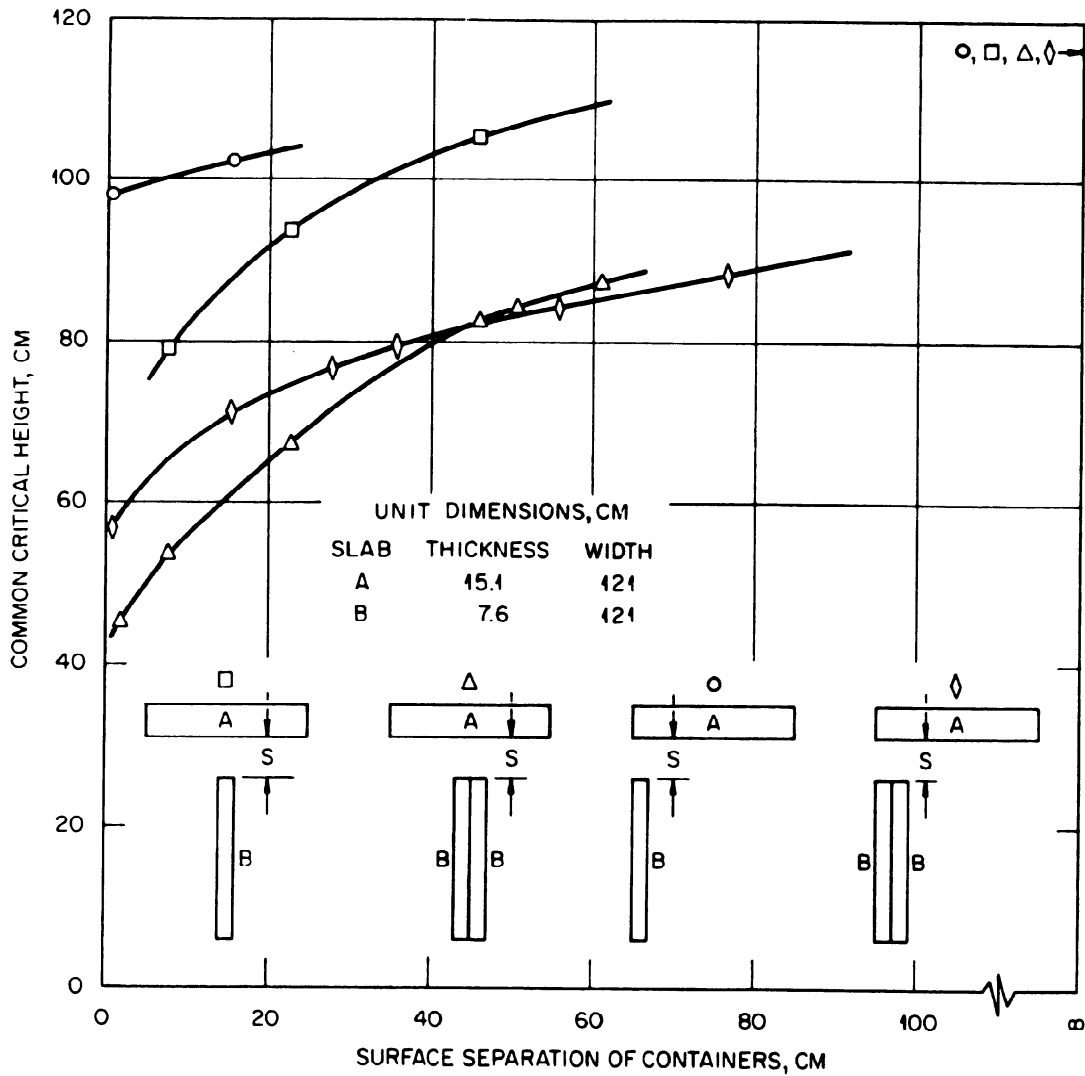


Fig. 82 - Unreflected and unmoderated planar arrays of slabs of aqueous $U(93.2)O_2F_2$ solution arranged in "T" and "L" geometries. U^{235} concentration: 79.2 g/liter; $H/U^{235} = 325$. Containers: 3.2-mm-thick aluminum.

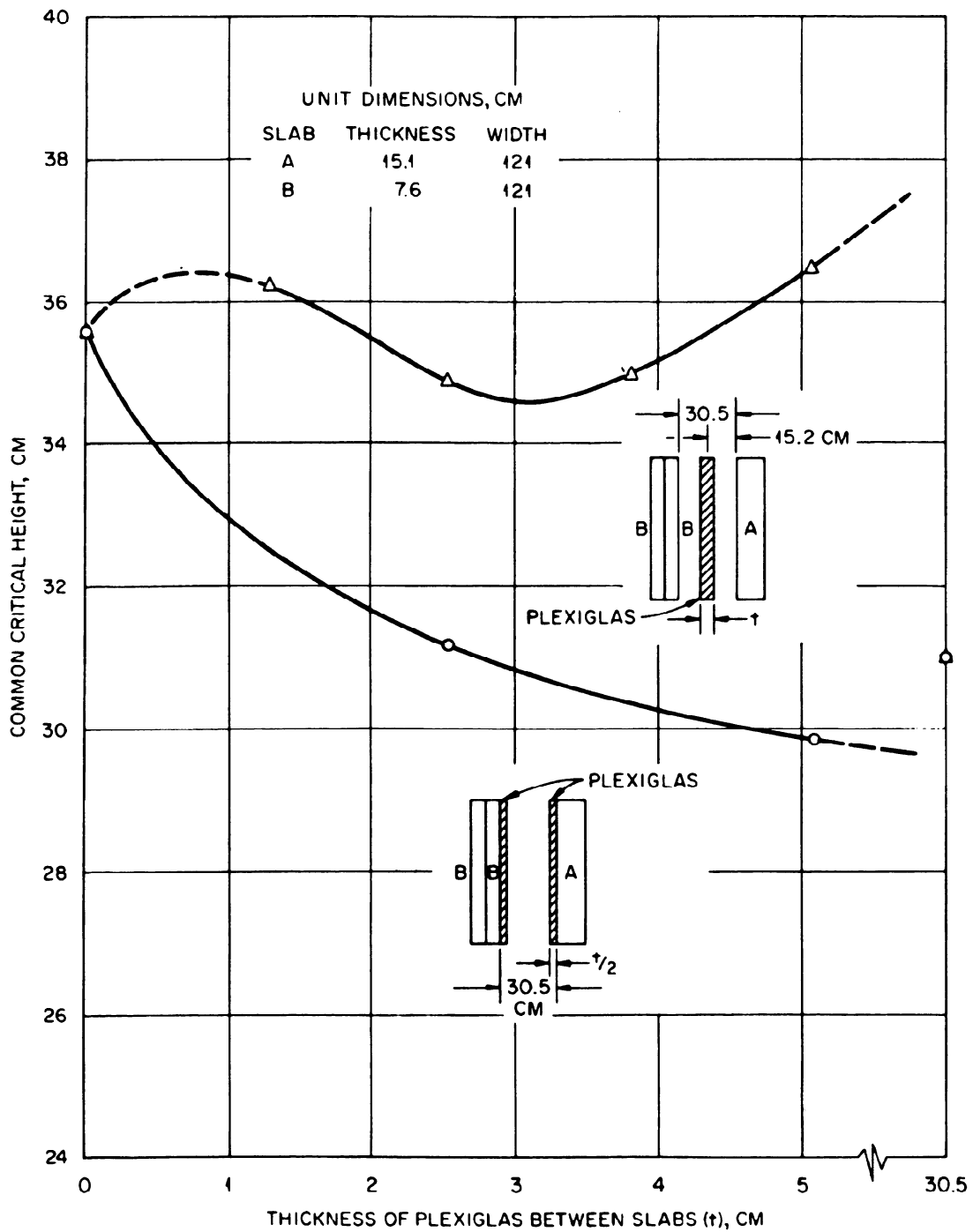


Fig. 83 - Unreflected, moderated planar arrays of parallel slabs of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 87.8 g/liter; $H/U^{235} = 293$. Containers: 3.2-mm-thick aluminum.

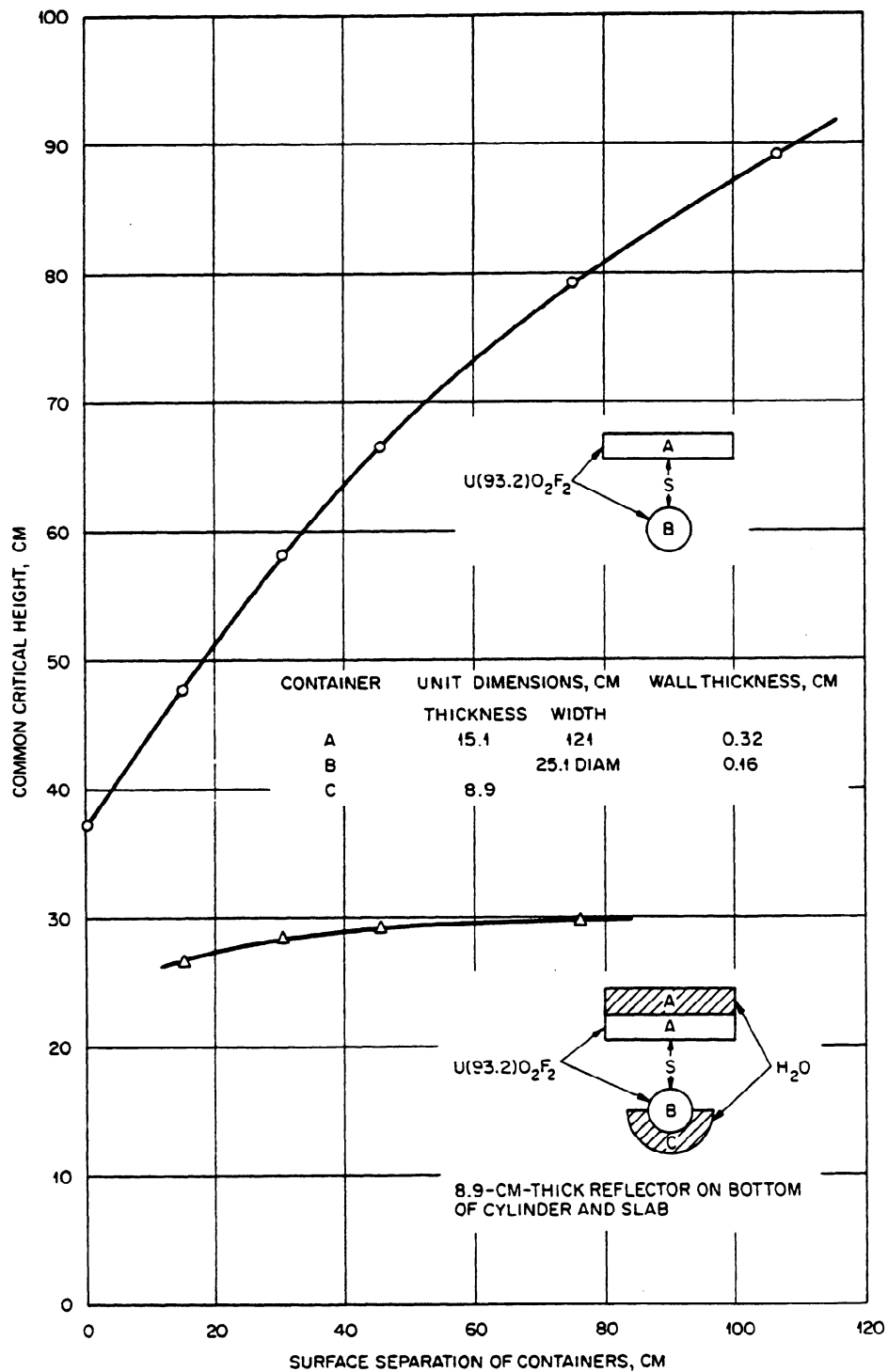


Fig. 84 - Unreflected and partially reflected planar arrays of a slab and a cylinder of aqueous $U(93.2)O_2F_2$ solution. U^{235} concentration: 77.9 g/liter; $H/U^{235} = 331$. Containers: slab, 3.2-mm-thick aluminum; cylinder, 1.6-mm-thick aluminum.

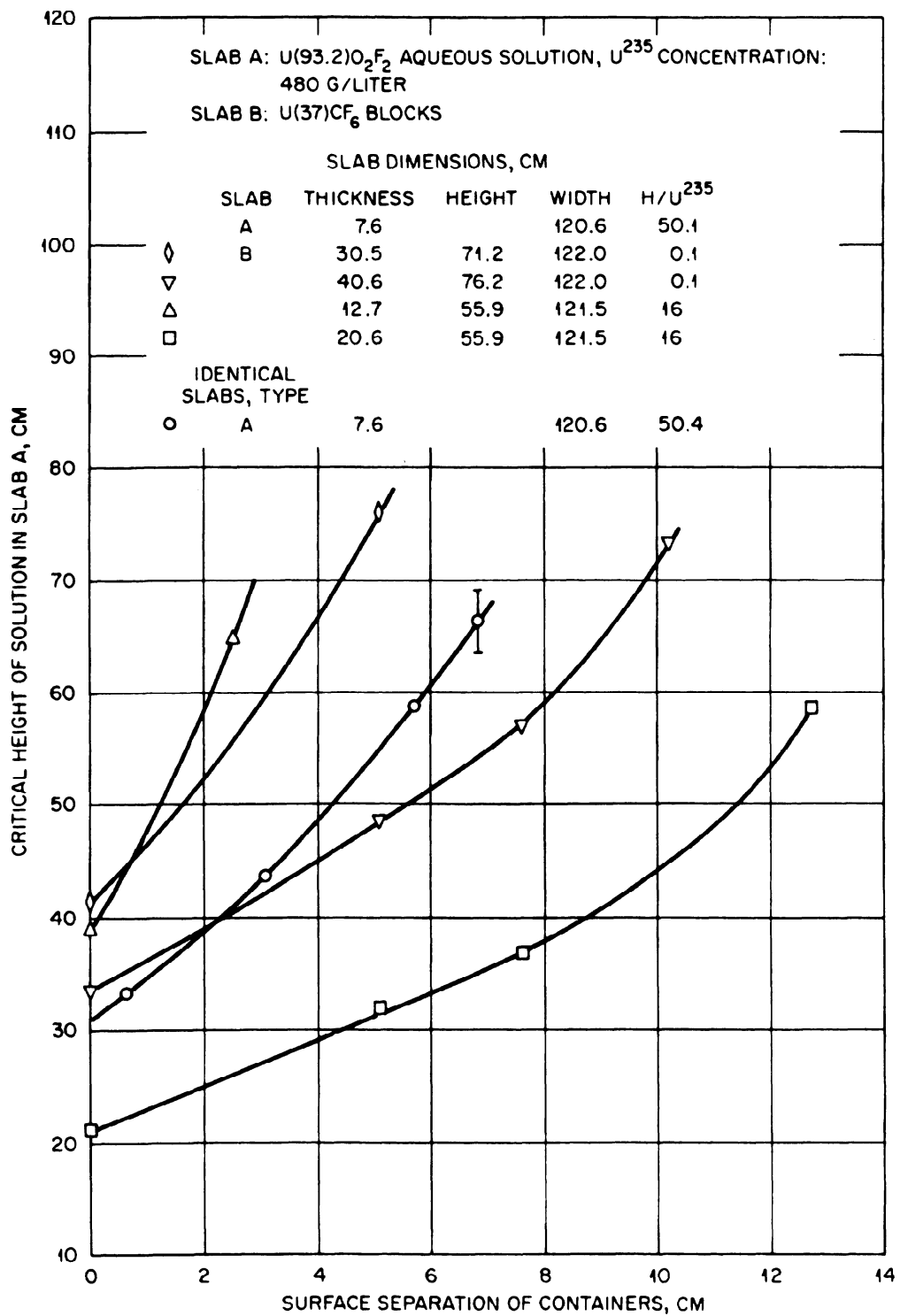


Fig. 85 – Unreflected planar arrays of a slab of aqueous $U(93.2)O_2F_2$ solution and a slab of $U(37)F_4-CF_2$ (3.2-mm-thick aluminum solution container).

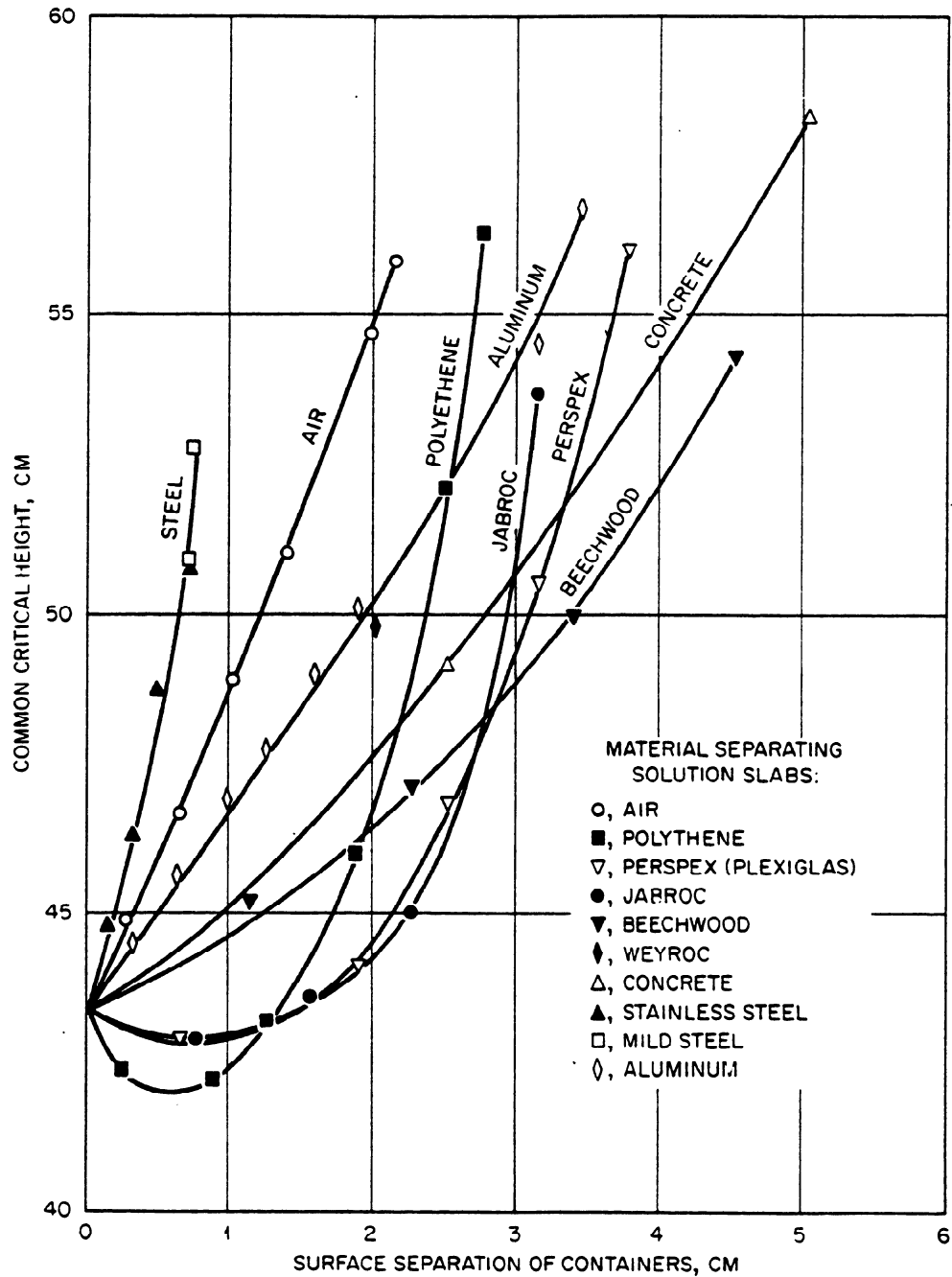


Fig. 86 - Unreflected planar arrays of two slabs of aqueous $U(30.45)O_2F_2$ solution separated by various materials. Jabroc is a wood product containing about 45% carbon, 6% hydrogen, and 37% oxygen; $\rho = 1.315 \text{ g/cm}^3$. Weyroc is also a wood product with a density of 0.72 g/cm^3 . U^{235} concentration: 207 g/liter ; $H/U^{235} = 130$. Dimensions of slabs: 6.09 cm thick and 120 cm wide.

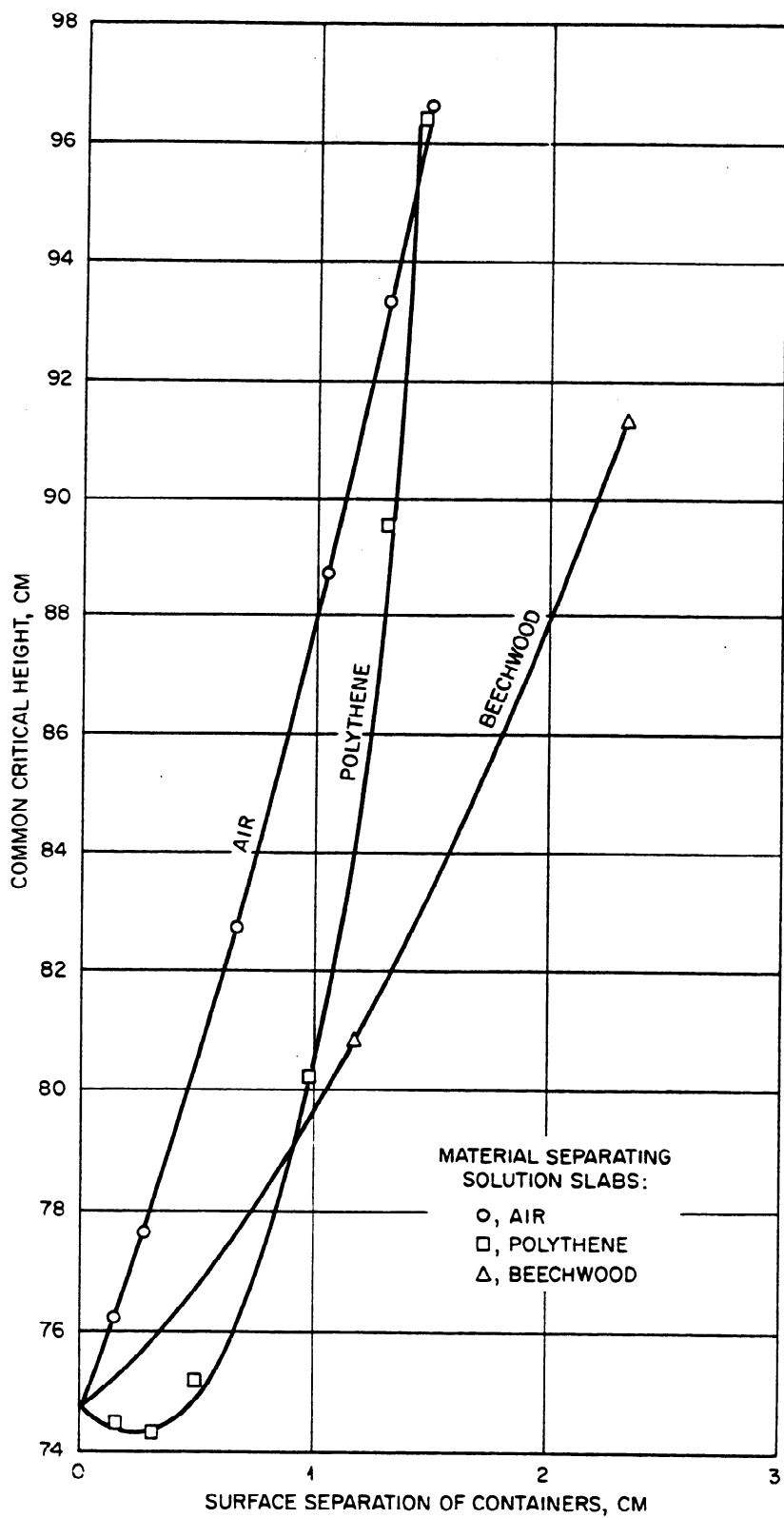


Fig. 87 - Unreflected planar arrays of two slabs of aqueous $U(30.45)O_2F_2$ solution separated by various materials. U^{235} concentration: 130 g/liter; $H/U^{235} = 214$.

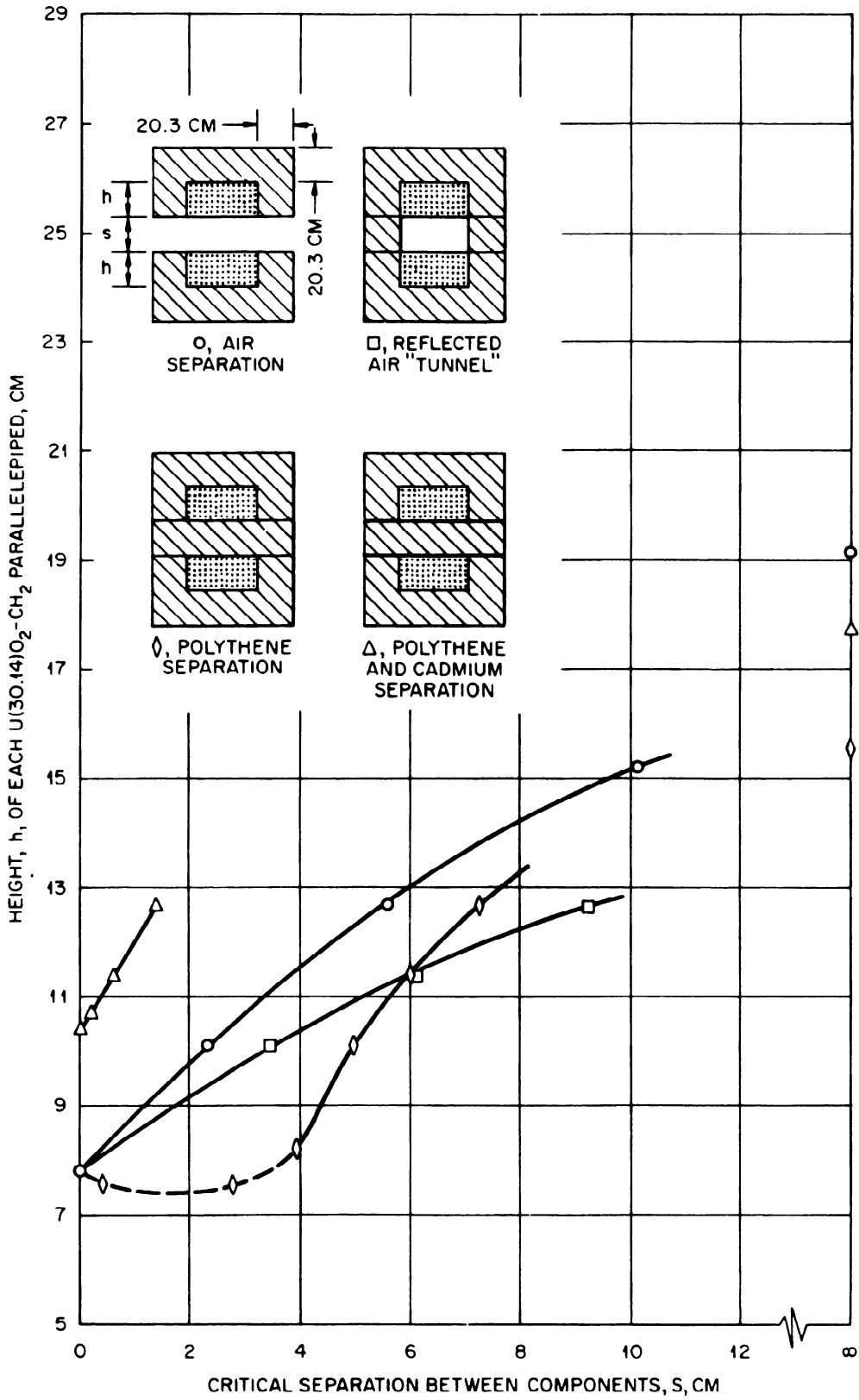


Fig. 88 - Reflected planar arrays of parallelepipeds of $U(30.14)O_2$ -paraffin (CH_2) separated by various materials. U^{235} concentration: 331 g/liter; $H/U^{235} = 81.8$.

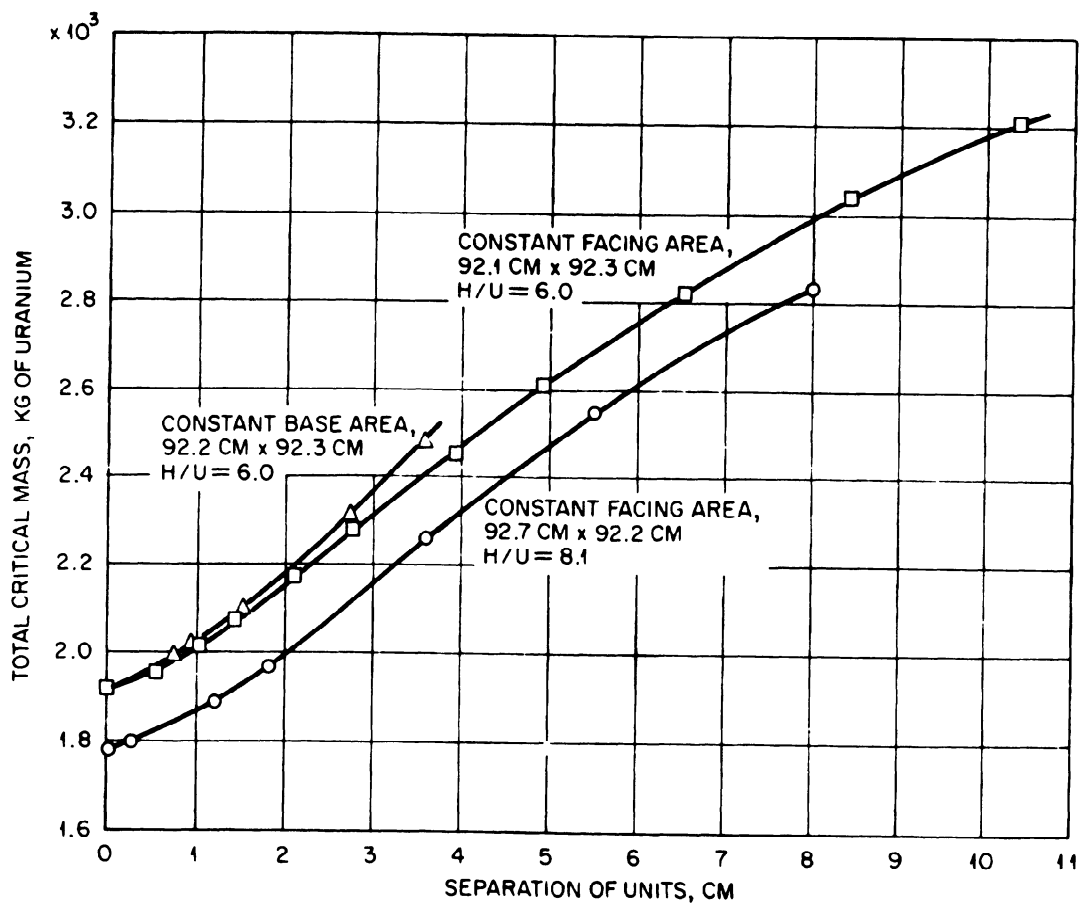


Fig. 89 – Reflected two-unit planar arrays of parallelepipeds of homogeneous $U(1.42)F_4$ -paraffin. Thick polyethylene reflector except on facing areas.

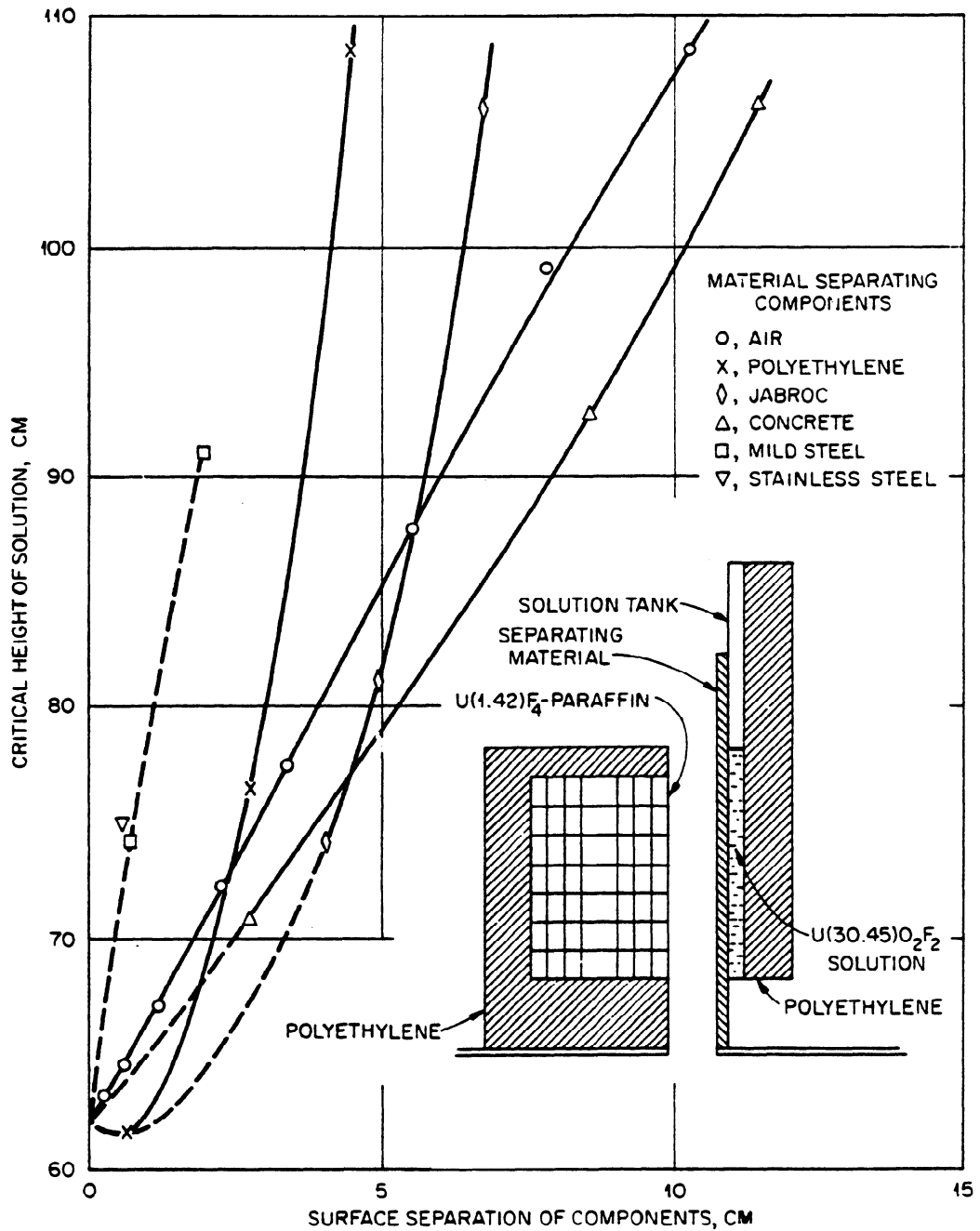


Fig. 90 - Reflected arrays of a parallelepiped of $U(1.42)F_4$ -paraffin and a slab of aqueous $U(30.45)O_2F_2$ solution separated by various materials. U^{235} concentration: solution, 204 g/liter; $H/U^{235} = 112$; blocks, 30.5 g/liter; $H/U^{235} = 572$. Component dimensions: solution, 6.09 cm thick, 120 cm wide; blocks; 61.6 cm thick, 123 cm wide.

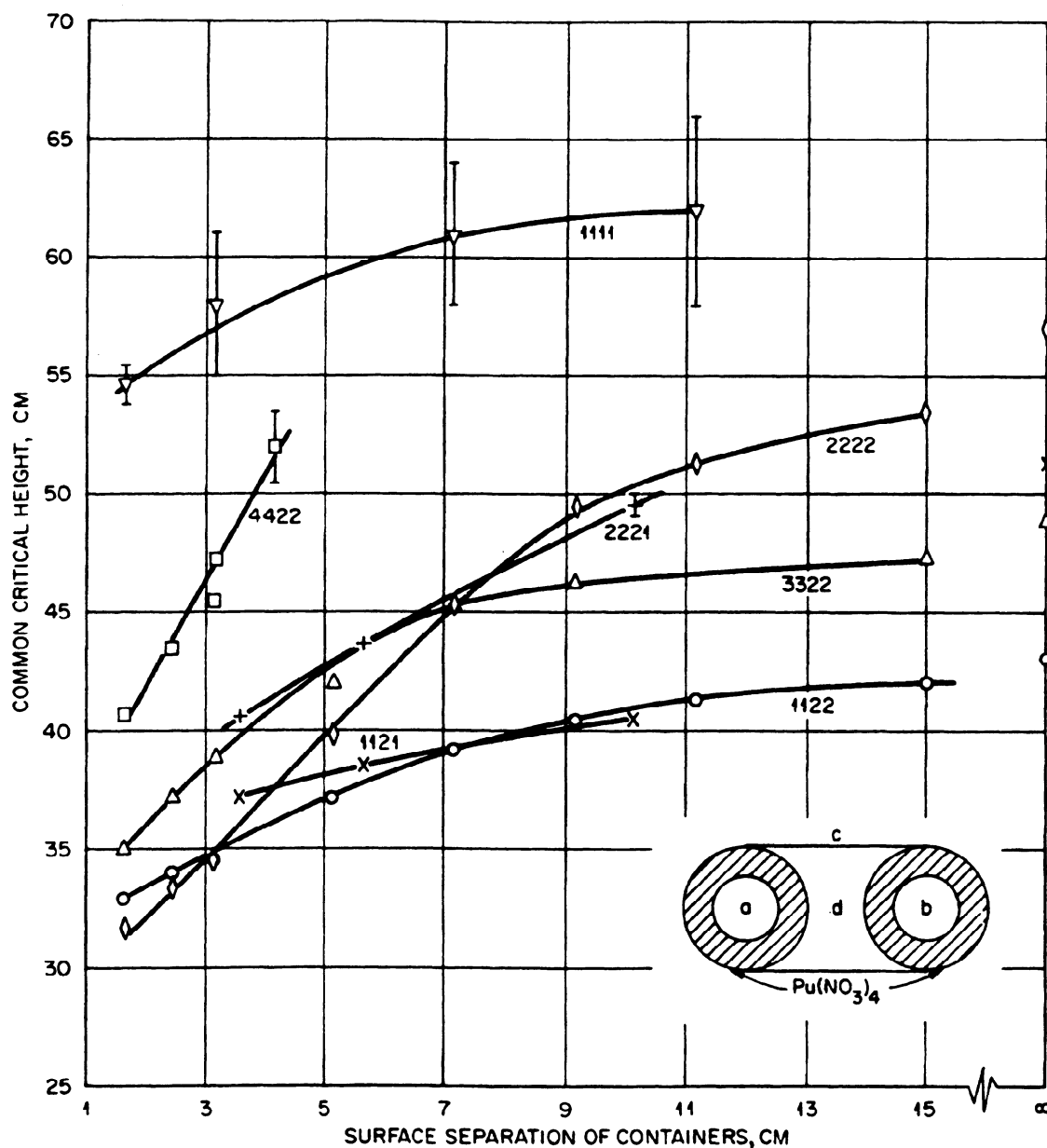


Fig. 91 - Two-unit planar arrays of identical annular cylindrical units of aqueous $Pu(NO_3)_4$ solution. Plutonium concentration: 74.2 g/liter ($Pu^{240} = 2.9\%$). Containers: internal diameter, 30.0 cm; external diameter 50.0 cm; 3-mm-thick stainless steel. Each region is designated by a letter. An array is described by four numerals in the order a,b,c,d, which indicate the content of each region as follows: (1) air; (2) water; (3) air with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container; and (4) water with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container.

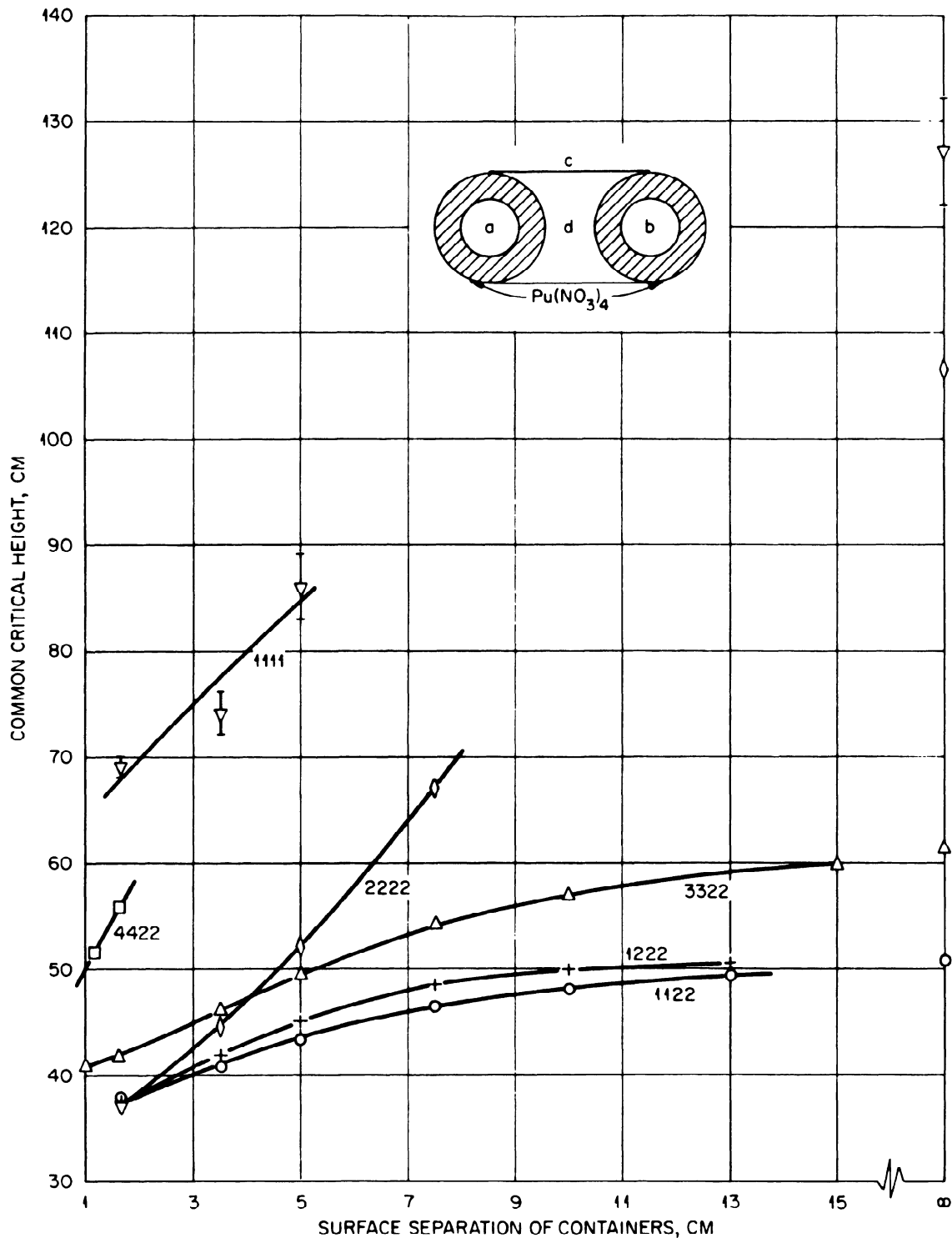


Fig. 92 – Two-unit planar arrays of identical annular cylindrical units of aqueous $Pu(NO_3)_4$ solution. Plutonium concentration: 56.1 g/liter ($Pu^{240} = 2.98\%$). Containers: internal diameter, 30.0 cm; external diameter, 50.0 cm; 3-mm-thick stainless steel. Each region is designated by a letter. An array is described by four numerals in the order a,b,c,d, which indicate the content of each region as follows: (1) air; (2) water; (3) air with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container; and (4) water with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container.

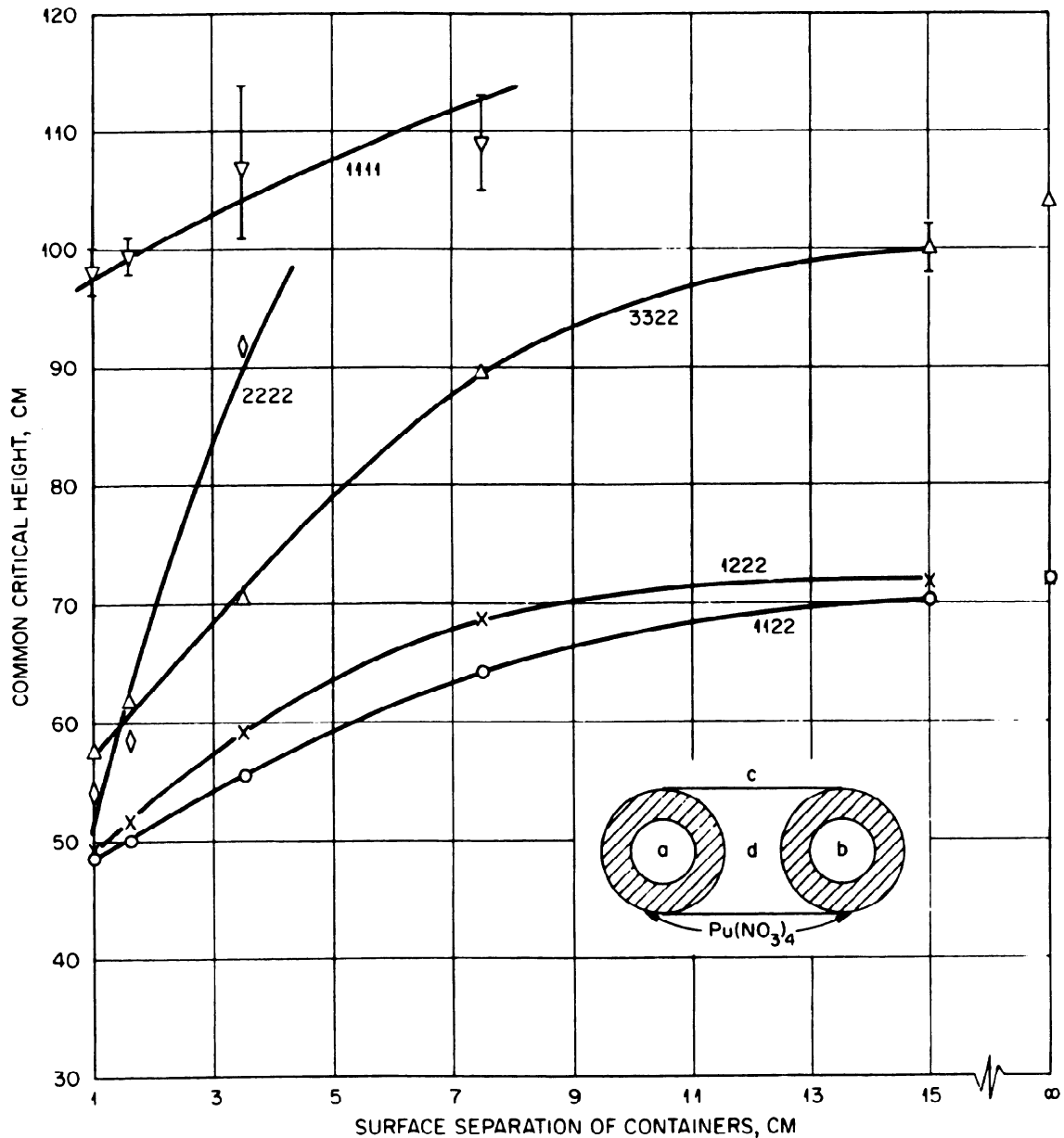


Fig. 93 – Two-unit planar arrays of identical annular cylindrical units of aqueous $\text{Pu}(\text{NO}_3)_4$ solution. Plutonium concentration: 40.6 g/liter ($\text{Pu}^{240} = 2.98\%$). Containers: internal diameter, 30.0 cm; external diameter, 50.0 cm; 3-mm-thick stainless steel. Each region is designated by a letter. An array is described by four numerals in the order a,b,c,d, which indicate the content of each region as follows: (1) air; (2) water; (3) air with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container; and (4) water with a 0.7-mm-thick cadmium foil attached to the inner wall of the solution container.

Metal Units

URANIUM

Planar Arrays

Neutron interaction between two^{124,125} and three¹²⁶ component systems was studied at ORNL utilizing uranium metal cylinders and slabs of varying thicknesses to determine the critical spacing of identical pieces. The units were U(93) metal with a density of 18.7 g/cm³. The critical separation between the large, parallel, flat surfaces of the units as a function of their geometry and thickness is shown in Fig. 94.

Spatial Arrays

A series of source neutron multiplication measurements with spatial arrays of U(93.4) metal slabs 2.54 × 20.3 × 25.4 cm, each containing 22.9 kg of U²³⁵, was also performed at ORNL.¹²⁷ Arrays reflected by Plexiglas ($\rho = 1.18$ g/cm³) were either unmoderated or were moderated by Plexiglas. Table 23 describes the experimental arrays and gives an estimate of the number of units required for critical arrays. Included in this table are the results of two additional experiments in which the void volume of a moderated 27-unit array first contained Styrofoam and then Foamglas.

Extensive studies at ORNL¹²⁸ utilized U(93.2) metal cylinders arranged in three-dimensional arrays. All these were constructed with the unit centers at the corners of rectangular parallelepipeds. In a few cases the pattern was cubic. Data obtained from critical arrays of 8, 16, 27, 45, and 64 units, both unreflected and reflected by various thicknesses of paraffin, are given in Table 24. The reflector was located at the lattice cell boundary of the peripheral units, where a lattice cell is occupied by a single unit. The average uranium density in an array is the mass of the metal unit divided by the volume of the lattice cell in which it is centered. The last column in the table gives a measure of the array shape.

The effect of a 15.2-cm-thick paraffin reflector on three sides of an array was also investigated.¹²⁸ Two arrays, each constituted of 20.9-kg units whose height-to-diameter ratio (h/d) was 0.94, were constructed, one containing 8 units and the other 27. The results indicated that the thick reflector on three sides of the arrays was slightly less effective than was the 2.5-cm-thick one completely surrounding the arrays.

Five arrays of the U(93.2) metal cylinders were constructed¹²⁸ with units at equal center spacing, in contrast to the usual equal surface spacing. These arrays are described in Table 25, which includes for comparison the critical dimensions of arrays of the same units located at equal surface spacing. The results suggest that, if the maximum achievable density with equal center spacing is less than the critical density with equal surface spacing, the array with equal center spacing will not be critical.

The effect of the array shape, as measured by the ratio of the height to the square root of the base area, on the critical uranium density was investigated¹²⁸ with 20.9-kg units with an h/d ratio of 0.94. The results are reported in Table 26 with, for comparison, the densities in arrays of the same number of identical units with the ratio of height to the square root of the base area of 0.95 from the data of Table 24.

In order to determine the effect of moderation on the critical uranium density of arrays of metal units, cylinders having a mass of 20.9 kg and an h/d of 0.94 were centered in Plexiglas boxes of various thicknesses and sizes; some of the arrays were surrounded with paraffin reflector.¹²⁸ Table 27 summarizes the results.

A set of experiments was performed¹²⁸ with a paraffin-reflected array of eight ~20.9-kg metal units to determine the relative reactivity associated with each of several thicknesses of Plexiglas separating the units. The results, summarized as follows, show that ~4.9-cm-thick Plexiglas provided optimum moderation of these metal units. (That the thickness of Plexiglas required for optimum moderation of fissile solution units is less than 4.9 cm is indicated in Table 14.)

Plexiglas thickness, cm	Reactivity, arbitrary units
2.5	0.05
3.8	1.79
4.5	2.16
4.8	2.30
5.1	2.29
7.6	0

Figure 95 illustrates that the individual effects of moderation and reflection may not be combined simply.¹²⁸

PLUTONIUM

Rocky Flats¹²⁹ is the source of subcritical measurements with 2-kg units of plutonium metal ($\rho = \sim 15.8 \text{ g/cm}^3$), each approximately 6.3 cm in diameter and 3.2 cm high, enclosed in thin-walled steel containers 10.2 cm in diameter and 6.8 cm high. These units were arranged with containers in contact on an effectively infinite concrete reflector. Extrapolation of source neutron multiplication data led to the following conclusions:

1. An infinite slab one unit thick, with an array density of $3.38 \text{ g of plutonium/cm}^3$, will be subcritical.
2. An infinitely high spatial array of units in a square pattern, three units on a side, with an array density of $2.9 \text{ g of plutonium/cm}^3$, will be subcritical.
3. A spatial array of units in a square pattern with four units on a side will be critical when the array is six units high.
4. A similar array with five units on a side will be critical at a height corresponding to 3.5 units.

The neutron interaction between two plutonium disks ($\rho = 15.8 \text{ g/cm}^3$) separated by and completely reflected by Plexiglas was investigated at Rocky Flats.¹³⁰ Both disks were 31.8 cm in diameter, were of equal but variable thickness, and were completely enclosed in a cylindrical Plexiglas reflector 7.6 cm thick on the lateral surfaces and 10.2 cm thick on the ends. Plexiglas also filled the space between the disks, which was varied from 0 to 15.2 cm. The critical thickness of the disks as a function of their separation, shown in Fig. 96, is the result of extrapolations of subcritical measurements.

Table 23 – REFLECTED SPATIAL ARRAYS OF 24.5-kg U(93.4) METAL SLABS

Units: U(93.4) metal slabs $2.5 \times 20.3 \times 25.4$ cm;
uranium density = 18.7 g/cm^3

Reflector: 2.5-cm-thick Plexiglas

Number of units assembled	Thickness of Plexiglas moderator centered between units, cm	Center spacing, cm	Source neutron multiplication	Estimated number of units in critical array
125	0	38.1	2.5	350–480
125	0	30.5	5.2	180–210
125	0	27.9	14.1	140–150
69	2.54	38.1	16.7	73–80
27	2.54	30.5	7.9	34–41
27	2.54	27.9	167	27.5
27	2.54*	30.5	3.7	34–41
27	2.54 [†]	30.5	1.2	155–215

*Styrofoam ($\text{C}_6\text{H}_5\text{CH:CH}_2$; $\rho = 0.024 \text{ g/cm}^3$) occupied 96.7% of the air space.

[†]Foamglas (borosilicate glass containing ~2% boron, $\rho = 0.141 \text{ g/cm}^3$) occupied 96.7% of the air space.

Table 24 – CRITICAL CONDITIONS FOR THREE-DIMENSIONAL ARRAYS OF U(93.2) METAL CYLINDERS WITH VARIOUS PARAFFIN REFLECTORS

Mass, kg	Unit		Array*	Paraffin reflector thickness, cm	Surface separation of units, † cm	Average uranium density in array, g/cm ³	Ratio of array height to $\sqrt{\text{base area}}$
	Diameter, cm	Height to diameter ratio					
10.48	11.51	0.47	2 × 2 × 2	0	0 [‡]	14.71	0.47
				1.3	0.23	13.56	0.48
				3.8	1.98	7.83	0.55
				7.6	3.42	5.35	0.59
				15.2	3.70	5.00	0.60
10.48	11.51	0.47	3 × 3 × 3	0	2.01	7.77	0.55
				1.3	2.99	5.95	0.58
				3.8	5.87	3.09	0.65
				7.6	8.26	1.97	0.69
				15.2	8.69	1.83	0.70
10.51	9.12	0.95	2 × 2 × 2	0	0 [§]	14.63	0.95
				1.3	0.60	12.04	0.95
				3.8	2.36	7.25	0.96
				7.6	3.97	4.87	0.96
				15.2	4.31	4.50	0.97
10.49	9.12	0.95	3 × 3 × 3	0	2.44	7.10	0.96
				1.3	3.43	5.53	0.96
				3.8	6.58	2.80	0.97
				7.6	9.02	1.81	0.97
				15.2	9.43	1.69	0.97
10.48	11.51	0.47	2 × 2 × 4	0	1.35	9.42	1.05
10.46	11.49	0.47	3 × 3 × 5	0	3.44	5.31	0.99
10.43	11.48	0.47	4 × 4 × 4	0	3.95	4.69	0.61
				15.2	12.36	1.04	0.74
15.69	11.49	0.70	2 × 2 × 2	0	0.90	11.37	0.73
				1.3	1.91	8.76	0.75
				3.8	4.96	4.45	0.79
				7.6	7.39	2.85	0.82
				15.2	7.82	2.65	0.82
15.68	11.49	0.70	3 × 3 × 3	0	4.20	5.19	0.78
				1.3	5.68	3.87	0.80
				3.8	10.19	1.83	0.84
				7.6	13.69	1.14	0.86
				15.2	14.19	1.07	0.87
20.81	11.46	0.94	2 × 2 × 2	0	2.22	8.56	0.95

Table 24 — (continued)

Mass, kg	Unit		Array*	Paraffin reflector thickness, cm	Surface separation of units, † cm	Average uranium density in array, g/cm ³	Ratio of array height to $\sqrt{\text{base area}}$
	Diameter, cm	Height to diameter ratio					
20.96	11.51	0.94	2 × 2 × 2	0	2.25	8.51	0.95
				1.3	3.68	6.30	0.95
				2.5	5.71	4.29	0.96
				3.8	8.21	2.84	0.96
				7.6	11.51	1.78	0.97
				15.2	11.99	1.67	0.97
20.88	11.48	0.94	3 × 3 × 3	0	6.36	3.83	0.96
				1.3	8.57	2.68	0.96
				3.8	14.76	1.19	0.97
				7.6	18.72	0.78	0.98
				15.2	19.15	0.74	0.98
26.22	11.51	1.17	2 × 2 × 2	0	3.54	6.81	1.18
				1.3	5.42	4.84	1.12
				3.8	11.53	1.98	1.09
				7.6	15.70	1.22	1.07
				15.2	16.38	1.13	1.07
26.11	11.49	1.17	3 × 3 × 3	0	8.49	2.98	1.10
				1.3	11.32	2.03	1.09
				3.8	19.61	0.82	1.06
				7.6	24.50	0.53	1.05
				15.2	24.99	0.51	1.05

*The horizontal and vertical dimensions, respectively, of the array are expressed in number of units.

† Errors on all surface separations are ± 0.01 cm for unreflected arrays and ± 0.03 cm for reflected arrays.

‡ Array was subcritical with an apparent neutron source multiplication of ~ 3 .

§ Array was subcritical with an apparent neutron source multiplication of ~ 10 .

Table 25 – COMPARISON OF URANIUM DENSITIES OF CRITICAL UNREFLECTED CUBIC AND RECTANGULAR PARALLELEPIPED ARRAYS OF U(93.2) METAL CYLINDERS

Mass, kg	Unit		Array*	Center spacing, cm	Surface spacing, [†] cm		Average uranium density in array, g/cm ³	Ratio of array height to $\sqrt{\text{base area}}$
	Diameter, cm	Height to diameter ratio			Horizontal	Vertical		
10.5	11.51	0.47	3 × 3 × 3	11.51	0	6.13	6.88 [‡]	1.00
10.5	11.51	0.47	3 × 3 × 3		2.01	2.01	7.77	0.55
15.7	11.49	0.70	2 × 2 × 2	11.49	0	3.42	10.33 [§]	1.00
15.7	11.49	0.70	2 × 2 × 2		0.90	0.90	11.37	0.73
21.0	11.51	0.94	2 × 2 × 2	13.50	2.00	2.74	8.51	1.00
21.0	11.51	0.94	2 × 2 × 2		2.25	2.25	8.51	0.95
20.9	11.48	0.94	3 × 3 × 3	17.60	6.12	6.84	3.83	1.00
20.9	11.48	0.94	3 × 3 × 3		6.36	6.36	3.83	0.96
26.2	11.51	1.17	2 × 2 × 2	15.78	4.27	2.32	6.68	1.00
26.2	11.51	1.17	2 × 2 × 2		3.54	3.54	6.81	1.13

*The horizontal and vertical dimensions, respectively, of the array are expressed in number of units.

[†] The error in the separation is ± 0.01 cm.

[‡] Array is subcritical; maximum apparent source neutron multiplication is ~ 70 .

[§] Array is subcritical; maximum apparent source neutron multiplication is ~ 81 .

Table 26 – EFFECT OF ARRAY GEOMETRY ON CRITICAL URANIUM DENSITIES IN UNREFLECTED ARRAYS OF 20.9-KG U(93.2) METAL UNITS

Dimension of array*	Surface separation of units, [†] cm	Ratio of array height to $\sqrt{\text{base area}}$	Average uranium density in arrays with equal numbers of identical units, g/cm ³	
			Ratio of array height to $\sqrt{\text{base area}}$ variable	Ratio of array height to $\sqrt{\text{base area}} = 0.95$
2 × 4 × 1	1.06 [‡]	0.35	12.23	8.51
3 × 3 × 1	0.66	0.31	12.40	7.83
3 × 3 × 2	4.64	0.64	5.21	4.97
2 × 2 × 4	3.91	1.91	6.01	5.38
2 × 4 × 2	3.89	0.67	6.03	5.38
4 × 4 × 1	1.52	0.24	10.06	5.38

*The horizontal and vertical dimensions, respectively, of the array are expressed in number of units. Unit dimensions: diameter, 11.48 cm; h/d, 0.94.

[†] The error in the separation is ± 0.01 cm.

[‡] This array consisted of two clusters of four units each with lateral surfaces in contact. This dimension is the horizontal separation between the two clusters.

Table 27 — CRITICAL CONDITIONS FOR MODERATED ARRAYS OF 20.9-KG CYLINDERS OF U(93.2) METAL

Number of units in array*	Plexiglas moderator thickness, † cm	Paraffin reflector thickness, cm	Surface separation of units, ‡ cm	Average uranium density in array, g/cm ³	Ratio of array height to $\sqrt{\text{base area}}$
8	1.3	0	4.24	5.64	0.95
		1.3	5.88	4.17	0.96
		7.6	12.57	1.55	0.97
		15.2	12.93	1.48	0.97
8	2.5	0	6.62	3.67	0.96
		1.3	8.61	2.67	0.96
		15.2	14.50	1.23	0.97
8	4.8	0	10.24	2.11	0.97
		15.2	16.45	0.99	0.97
27	4.8	0	16.29	1.00	0.97

*The 8-unit arrays were constructed with two units in each dimension; the 27-unit array had three units in each dimension. Unit dimensions: diameter, 11.48 cm; h/d, 0.94.

† Each unit was centered in approximately cubic Plexiglas containers having wall thicknesses one-half the designated moderator thickness.

‡ The error in the separation of the units in the unreflected arrays is ± 0.01 cm; in the reflected arrays it is ± 0.03 cm.

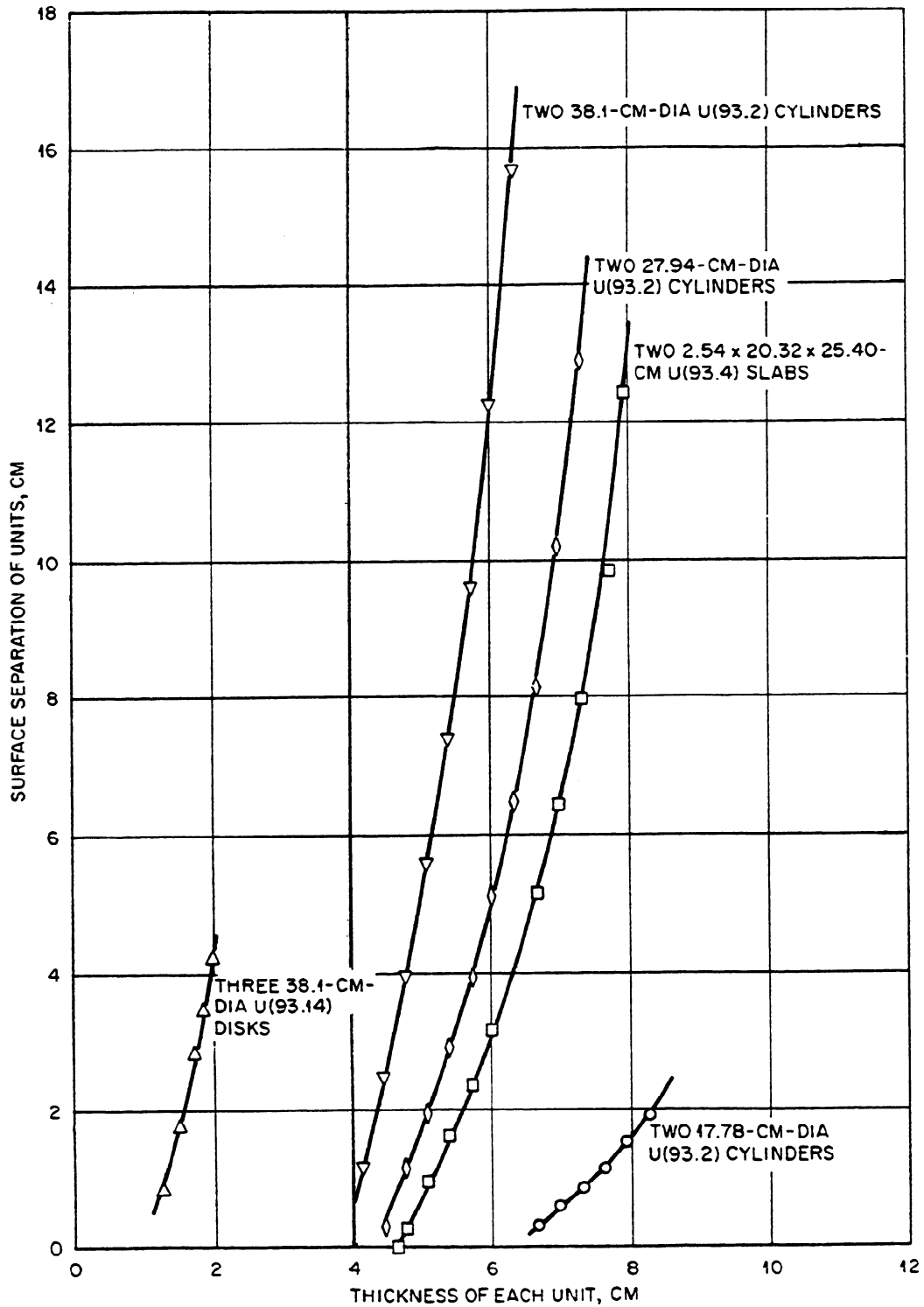


Fig. 94 - Unreflected and unmoderated critical planar arrays of U(93) metal.

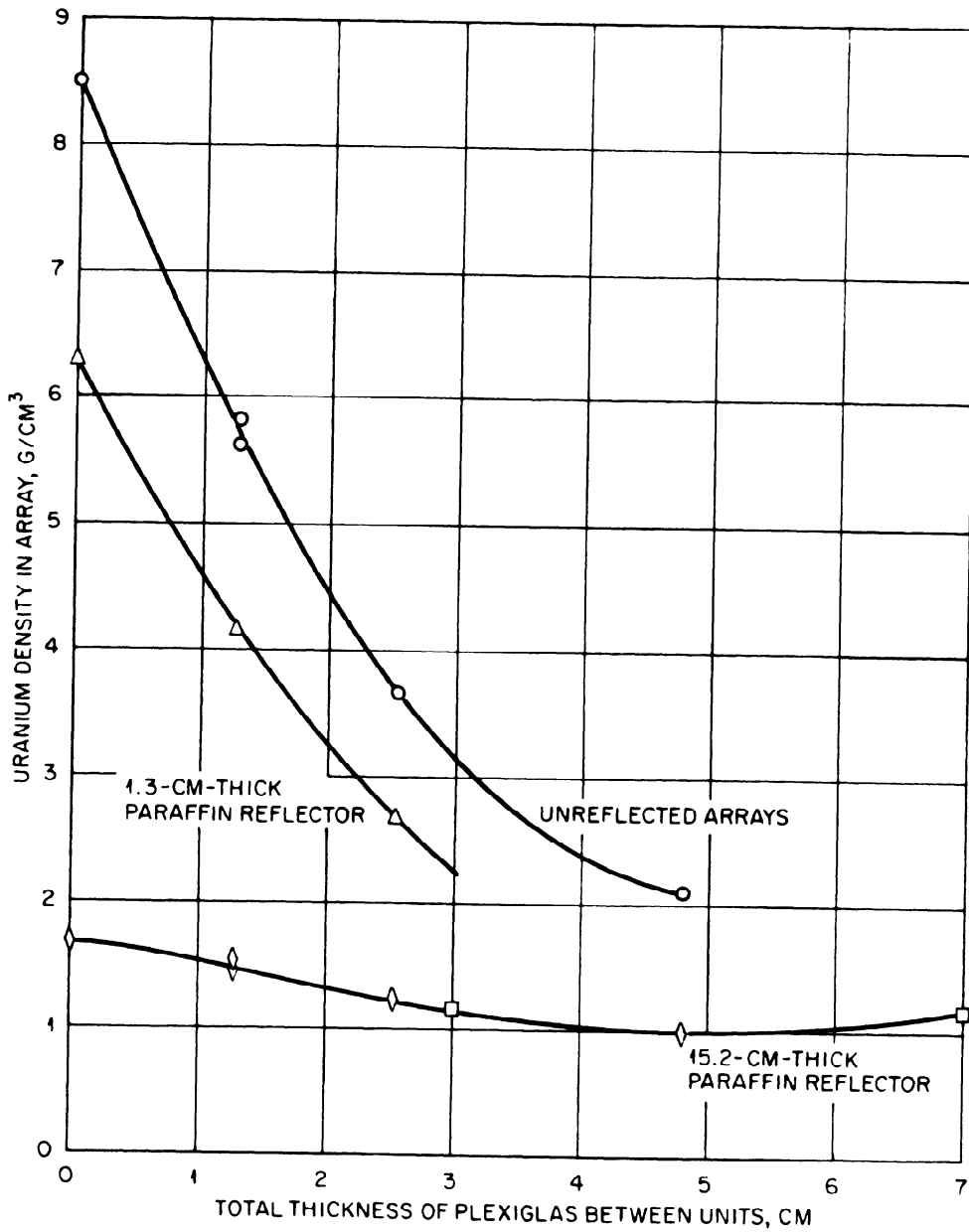


Fig. 95 – Effect of Plexiglas as a moderator and paraffin as a reflector on the critical density of an eight-unit array of 20.9-kg U(93.2) metal units.

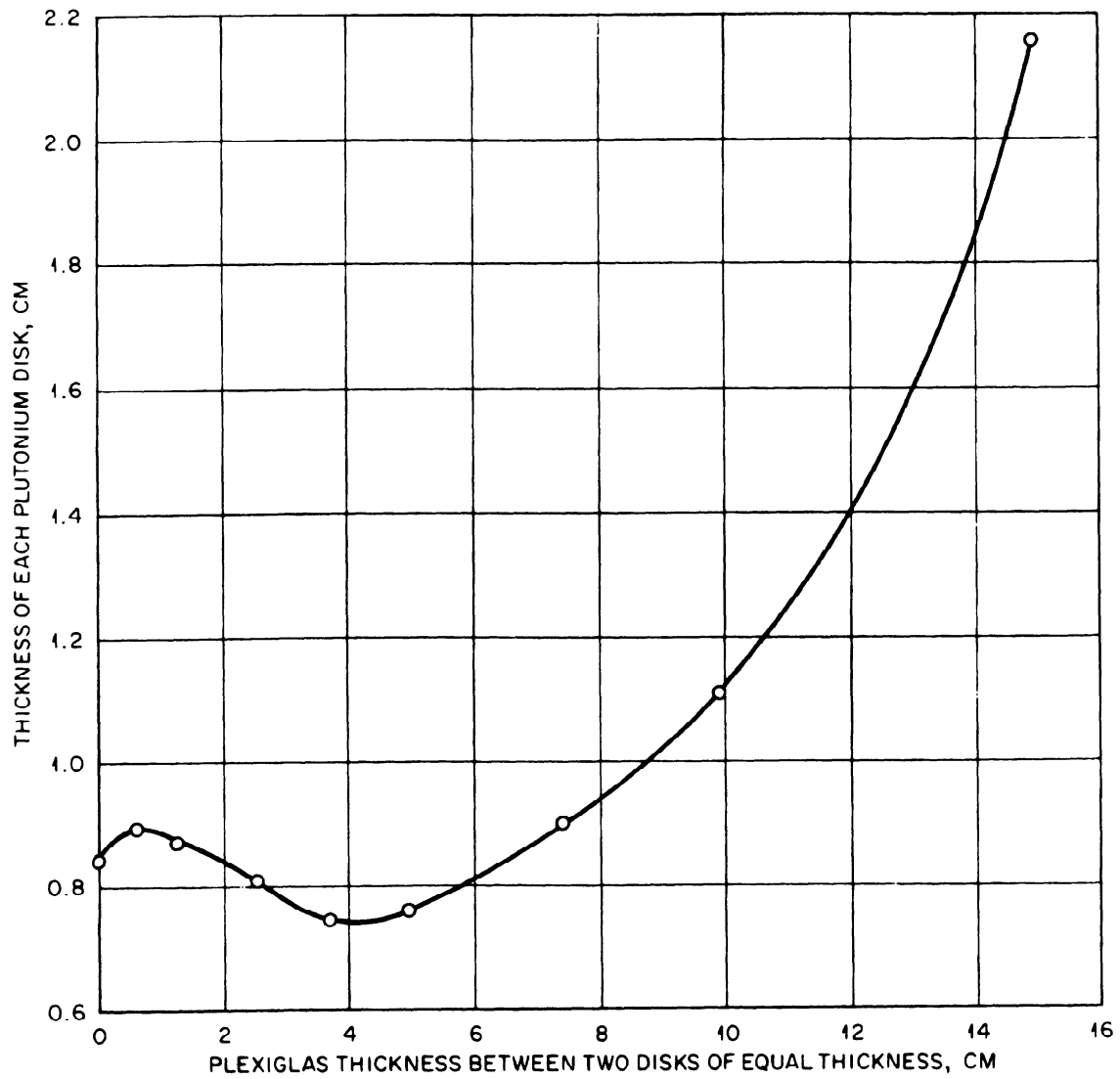


Fig. 96 – Critical thickness of each of two plutonium metal disks separated by Plexiglas. Values estimated from neutron multiplication measurements.

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